JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 874 - 882
https://publishoa.com
ISSN: 1309-3452

Comparative Analysis of Defuzzification Techniques for Fuzzy Output

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ABSTRACT

In many situations, a single crisp value is required as a fuzzy process's output instead of a fuzzy output. A defuzzification operation is one that works with a fuzzy output and transforms it into a definite and decisive crisp value. In contrast, fuzzification is an operation which helps to convert a sharp value into a fuzzy one. For a given fuzzy operation, the output may be analytical combination of two or more than two membership functions elucidated on universal set of the output varying quantity. This paper focuses on the study of the various methods used to defuzzify the fuzzy output. The method of defuzzification is to turn a value quantity into a concrete value.

Keywords: Defuzzification, Crisp value, Fuzzy number, Fuzzy output.

I. INTRODUCTION

The technique used for transforming a fuzzy output in a single crisp value for a fuzzy set is termed as Defuzzification. Fuzzification, inference, and composition techniques are employed in the evaluation of linguist if-then rules via systems based on fuzzy rules. They give hazy results that must usually be converted to a sharp output. Defuzzification is used to turn the fuzzier results into crisper ones [1]. The defuzzified value indicates the measure of power in the FLC (Fuzzy Logic Controller). In the literature, there are several approaches for the defuzzification process. The centroid approach, as well as the center of sums and mean of maxima, are widely used methods [2]. Because the defuzzification issue appears to be ambiguous and depending on implementation, customization is deemed to be a needed function [3].

The defuzzification stage in proposed fuzzy systems entails selecting one value as the controller's output. For defuzzification, we have numerous ways [4]. The centre of area (COA) approach is employed to evaluate the membership function's center of area [5]. The mode rule involves determining the maximum support value for the membership function. The Average of the elements that achieve highest membership degree is utilized by mean of maximum technique as its defuzzified value [6]. The median rule entails selecting the median which splits the region of membership function in two equivalent portions [7]–[10]. Various strategies for comparing fuzzy numbers have been used in the literature. Dubois and Prade, who ordered fuzzy numbers using maximising sets [11]. Abbas-bandy and Asady, take into account a source (fuzzy) for fuzzy numbers, then rank them based on the distance between fuzzy numbers and that source [6], [12]. Wang et al. gave accurate centroid equations for fuzzy quantities and defend those equations using analytical geometry [13]. Defuzzification is a technique for substituting a fuzzy set with a crisp representation [14], [15].

II. PRELIMINARIES

A. Defuzzification

In many cases, if the output is represented as a single scalar quantity, it is easier to make a crisp judgement for a system with a fuzzy output [16]. The process of defuzzification is the inverse of fuzzification and it involves taking fuzzy quantity and converting that into a single crisp quantity.

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B. Fuzzy Set Theory

Fuzzy sets, which have a partial composition ranging from 0 to 1, have broadened and extended the notion of crisp sets [17]. Fuzzy sets are sets with imprecise/vague boundaries and they act as a potential tool for handling imprecision and uncertainties being a broad notion of classical set.

As a result, fuzzy set is a well-defined mapping from universal set X to [0,1] and it can be represented as

$\tilde{A}$ = {(a, $\mu_{\tilde{A}}$ (a)) | a ∈X}

where element a ∈ X, which is universal set and $\mu_{\tilde{A}}$ (a) denotes the degree of belongingness.

C. Membership Function

Membership function represents the actual degree of belongingness, i.e., the similarity of an element to a particular class and it is similar for both discrete and continuous items [16], [17]. Membership function value is a subset of positive reals, lying in interval [0,1]. A membership function for set $\tilde{A}$ is commonly represented by $\mu_{\tilde{A}}$ (x) to membership range.

$$\mu_{\tilde{A}}(x): X \to M$$

The basic characteristics that characterize a membership function are listed below.

• $\boldsymbol{\alpha - cut}$

It is the collection of all a ∈ X with membership values either equal to or greater than α, i.e.,

$$\alpha_{\mu_{\tilde{A}}}(a) = \{a \mid \mu_{\tilde{A}}(a) \geq \alpha\}$$

• **Support**

It is defined as the collection of all $a \in X$ with positive membership value i.e., $\mu_{\tilde{A}}$ (x) > 0

$$S(\mu) = \{a \in X \mid \mu_{\tilde{A}}(x) > 0\}$$

• **Core**

It is the collection of all $a \in X$ with membership grade equal to exactly one. Core being empty is a possibility.

