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Analysis of Arithmetic Operations for Fuzzy Numbers

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ABSTRACT

This paper deals with fundamental arithmetic operations on fuzzy numbers and use of these operations for carrying out various fuzzy processes. When modelling specific problems in the field of sciences as well as engineering, it's common to notice that the problem's characteristics aren't known precisely and instead fall into a range. Interval arithmetic, which permits mathematical evaluations (operations) to be conducted on intervals in order to produce significant estimations of required values in terms of intervals, has already been used to deal with similar problems. Fuzzy arithmetic can be thought of as an extension of arithmetic interval, in which numerous levels in [0,1] are considered instead of only one (constant) level.

Keywords: Arithmetic Operations, Crisp value, Fuzzy number, Fuzzy output.

I. INTRODUCTION

Fuzzy subgroups of the real numbers that meet certain additional criteria are fuzzy numbers. Non-probabilistic uncertainty can be easily modelled using fuzzy numbers. The extension principle or interval arithmetic have been used to build arithmetic operations on fuzzy integers (Kumar & Dhiman, 2021; Lynch, 1985). The structure of the membership functions of fuzzy numbers has a significant impact on the outcome of our computations when working with them. Calculations become more complex as membership functions are less regular (Dhiman & Kumar, 2018). Furthermore, fuzzy numbers with a simpler membership function form are generally more logical and natural to read.

In several scientific fields, such as systems evaluation and operations research, a model must be built with data that is only known to be approximated. This is made possible by Zadeh's (1965) fuzzy sets theory (Zadeh, 1965). Fuzzy numbers are fuzzy partitions of the real line that can be used to describe fuzzy numerical values. Only if we can readily execute algebraic operations on them will this technique be useful (Gaines, 1978; Lynch, 1985; Sugeno & Nishida, 1985). Tolerance analysis is generalised using fuzzy sets theory, which differs significantly from a probabilistic method. The literature is well-versed in representation of fuzzy sets in two fuzzy numbers. Fuzzy numbers as well as some arithmetic operations on them were introduced by Dubois and Prade (DUBOIS & PRADE, 1978). The fundamental advantage of employing fuzzy set theory for fuzzy control is that it allows us to utilise fuzzy logic to evaluate fuzzy rules, whereas traditional control systems use "classical" arithmetic to compute the controller output for a particular input. Combining the benefits of fuzzy logic with the versatility of arithmetic operations is appealing (Dhiman & Kumar, 2021; SLOAN, 1992). A function f(x) in the conclusion section of a fuzzy rule brings the versatility of conventional arithmetic, as explained by the Sugeno-type fuzzy controller (Sugeno & Nishida, 1985). If the representation is extended to a quintuple of parameters on which the operations are conducted, both arithmetic and logic operations can be defined inside this class of fuzzy numbers.

II. PRELIMINARIES

A. Crisp Number

The number which are having the grade of membership as either 1 or 0 are well known as crisp numbers (Kumar & Dhiman, 2021).

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$$\chi_B(a) = \begin{cases} 1 \text{ if a } \in \text{ B} \\ 0 \text{ if a } \notin \text{ B} \end{cases}$$

B. Fuzzy Set Theory

The definition of crisp set has expanded and extended by having a partial composition, i.e., from 0 to 1, well known as Fuzzy sets. It determines membership function's degree that takes values between 0 and 1 and is associated to that set referred to in [0, 1], in which 0 stands for "no" and 1 stands for "yes." (Dhiman & Kumar, 2018, 2021) Thus, based on it, fuzzy set is a mapping from universal set X to the interval [0, 1] i.e., $\tilde{F}$ = {(z, $\mu_{\tilde{F}}$ (z)) | z ∈ X} where z is an element of X and $\mu_{\tilde{F}}$ (z) represents membership's grade of element x in fuzzy set $\tilde{F}$. Certainly $\mu_{\tilde{F}}$ (z) ∈ [0, 1].

C. Classical Set

Consider the universe of discourse, A, as a well-defined collection of objects with similar properties. Individual elements in that universe of discourse will be depicted by the small letter 'a'. Elements in A can have discrete as well as continuous valued quantities as characteristics on the real line (Bělohlávek et al., 2002; Wu & Gong, 2001). As a collection of objects with comparable qualities, define a universe of discourse, B. Every element in the universe of discourse B will be denoted by the letter b. The membership for the element in the set will be 1 if it belongs to that set otherwise it will be zero.

