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Application of RAM in Dual-Hesitant Fuzzy Transportation Problem

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ABSTRACT
Dual hesitation fuzzy sets are applied to handle **inaccurate, preliminary** or **incomplete** information and knowledge **situations** in **actual** operational **investigation situations. This** work **presents** a **new** method called Russell's Approximation Method (RAM) **to solve the dual hesitant** fuzzy **transport problem. This procedure** is **illustrated using** a numerical **example,** and the **results achieved by** this **procedure are** compared **to** the result obtained from North West corner method.

Keyword: dual-hesitant fuzzy transportation problem, score function, RAM, NWC, dual-hesitant fuzzy numbers.

Introduction

Transportation is a close agreement to put together ways to make goods available to customers, especially with skillful techniques. They guarantee the competent development and agile openness of raw materials and finished products. Hitchcock [5] was the first to develop the TP in 1941. Charnes [1] in 1953 and Dantzig [2] (Primal-Simplex-Transport) in 1963 have studied transport (TP) problems and provided various methods for solving TP.It is not possible to explicitly recognize all the limitations of TP because of the insurmountable factors in existing relationships. It is not possible to explicitly present such ambiguous information by selecting a random variable from a probability distribution**.**

To master this situation, Zadeh [12] introduced fuzzy numbers in 1965. Zimmermann predicted in 1978 [14] that the explanations obtained by fuzzy linear programming would be forever clever. A new category of fuzzy, more precisely dual fuzzy numbers is introduced to restore complexity in realistic situations that carefully represent the situation.

The idea of hesitant fuzzy sets (HFS) was mainly proposed by Torra and Narukawa (2009, 2010) [10, 11]. To overcome the difficulties in real life, we will incorporate a new class of fuzzy, or double resistance fuzzy numbers. This explains the

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situation well and gives better results compared to existing studies on fuzzy. A constrained, doubly hesitant fuzzy transport problem was proposed by Gurupada Maityetal. (2019) [3]. The Dual Hesitant Fuzzy Set (DHFS) is an extension of HFS that covers fuzzy sets, intuitive fuzzy sets, HFS, and fuzzy multi sets as a special case presented by Zhu et al (2012) [13].Singh (2014)[9] proposed a study on deciphering assignment problems with DHFSs.
Saad & Abbas in (2003) [8] extended an algorithm for discovering the elucidation for the TPs in fuzzy environment. Fuzzy zero point method was projected by Pandian & Natrajan in (2010) [9,10] to solve TPs. Hajjari & Abbasbandy in (2011)[4] projected a promoter operator for defuzzification methods with method of magnitude.
In this research work we have considered the same example proposed by Gurupada Maity et al for solving a dual-hesitant fuzzy transportation problem which is solved by using RAM. A preface to fuzzy sets and DHFs are presented in section 2. Section 3 an algorithm is proposed. In section 4 a numerical example is illustrated. Conclusion of the work is given in section 5.

2. PREFACE

2.1 Hesitant fuzzy sets

Definition 2.1.1 (Torra [10,11]).

A HFS *H* on *X* is defined in terms of a function $h(x)$ that returns a subset of values in the interval [0, 1] once it is applied to *X*, i.e., an element of its power set: $h:X\rightarrow \wp([0,1])$ .Thereafter, Xu and Xia (2011) described the definition of HFS in a compact form by including the mathematical representation of a HFS.

Definition 2.1.2 (Xu and Xia [13]).

A HFS is stated mathematically in the following way: $H=\{(x_i,h(x_i)): x_i \in X\}$where $h(x_i)$ is a set of several different values in the interval [0, 1] for each $x_i \in X$, which denotes the possible membership degree of the element $x_i \in X$ in the set *H*. In the usual sense, each member of $h(x_i)$ is called a *Hesitant Fuzzy Element (HFE)*, denoted by h_i.

2.2 Dual hesitant fuzzy sets

Definition 2.2.1

Let *X* be a fixed set; then a DHFS *D* on *X* is defined as follows: $D= \{(x, h(x), g(x) : x\in X\}$ where $h(x)$ and $g(x)$ are mappings that take set-values in [0, 1]; they are denoted as the possible membership degree and non-membership degree of any element $x \in X$, to the set D ,respectively, with the conditions $0\leq h_D, g_D \leq 1$, $0 \leq h_D+g_D \leq 1$, for any $h_D\in h(x)$; $g_D \in g(x)$.
A Dual-Hesitant Fuzzy element (DHFE) is understood as the pair d(x) = ((h(x), g(x)), and it is denoted in the functional form as d = (h, g).