• **Height**

The maximum membership grade attained by an element in a given fuzzy set is the height h (μ) of that fuzzy set [18].

$$h(\mu) = \sup \mu(a) \text{ for all a} \in X$$

For a normal fuzzy set, h $(\mu) = 1$ and for a subnormal fuzzy set, h (μ) is less than 1.

III. NUMERICAL EXAMPLE

In a specific region of the country, a train authority aims to establish a major railway system. For right-of-way concerns, the entire region through which the new line passes must be bought It has been polled thrice and the results are compiled for further analytical study [5]. The sets $\tilde{B}_1$, $\tilde{B}_2$, and $\tilde{B}_3$ (as shown in figure 1.) describe the reported data for the roads which are specified on the universal set of widths of right of way (in metres).

$\tilde{B}_1$

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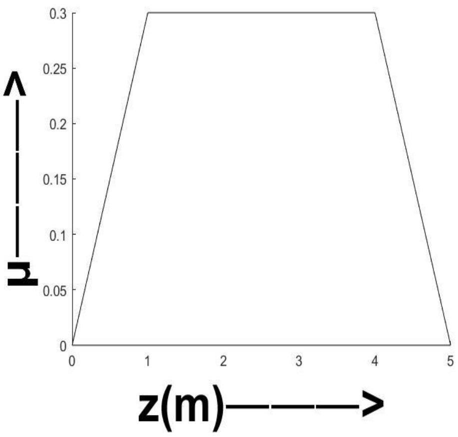
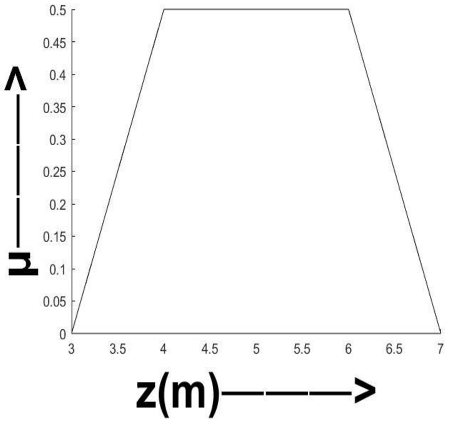
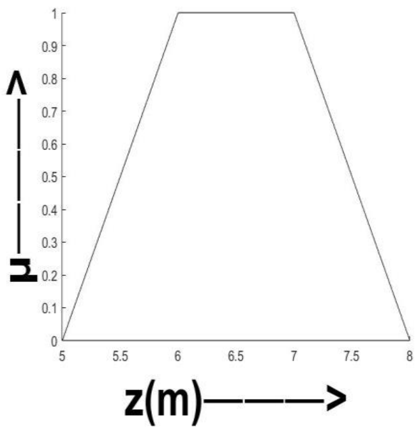


1A. Fuzzy set $\tilde{B}_1$
$\tilde{B}_2$



1B. Fuzzy set $\tilde{B}_2$
$\tilde{B}_3$



1C. Fuzzy set $\tilde{B}_3$

Figure 1. Plots created using MATLAB

We now wish to combine the results of these three surveys to determine which is the best suitable width of right-of-way (z) so that the train firm can generate a rough estimate of how much it will cost to buy the right-of-way. We wish to locate $z*$ using various defuzzification approaches and the three prior fuzzy sets.

A. Center of Area Method

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This technique is also known as the center of gravity method or the center of area method. Sugeno invented this technique in 1985. This is the most widely used method. The only drawback of this method is that it is computationally challenging for membership functions that are complex. The algebraic expression for this process (also known as centroid method) is aesthetically pleasing among most of the known defuzzification strategies [2], [19], [20].

$$z^* = \frac{\int \mu_{\tilde{c}}(z).zdz}{\int \mu_{\tilde{c}}(z)dz},$$

An algebraic integration is denoted by $\int$. Using the centroid approach (as shown in figure 2.), the above equation can be used to find z∗.