D. Fuzzy Numbers

A fuzzy set F defined on R must have following properties for being treated as a fuzzy number:
a) The fuzzy set F should be normal.
b) Alpha cut of F should be an interval which is closed for every alpha in (0,1].
c) Strong alpha cut of a must be bounded.

E. Membership Function

It well represents the extent of fuzziness in a fuzzy set, which is same for both type of elements like discrete or continuous. The membership value is within the range [0, 1] i.e., a subset of positive real numbers including "0" with a finite supremum as "1" is well known as membership function. It is usually indicated for fuzzy set $\tilde{A}$ by $\mu_{\tilde{A}}$ (x) to the membership range (Dhiman & Kumar, 2018; Kumar & Dhiman, 2021).

$$\mu_{\tilde{A}} \text{ (x)}: X \to M$$

F. Alpha Cut

The most fundamental and extensively utilised notion in fuzzy set theory (Given by Zadeh) is " α−cut of set A" (Zadeh, 1965). For the given fuzzy set A, the strong alpha cut and the weak alpha cut can be represented respectively as
$A(\alpha) = \{a|\mu_A(a) > \alpha\};\ \alpha \epsilon\ [0,1)$
$A(\alpha) = \{a|\mu_A(a) \geq \alpha\};\ \alpha \epsilon\ [0,1)$

III. FUNDAMENTAL ARITHMETIC OPERATION ON FUZZY SET

In order to analyse the fundamental concepts of fuzzy arithmetic, we should first recite the arithmetic of intervals as well as how to "add", "subtract", "multiply", & "divide" two closed intervals in ℝ. A closed interval, a subset of real numbers is also known as an interval of confidence in this sense since it confines the ambiguity of data to an interval (DUBOIS & PRADE, 1978; Gupta & Qi, 1991). Two closed intervals in ℝ are $Z_1 = [\varepsilon_1, \varepsilon_2]$ and $Z_2 = [\alpha_1, \alpha_2]$. Then there's the following list of definitions:

A. Addition (+)

If $\varepsilon \in [\varepsilon_1, \varepsilon_2]$ and $\alpha \in [\alpha_1, \alpha_2]$, then $\varepsilon + \alpha \in [\varepsilon_1+\alpha_1,\ \varepsilon_2+\alpha_2]$. As a result, we will obtain the addition of two given sets Z_1 and Z_2, namely Z_1 (+) Z_2, which can be mathematically stated as
Z_1 (+) Z_2 = $[\varepsilon_1, \varepsilon_2]$ (+) $[\alpha_1, \alpha_2]$= $[\varepsilon_1+\alpha_1,\ \varepsilon_2+\alpha_2]$.

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B. Subtraction (-)

If $\varepsilon \in [\varepsilon_1, \varepsilon_2]$ and $\alpha \in [\alpha_1, \alpha_2]$, then ε - $\alpha \in [\varepsilon_1$-$\alpha_2$, ε_2-$\alpha_1]$. As a result, we will obtain the subtraction of two given sets Z_1 and Z_2, namely Z_1 (-) Z_2, which can be mathematically stated as
Z_1 (-) Z_2 = $[\varepsilon_1, \varepsilon_2]$ (-) $[\alpha_1, \alpha_2]$= $[\varepsilon_1$-α_2, ε_2-$\alpha_1]$.

C. Multiplication (·)

For the given two closed intervals $Z_1 = [\varepsilon_1, \varepsilon_2]$ and $Z_2 = [\alpha_1, \alpha_2]$ of $\mathbb{R}$, their multiplication, namely Z_1 (·) Z_2, can be mathematically stated as
Z_1 (·) Z_2 = $[\varepsilon_1, \varepsilon_2]$ (·) $[\alpha_1, \alpha_2]$.
= [min $(\varepsilon_1\alpha_1, \varepsilon_1\alpha_2, \varepsilon_2\alpha_1, \varepsilon_2\alpha_2)$,
max $(\varepsilon_1\alpha_1, \varepsilon_1\alpha_2, \varepsilon_2\alpha_1, \varepsilon_2\alpha_2)$].