2.3 Arithmetic operations on DHFEs

Let $d_1 = \{ h_{d_1}, g_{d_1}\}$ and $d_2=\{ h_{d_2}, g_{d_2}\}$ represent two DHFEs; then addition and subtraction is given by,

Addition, $d_1 \oplus d_2 = \{ h_{d_1,} \oplus h_{d_2}, g_{d_1,} \ominus g_{d_2}\}$
Subtraction, $d_1 \ominus d_2 = \{ h_{d_1,} \ominus h_{d_2}, g_{d_1,} \oplus g_{d_2}\}$

2.4 Ranking of dual hesitant fuzzy sets

Let $D= \{(x, h(x), g(x) : x\in X\}$ be a DHFS, where $X =\{x_1,x_2,x_3,\ldots x_n\}$ and d = (h, g) be a DHFE. We define a score function S_d on the DHFS, represented as follows:
$s_d=\left|\frac{1}{k}\sum_{i=1}^{k} h_d(x_i) - \frac{1}{k}\sum_{i=1}^{k} g_d(x_i)\right|$
Let d_1 and d_2 be any two DHFSs.

JOURNAL OF ALGEBRAIC STATISTICS
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With regard to a given score function, Zhu et al (2012) defined order relations as follows:
Case 1: If $s_{d_1} > s_{d_2}$,then d1 is called superior to d2, denoted by d1 > d2.
Case 2: If $s_{d_1} < s_{d_2}$,then h1 is called inferior to h2, denoted by d1< d2.

Case 3: If $s_{d_1} = s_{d_2}$, then h1 is called indifferent from h2, denoted by $d_1 \backsim d_2$

3 PROBLEM FORMULATION

The balanced fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:

$$minimize \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} * x_{ij}$$

Subject to $\sum_{j=1}^{q} x_{ij} = \tilde{a}_i$, i = 1,2,3,………,p
$\sum_{i=1}^{p} x_{ij} = b_j$,j = 1,2,3,………,q
$\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j$
X_{ij} is a non- negative trapezoidal fuzzy number,
Where P = total number of sources
Q = total number of destinations
a_i = the fuzzy availability of the product at i^{th} source
b_j = the fuzzy demand of the product at j^{th} destination
c_{ij} = the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination
x_{ij} = the fuzzy quantity of the product that should be transported from i^{th} source to j^{th} destination to minimize the total fuzzy transportation cost.
we use the simplest form of notation for dual-hesitant fuzzy cost as $\widetilde{c_{ij}} = (h_{ij}, g_{ij})$ in the rest of our discussion. Hence, the dual-hesitant fuzzy cost $\widetilde{c_{ij}}$ is introduced in order to design the mathematical model of the TP; we name it as aDHFTPR and we put this in the following way:

$$optimize \sum_{i=1}^{p}\sum_{j=1}^{q} \widetilde{c_{ij}} * x_{ij}$$

Subject to $\sum_{j=1}^{q} x_{ij} = \tilde{a}_i$, i = 1,2,3,………,p
$\sum_{i=1}^{p} x_{ij} = b_j$,j = 1,2,3,………,q
$\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j$
In fact, the objective function (z) in problem is designed based on dual-hesitant fuzzy costs $\widetilde{c_{ij}}$.

4. Algorithm for Russell's Approximation Method (RAM)

RAM for solving transportation problems to minimize the cost is illustrated below.
Step-1: Construct a Dual-Hesitant Fuzzy Transportation Table (DHFTT) from the specified transportation problem.
Step-2: Make sure if the TP is balanced or not, if not, make it balanced.
Step-3: For each source row, determine its $\bar{U}_i$ (largest cost in row i).

Step-4: For each destination column, determine its $\bar{V}_j$ (largest cost in column j).

Step-5: For each variable, calculate $\Delta_{ij} = c_{ij} - (\bar{U}_i + \bar{V}_j)$.

Step-6: Select the variable having the most negative Δ value, break ties arbitrarily.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 924 - 930
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Step-7: Allocate as much as possible. Eliminate necessary cells from consideration.

Step-8: Re iterate from Step 3 until the demand and supply are exhausted.

Step-9: Now shift this allocation to the original DHFTT.

Step-10: To conclude, compute the total profit of the DHFTT. This calculation is the sum of the product of cost and resultant allocated value of the DHFTT.

5. NUMERICAL EXAMPLE

Consider Dual-Hesitant Fuzzy Transportation problem with three sources that is S_1, S_2, S_3 and three destinations D_1, D_2, D_3.

Table 1: Formulating dual-hesitant fuzzy cost

By using the score function we convert the given dual -Hesitant Fuzzy Transportation problem into crisp values, we get the following table.

	D1	D2	D3	ai
S1	{{0:5; 0:4; 0:1},{0:4; 0:5; 0:9}}(20, 25, 30)	{{0:7; 0:6; 0:5}; 0:2} ,{ 0:1; 0:3; 0:4; 0:5}} (14, 16, 20, 35)	{{*0:6; 0:4; 0:3 },{ 0:2; 0:6; 0:7}}*(22, 25, 36)	20
S2	{{0:4;0:2},{0:3;0:5}} (12, 15)	{{0:7; 0:6; 0:3}, {0:1; 0:3; 0:6}} (30, 35, 40)	{{0:6; 0:5; 0:3},{ f0:2; 0:3; 0:5}} (22, 27, 30)	24
S3	{{0:3; 0:2; 0:1},{0:2; 0:6; 0:7}} (30, 40, 45)	{{0:2; 0:1}; {0:5; 0:9}} (25, 32)	{{0:6; 0:5; 0:3; 0:2}; {0:2; 0:3; 0:5; 0:7}} (32, 35, 40, 50)	35
bj	35	24	20	

Table 2: Score value and fuzzy cost ranking value

	D1	D2	D3	
S1	0.27 / 25	0.175 /21.25	0.07 / 27.67	20
S2	0.1 / 13.5	0.2 / 35	0.14 / 26.33	24
S3	0.3 /38.33	0.55 / 28.5	0.025 /39.25	35
bj	35	24	20	

JOURNAL OF ALGEBRAIC STATISTICS
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Table 3. Initial iteration of cost matrix

From Table 3, it is found that the DHFT is balanced.