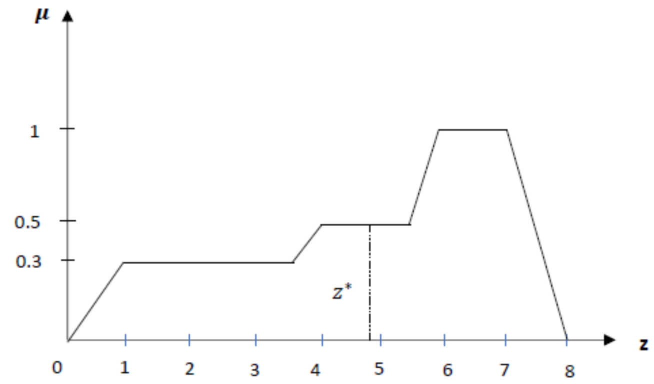

Figure 2. The Centre of Area technique.

$$z^* = \frac{\int \mu_{\tilde{B}}(z).zdz}{\int \mu_{\tilde{B}}(z)dz},$$

$$= [\int_0^1 (0.3z)zdz + \int_1^{3.6} (0.3)z\,dz + \int_{3.6}^4 \frac{(z-3.0)}{2} z\,dz + \int_4^{5.5} (0.5)z\,dz$$

$$+ \int_{5.5}^6 (z-5)z\,dz + \int_6^7 zdz + \int_7^8 (8-z)zdz\,]$$

$$\div [\int_0^1 (0.3z)dz + \int_1^{3.6} (0.3)dz + \int_{3.6}^4 \frac{(z-3.0)}{2} dz + \int_4^{5.5} (0.5)dz$$

$$+ \int_{5.5}^6 (z-5)dz + \int_6^7 dz + \int_7^8 (8-z)dz\,]$$

$$= 4.9\, m.$$

B. Weighted Average Method

Because it is one of the far most operationally efficient methods, in fuzzy applications, the weighted average approach is the most often used. Regrettably, it's frequently limited to output membership functions with symmetrical outputs (as shown in figure 3.) [19], [20]. The mathematical expression provides the answer,

$$z^* = \frac{\sum \mu_{\tilde{B}}(\bar{z}).\bar{z}dz}{\sum \mu_{\tilde{B}}(\bar{z})dz},$$

where the centroid of every proportional membership function is $\bar{z}$ and $\sum$ signifies the algebraic sum. The above equation can be used to find z∗ utilizing the weighted average strategy.

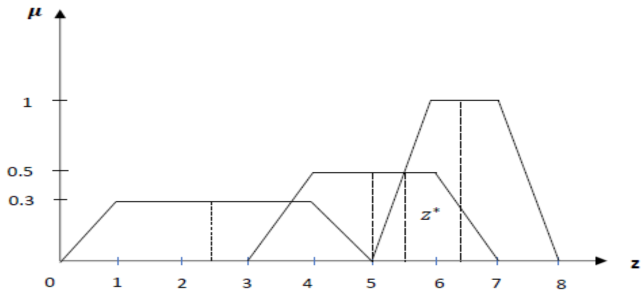

Figure 3. The weighted average technique.

$$z^* = \frac{\int \mu_{\tilde{B}}(\bar{z}).\bar{z}dz}{\int \mu_{\tilde{B}}(\bar{z})dz},$$

$$z^* = \frac{(0.3 \times 2.5) + (0.5 \times 5) + (1 \times 6.5)}{0.3 + 0.5 + 1} = 5.41m.$$

C. Middle of Maxima Method

Mean maximum membership is another name for this strategy. The maximum membership locations in this method can't be one-of-a-kind (i.e., rather than a single point, maximum membership might be a flat area) [19], [20]. The mathematical relation for this approach can be expressed as

$$z^* = \frac{a + b}{2}$$

where a and b are the values shown in Figure 4. The above equation can be used to find z^* using the Middle of Maxima Method.