D. Division

For the given two closed intervals $Z_1 = [\varepsilon_1, \varepsilon_2]$ and $Z_2 = [\alpha_1, \alpha_2]$ of $\mathbb{R}$, their division, namely Z_1 (/) Z_2, can be mathematically stated as

Z_1 (/) Z_2 = $[\varepsilon_1, \varepsilon_2]$ (/) $[\alpha_1, \alpha_2]$.
= [min $(\varepsilon_1/\alpha_1, \varepsilon_1/\alpha_2, \varepsilon_2/\alpha_1, \varepsilon_2/\alpha_2)$, max $(\varepsilon_1/\alpha_1, \varepsilon_1/\alpha_2, \varepsilon_2/\alpha_1, \varepsilon_2/\alpha_2)$].

E. Scalar multiplication

Let $Z_1 = [\varepsilon_1, \varepsilon_2]$ be a subset of positive reals and c $\in \mathbb{R}^+$. Manipulating the scalar 'c' as the closed interval
[c, c], then scalar multiplication c · Z_1 can be mathematically stated as
c · Z_1 = [c, c] (·) $[\varepsilon_1, \varepsilon_2]$ = [c ε_1, c ε_2].

F. Inverse

Let $Z_1 = [\varepsilon_1, \varepsilon_2]$ be a subset of positive reals, if $\varepsilon \in [\varepsilon_1, \varepsilon_2]$ and $0 \notin [\varepsilon_1, \varepsilon_2]$ then $(1/\varepsilon) \in [1/\varepsilon_2, 1/\varepsilon_1]$. Therefore, the inverse of Z_1, denoted by Z_1^{-1}, can be expressed mathematically as
Z_1^{-1}= $[\varepsilon_1, \varepsilon_2]^{-1}$ = $[1/\varepsilon_2, 1/\varepsilon_1]$,
provided $0 \notin [\varepsilon_1, \varepsilon_2]$.

J. Minimum

The minimum of two given intervals $Z_1 = [\varepsilon_1, \varepsilon_2]$ and $Z_2 = [\alpha_1, \alpha_2]$ of $\mathbb{R}$, namely Z_1 (∧) Z_2, can be mathematically stated as
Z_1 (∧) Z_2 = $[\varepsilon_1, \varepsilon_2]$ (∧) $[\alpha_1, \alpha_2]$ = $[\varepsilon_1 \wedge \alpha_1, \varepsilon_2 \wedge \alpha_2]$.

K. Maximum

The maximum of two given intervals $Z_1 = [\varepsilon_1, \varepsilon_2]$ and $Z_2 = [\alpha_1, \alpha_2]$ of $\mathbb{R}$, namely Z_1 (∨) Z_2, can be mathematically stated as
Z_1 (∨) Z_2 = $[\varepsilon_1, \varepsilon_2]$ (∨) $[\alpha_1, \alpha_2]$ = $[\varepsilon_1 \vee \alpha_1, \varepsilon_2 \vee \alpha_2]$.

Illustrative Examples:

Let us consider two intervals P and Q such that P= [2,3] and Q= [-2,1]. Now let us consider the aforementioned operations with intervals P and Q:
P (+) Q = [2-2,3+1] = [0,4]
P (-) Q = [2-1,3+2] = [1,5]

P (·) Q = [min (-4,2, -6,3), max (-4,2, -6,3)] = [-6,3]
P (/) Q = [min (-1,2, -3/2,3), max (-1,2, -3/2,3)]
= [-3/2,2]
4 · Q = [4 · 2, 4 · 3] = [8,12]
P^{-1} = [1/3, 1/2], Y^{-1} = [-1/2, 1]
P (∧) Q = [2∧-2, 3∧1] = [-2, 1]
P (∨) Q = [2∨-2, 3∨1] = [2,3]