Table-4: Calculate ¯*Ui* and ¯*Vj*

	D1	D2	D3	SUPPLY	¯*Ui*
S1	0.27	0.18	0.07	20	0.27
S2	0.1	0.2	0.14	24	0.2
S3	0.3	0.55	0.03	35	0.55
DEMAND	35	24	20		

	D1	D2	D3		
S1	0.27	0.175	0.07	20	
S2	0.1	0.2	0.14	24	
S3	0.3	0.55	0.025	35	
bj	35	24	20		
¯*Vj*	0.3	0.55	0.14		

Compute reduced cost of each cell Δ*ij*, where Δ*ij*=*cij* - (¯*Ui*+¯*Vj*)

1.$\Delta_{11}= c_{11}-(\bar{U}_1+\bar{v}_1)=0.27-(0.27+0.3)=-0.3$

2. $\Delta_{12}= c_{12}-(\bar{U}_1+\bar{v}_2)=0.18-(0.55+0.2)=-0.64$

3. $\Delta_{13}= c_{13}-(\bar{U}_1+\bar{v}_3)=0.07-(0.14+0.27)=-0.34$

4. $\Delta_{21}= c_{21}-(\bar{U}_2+\bar{v}_1)=0.1-(0.3+0.27)=-0.4$

5. $\Delta_{22}= c_{22}-(\bar{U}_2+\bar{v}_2)=0.2-(0.55+0.2)=-0.55$

6. $\Delta_{23}= c_{23}-(\bar{U}_2+\bar{v}_3)=0.14-(0.14+0.27)=-0.2$

7. $\Delta_{31}= c_{31}-(\bar{U}_3+\bar{v}_1)=0.3-(0.27+0.3)=-0.55$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 924 - 930
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8. $\Delta_{32} = c_{32} - (\bar{U}_3 + \bar{v}_2) = 0.55 - (0.55 + 0.2) = -0.55$

9. $\Delta_{33} = c_{33} - (\bar{U}_3 + \bar{v}_3) = 0.03 - (0.14 + 0.27) = -0.66$

Table 5 : Calculation of Δij

	D1	**D2**	**D3**	**SUPPLY**	**$\bar{U}i$**
S1	$0.27(-0.3)$	$0.18(-0.64)$	$0.07(-0.34)$	20	0.27
S2	$0.1(-0.4)$	$0.2(-0.55)$	$0.14(-0.2)$	24	0.2
S3	$0.3(-0.55)$	$0.55(-0.55)$	$0.03(-0.66)$	35	0.55
DEMAND	35	24	20		
$\bar{V}j$	0.3	0.55	0.14		

The most negative Δij is -0.66 in cell $S3D3$
The allocation to this cell is min(35,20) = **20**.
This satisfies the entire demand of $D3$ and leaves 35 - 20=15 units with $S3$

Table 6: First Allocation of values

	D1	**D2**	**D3**	**SUPPLY**
S1	0.27	0.18	0.07	20
S2	0.1	0.2	0.14	24
S3	0.3	0.55	0.03(20)	15
DEMAND	35	24	0	

Delete the column D3. Proceeding above the final allocation is given below

Table 7: Final Allocation of values

	D1	**D2**	**D3**	**SUPPLY**
S1	**0.27**	**0.18(20)**	**0.07**	**20**
S2	**0.1(20)**	**0.2(4)**	**0.14**	**24**

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 924 - 930
https://publishoa.com
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S3	**0.3(15)**	**0.55**	**0.03(20)**	**15**
DEMAND	**35**	**24**	**0**	

The minimum total transportation cost =
(21.25×20)+(13.5×20)+(35×4)+(38.33×15)+(39.25×20)=425+270+140+574.95+785=**2194.95**
Here, the number of allocated cells = 5 is equal to m + n - 1 = 3 + 3 - 1 = 5
∴ This solution is non-degenerate

This procedure of applying ATM is comparatively better than North West corner where the cost is **2230.**

5. CONCLUSION

RAM is applied in a Dual-Hesitant Fuzzy Transportation problem in this paper. This method is easier when compared to other methods. Computing the same problem using North West corner method we get a better approximation. This technique can be used to solve all types of Dual-Hesitant Fuzzy Transportation problems. Consequently this scheme can be utilized to resolve the real-life problems including Transshipment and supply chain Problems.

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