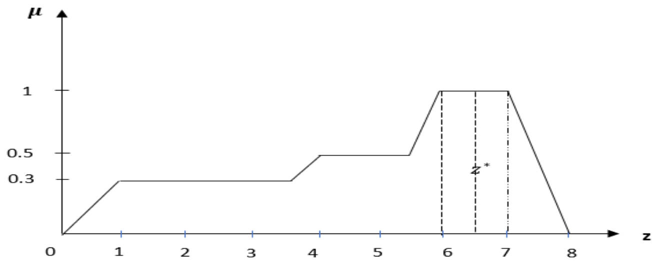

Figure 4. The Middle of Maxima technique.

$$z^* = \frac{a + b}{2} \Rightarrow z^* = \frac{6 + 7}{2} = 6.5m.$$

D. Center of Sums

This strategy outperforms many commonly used defuzzification techniques, and it isn't limited to homogeneous membership functions. Instead of combining individual output fuzzy sets, such as $\tilde{B}_1$ and $\tilde{B}_2$, this procedure uses the algebraic sum of them (as shown in figure 5). The interweaving regions are added two times in this procedure, and it is also necessary to calculate the centroid for individual membership function [5], [19], [21]. As follows, we have defuzzified value z∗:

$$z^* = \frac{\sum_{k=1}^{n}(\mu_{\tilde{B}}(z) \int_z \bar{z}\, dz}{\sum_{k=1}^{n}(\mu_{\tilde{B}}(z) \int_z dz}$$

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Volume 13, No. 2, 2022, p. 874 - 882
https://publishoa.com
ISSN: 1309-3452

where $\bar{z}$ symbolizes the distance from each membership function's centroid. The above equation can be used to find z∗ utilizing the center of sums strategy.

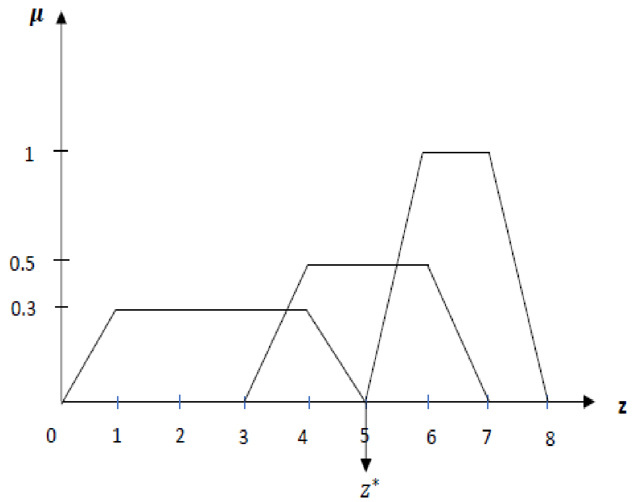

Figure 5. Center of sums result

$$z^* = \frac{\begin{bmatrix} 2.5 \times 0.5 \times 0.3(3+5)+ \\ 5 \times 0.5 \times 0.5(2+4) + 6.5 \times 0.5 \times 1(3+1) \end{bmatrix}}{\begin{bmatrix} 0.5 \times 0.3(3+5)+ \\ 0.5 \times 0.5(2+4) + 0.5 \times 1(3+1) \end{bmatrix}}$$

$= 5.0\ m.$

E. Center of Largest Area

If minimum two convex subregions exist in the ultimate fuzzy set, then the center of gravity (i.e., the centroid approach is used to calculate z*) of the convex subregion with maximum area utilized to determine the z* value of the output [15], [19]. This is written in algebraic form as

$$z^* = \frac{\int \mu\tilde{c}_m(z)zdz)}{\int \mu\tilde{c}_m zdz}$$

where $\tilde{c}_m$ is the convex segment with the maximum area making up $\tilde{c}_k$.When the entire output $\tilde{c}_k$ is nonconvex, this condition is true. Whenever $\tilde{c}_k$ is convex, z* value is same as that evaluated by utilizing centroid technique (since there is just unique convex zone). The above equation can be used to find z∗ using the center of largest area approach. Because entire resulting fuzzy set is convex, the center of biggest area approach yields the same result as the centroid method, resulting in z∗= 4.9.