Let us discuss one more example

Let us consider two intervals C and D such that C= [-3,4] and D= [1,2]. Now let us consider the aforementioned operations with intervals C and D:
C (+) D = [-3+1, 4+2] = [-2,6]
C (-) D = [-3-2, 4-1] = [-5,3]
C (·) D = [min (-3, -6, 4, 8), max (-3, -6, 4, 8)] = [-6,8]
C (/) D = [min (-3, -3/2, 4, 2), max (-3, -3/2, 4, 2)]
= [-3,4]
4 · C = [4 · -3, 4 · 4] = [-12,16]
C^{-1} = [-1/3, 1/4], D^{-1} = [1/2,1]
C (∧) D = [-3∧1, 4∧2] = [-3, 2]
C (∨) D = [-3∨1, 4∨2] = [1,4]

IV. OPERATIONS ON FUZZY SETS

On the universal set X, construct the fuzzy sets $\tilde{A}$, $\tilde{B}$, and $\tilde{C}$. For the set based theoretic operations of union, intersection, & complement on a given member ‘a’ of the universal set, following are the function-theoretic operations for the fuzzy sets $\tilde{A}$, $\tilde{B}$, and $\tilde{C}$ on X are established:

Union $\mu_{\tilde{A}\vee\tilde{B}}(a) = \mu_{\tilde{A}}(a) \vee \mu_{\tilde{B}}(a).$ [1]

Intersection $\mu_{\tilde{A}\wedge\tilde{B}}(a) = \mu_{\tilde{A}}(a) \wedge \mu_{\tilde{B}}(a).$ [2]

Complement $\mu_{\bar{A}}(a) = 1 - \mu_{\tilde{A}}(a).$ [3]

Related diagrams are shown below (see Fig 1-3) for these operations, extended to include fuzzy sets (Ross, 2010). The conventional fuzzy operations are defined as the operations described in Equations [1]-[3]. A discussion of these and other fuzzy operations is included later in this paper.
A subset of a universe X will be any fuzzy set $\tilde{A}$ defined on that universe. Likewise, as with classical sets, any element ‘a’ of the null set Ø is equal to 0, and an element ‘a’ in the entire set X is equal to 1. These ideas may be expressed in the following manner:

$$X \supseteq \tilde{A} \Longrightarrow \mu_X(a) \geq \mu_{\tilde{A}}(a)$$

For all $a \in X, \mu_{\emptyset}(a) = 0.$

For all $a \in X, \mu_X(a) = 1.$

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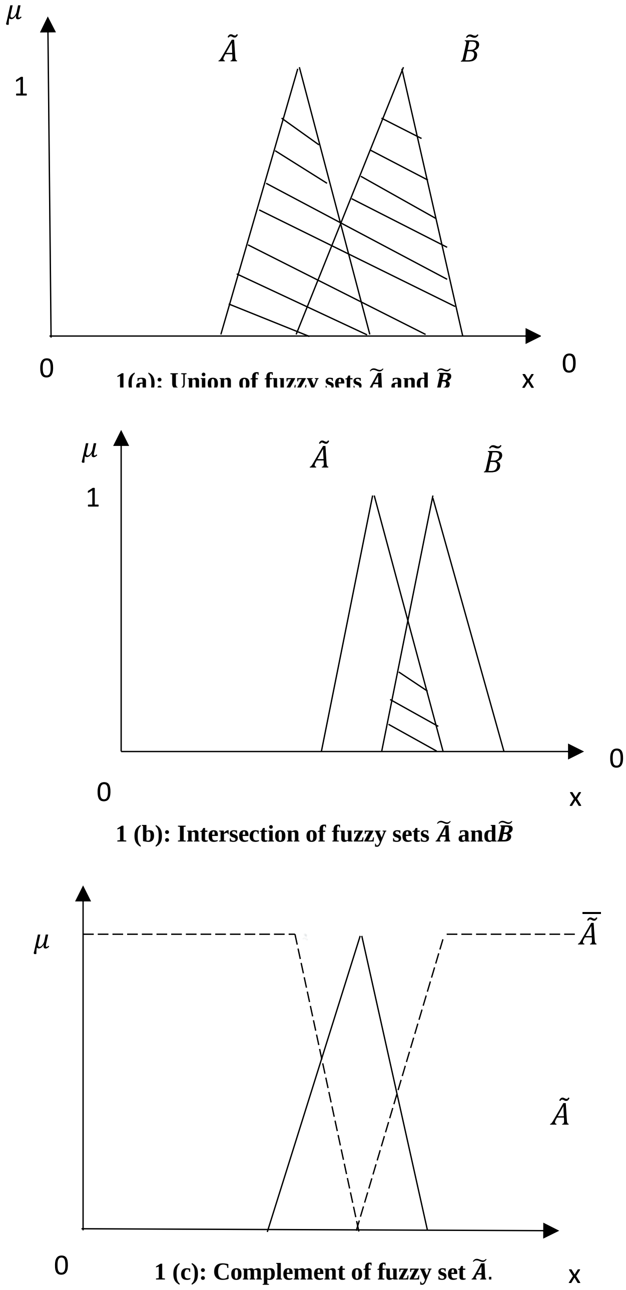

1(a): Union of fuzzy sets $\tilde{A}$ and $\tilde{B}$

1 (b): Intersection of fuzzy sets $\tilde{A}$ and$\tilde{B}$

1 (c): Complement of fuzzy set $\tilde{A}$.

Figure 1: Union, Intersection and complement of a fuzzy set.

All fuzzy sets as well as relative subsets on X are gathered together to compute the power set (fuzzy) P ($\tilde{X}$), Given that all fuzzy sets can coincide, it should be evident that the fuzzy power set's cardinality, $n_{P(X)}$, is infinite, implying that $n_{P(X)}$= ∞ (Menger, 1942; Negoita, 1988; Ross, 2010).

For fuzzy sets, we can apply the principles given by De Morgan, as indicated by the following expressions:

$\overline{\tilde{A}\cap\tilde{B}} = \bar{\tilde{A}} \cup \bar{\tilde{B}}$

$\overline{\tilde{A}\cup\tilde{B}} = \bar{\tilde{A}} \cap \bar{\tilde{B}}$

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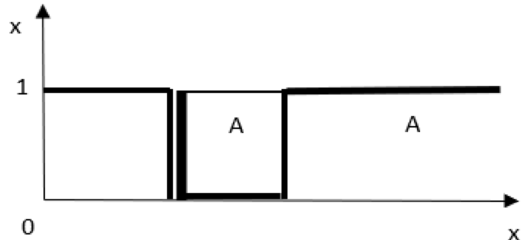
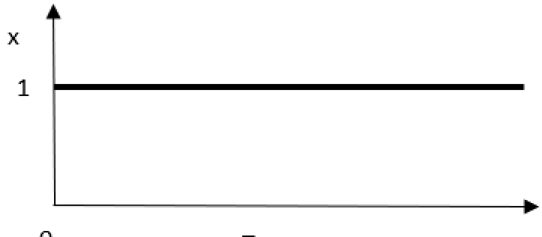
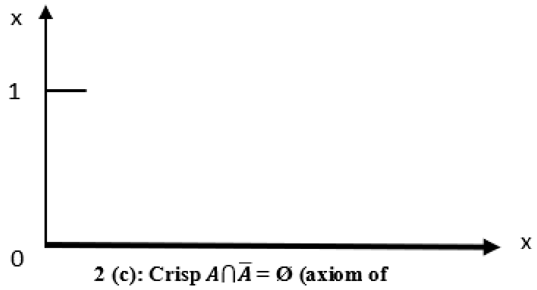


2 (a): Crisp set A and its complement

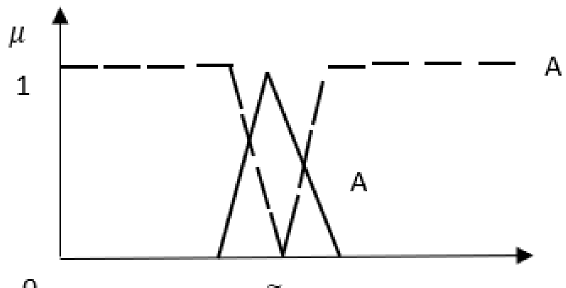


2 (b): Crisp $A \cup \overline{A}$ = X (axiom of excluded middle)