IV. COMPARISON AND EVALUATION OF DEFUZZIFICATION METHODS

The query is a natural one to raise: Amongst the five fuzzy system interference system strategies provided here, which is the best one? One simple solution to the question is that is it depends on the situation or difficulty. Hellendoom and Thomas (1993) defined five parameters by which to evaluate the approaches to provide a more detailed answer to this question. These parameters will be restated here for the convenience of the reader who is also considering the upsides and downsides of various approaches (Table 1.) as shown below:

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A. Continuity

A minor change in the fuzzy controller's input should not lead to a significant change in its output. In a particular instance of a fuzzy controller with one output and two input, for example, if the inputs (α_1^*, γ_1^*) and (α_2^*, γ_2^*) vary slightly, then the output values so obtained $\dot{z}_1^*$ and $\dot{z}_2^*$ must vary slightly [21].

$$\forall \varepsilon > 0 \, \exists \, \delta > 0 : |\alpha_1^* - \alpha_2^*| < \delta \wedge |\gamma_1^* - \gamma_2^*| < \delta \Rightarrow |\dot{z}_1^* - \dot{z}_2^*| < \varepsilon$$

B. Dis-ambiguity

If the proposed technique for determining z* is really well defined, then defuzzification technique is free from ambiguity [1], [21]. Centre of largest area doesn't satisfy this criterion in the sense that whenever the biggest membership functions have equal area, ambiguity will be there in choosing z*.

C. Plausibility

Plausibility is the third requirement. For being plausible, z* should be located about in the midway of $\tilde{c}_k$ sustenance region and have a high level of involvement in $\tilde{c}_k$. The centroid technique is not plausible in the instance depicted because, while z* is in the middle of $\tilde{c}_k$'s support, it has a low degree of membership [22].

D. Complexity in Computation

It suggests that longer intensive technique is, less worth it ought to have for an exceedingly computational system. The complexity in competition for center of sums is determined by output membership functions' shape as well as type of inference used, whether scaled inference or maximum - minimum based inference is used [5]. In this scenario, the Middle of Maxima is a fast technique than Center of Area Approach.

E. Counting of Weight

In this fifth and final criterion, the resulting fuzzy sets are given a weighted average. The difference between the centroid approach, weighted average approach, and center of sum approach is defined by this criterion [21]. The fifth criterion has the disadvantage of being problem-specific, because there are few criteria by which to determine the optimum weighting method. Center of sums is, however, weight counting.

	Center of Area	**Weighted Average Method**	**Middle of Maxima**	**Center of Sums**	**Center of Largest Area**
Continuity	+ +	+	− −	+ +	0
Dis-ambiguity	+ +	+ +	0	+ +	− −
Plausibility	0	− −	0^*	+	+ +
Complexity in Computation	− −	− −	+	0	− −
Counting of Weight	− −	0	− −	+ +	− −

Table 1. Defuzzification methods and their criteria.

+, good, ++, very good; etc.

*0 only in the condition of scaled inference.

The analysis of the case study related to real life problem is shown as under (Table 2.): -

Defuzzification Method	Defuzzified Value
Center of Area	4.9
Weighted Average method	5.41
Middle of Maxima	6.5
Center of Sums	5.0
Center of Largest Area	4.9

Table 2. Defuzzification methods and their corresponding

defuzzified values related to real life problem.

The accompanying table plainly shows that the center of area, center of largest area and center of sums get nearly identical results. Other procedures, on the other hand, produce a wide range of results. So, in a privately owned area, the right-of-way path that the government has to purchase is approximately 4.9 meters.

VI. CONCLUSION

The paper's main goal, however, has been to explain how to turn fuzzy output into crisp or sharp representations a strategy known as defuzzification. This approach is mandatory in view of the fact that one cannot instruct a machine for raising its voltage "slightly", even when the voltage originates from a fuzzy converter, one must modify the given voltage by a fixed number. It is both a natural and required process. In true terms, in field of mathematics, we use to solve a complicated problem in Argand Plane by first establishing the solution's real and imaginary components, the imaginary conclusion is then decomplexified and returned to the real domain [3]. There are a variety of alternative defuzzification strategies, and five of them have been tested. The process of defuzzification must be evaluated in terms of the quality of the response in the purpose of obtaining information supplied, as with many other topics in fuzzy logic [21].

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