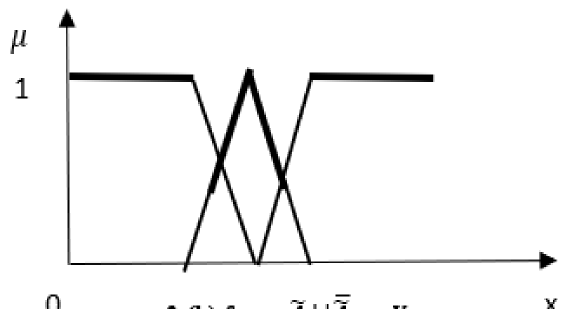


2 (c): Crisp $A \cap \overline{A}$ = Ø (axiom of contradiction)

Figure 2: Excluded middle axioms for crisp sets



3 (a) Fuzzy set $\widetilde{A}$ along with its complement



3 (b) fuzzy $\widetilde{A} \cup \overline{\widetilde{A}} \neq X$

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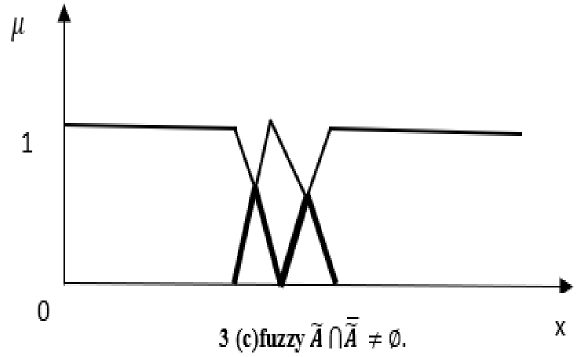


Figure 3: Excluded middle axioms for fuzzy sets are not valid

In addition to all operations on classical sets, all other operations on fuzzy sets are also valid, excluded middle axioms is an exception. As fuzzy sets do not have the same axiomatic structure as formally defined sets, these axioms do not hold for them (see Gaines, 1978). A set and its complement can also overlap due to the overlap of fuzzy sets (Grattan-Guinness, 1982). The following are the exempted middle axioms, which are generalised to fuzzy sets:

$$\bar{\tilde{A}} \cup \tilde{A} \neq X.$$

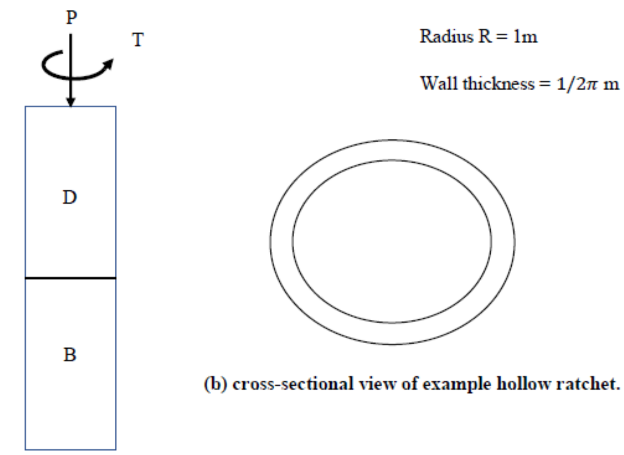
$$\bar{\tilde{A}} \cap \tilde{A} \neq \emptyset.$$

Here, we compare exempted middle axioms for classical or crisp and fuzzy sets using enlarged venn diagrams.

V. FUZZY SET CHARACTERISTICS

Crisp sets have the same features as fuzzy sets. Classical sets can be regarded of as a particular instance of fuzzy sets due to this fact and the fact that the membership grades of a crisp set are contained in the interval [0,1]. As a result, the characteristics are the same as for fuzzy sets (Ross, 2010).

Imagine a simple hollow ratchet with a radius of about 1m and thickness of the wall being 1/(2πm). As shown below (see Fig 4), the ratchet is constructed by layering a ductile segment, D, of the suitable cross segment over a brittle segment, B. A force P acting downward and a torque T are imparted to the ratchet simultaneously (Lynch, 1985; Zadeh, 1965). The hypothetical shear stress on any component in the ratchet is T (pascals) due to the parameters employed, and the hypothetical vertical component of stress in the ratchet is P (pascals). We suppose that neither B nor D's failure properties are known with absolute knowledge.



(a) Axial view

(b) cross-sectional view of example hollow ratchet.

Figure 4.

Utilising the fault function, μ_A= f ([P2 + 4T2]1/2) as a metric, we identify the fuzzy set $\tilde{A}$ to be the portion in (P, T) space within which component D will be "safe." In a Similar way, we identify the set $\tilde{B}$ as the portion in (P, T) space within which component B will be "safe," Utilising the fault function μ_B = g (P − β|T |), as a metric (Menger, 1942; Zadeh, 1965). Indeed, f and g are membership functions well defined on the interval [0, 1]. At this particular point, it is

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insignificant to know exactly what they are. However, before specifying f and g, it is useful to go over the fundamental set operations in this framework. The following is a synopsis of the discussion:

1. The loading's set for which either component B or component D is expected to be "safe" is denoted by $\tilde{A} \cup \tilde{B}$.
2. $\tilde{A} \cap \tilde{B}$ denotes the loading's set for which both component B and component D are expected to be "safe."
3. The loading's set for which component D and component B are unsafe are $\bar{\tilde{A}}$ and $\bar{\tilde{B}}$, respectively.
4. The loading's set within which the flexible component is safe but the inflexible component is in risk is $\bar{\tilde{A}} \mid \bar{\tilde{B}}$.
5. The loading's set within which the inflexible component is safe but the flexible component is in hazard is described as $\bar{\tilde{B}} \mid \bar{\tilde{A}}$.
6. According to De Morgan's law $\overline{\tilde{A} \cap \tilde{B}} = \bar{\tilde{A}} \cup \bar{\tilde{B}}$, the loadings which are not safe for either material is the union of those which are unsafe for the brittle component and those that are unsafe for the ductile component.
7. According to De Morgan's principle $\overline{\tilde{A} \cup \tilde{B}} = \bar{\tilde{A}} \cap \bar{\tilde{B}}$, the loads that are neither safe for both component D & B is the intersection of those that are unsafe for material D and those that are unsafe for component B.

Let us use the following numerical example to illustrate the idea:

$\tilde{A} = \left\{\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\right\}$

$$\tilde{B} = \left\{\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}\right\}.$$

Several of the operations just outlined can now be calculated (membership for element 1 in each $\tilde{A}$ and $\tilde{B}$ is implicitly 0):

Complement

$$\bar{\tilde{A}} = \left\{\frac{1}{1} + \frac{0}{2} + \frac{0.5}{3} + \frac{0.7}{4} + \frac{0.8}{5}\right\}$$

$$\bar{\tilde{B}} = \left\{\frac{1}{1} + \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5}\right\}$$

Union

$\tilde{A} \vee \tilde{B} = \left\{\frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5}\right\}.$

Intersection

$$\tilde{A} \wedge \tilde{B} = \left\{\frac{0.5}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5}\right\}.$$

Difference

$$\bar{\tilde{A}} \mid \bar{\tilde{B}} = \left\{\frac{0.5}{2} + \frac{0.3}{3} + \frac{0.3}{4} + \frac{0.2}{5}\right\}.$$

$$\bar{\tilde{B}} \mid \bar{\tilde{A}} = \left\{\frac{0}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.4}{5}\right\}.$$

De Morgan's principles

$\overline{\tilde{A} \vee \tilde{B}} = \bar{\tilde{A}} \wedge \bar{\tilde{B}} = \left\{\frac{1}{1} + \frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5}\right\}.$

$\overline{\tilde{A} \wedge \tilde{B}} = \bar{\tilde{A}} \vee \bar{\tilde{B}} = \left\{\frac{1}{1} + \frac{0.5}{2} + \frac{0.5}{3} + \frac{0.8}{4} + \frac{0.8}{5}\right\}.$

VI. CONCLUSION

Most of the other operational activities described here are applicable to fuzzy sets as well as crisp sets; however, a few of the operations permitted in fuzzy sets, such as accumulation and averaging, have no analogues in classical set concept (Ross, 2010). As a result of the excluded middle axioms, thinking in any scenario is restricted. Non-equivalent to binary, set membership is a concept that can have an infinite range of values. Fuzzy sets are managed and mathematically processed in the same way that crisp sets are. Sets are said to be non-interactive, which is equivalent to the assumption of isolation in probability models. When inputs from several universes are merged in fuzzy systems modelling, noninteractive fuzzy sets become necessary.

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