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Solving Intuitionistic Fuzzy Linear Programming Problems Using Singularly Perturbed Differential Equations Of Reaction-Diffusion Type

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ABSTRACT

In this paper, fully fuzzy linear programming problem under Intuitionistic fuzzy environment is considered and a new approach for solving Intuitionistic Fuzzy Linear Programming Problems with Triangular Intuitionistic fuzzy numbers using Singularly Perturbed Differential Equations of Reaction-Diffusion type is proposed. The exact solution of Singularly Perturbed Differential Equation is used to derive the decision maker weights and it is applied in solving Fuzzy Linear Programming problems under Intuitionistic fuzzy environment. A Numerical example is provided to illustrate the proposed approach and to show the efficiency of the proposed approach and finally the solution is compared with the existing approach.
KEYWORDS: Intuitionistic fuzzy number, Triangular Intuitionistic fuzzy number, Intuitionistic fuzzy linear programming problem, Singular perturbation problem.

Mathematics Subject Classification: 34M60 , 34D15 ,90C05 ,03E72

1. INTRODUCTION

Fuzzy theory was popularised in the "context of optimization problems by Bellman and Zadeh (1970)". The idea "Intuitionistic fuzzy set (IFS) by Atanassov (1986)" was shown to be fairly helpful in dealing with ambiguity. "The concept of Fuzzy Linear Programming (FLP) was first formulated by Zimmermann [26]". "Many authors such as Mahapatra et al.(2010), Nachammai (2013) and Nagoorgani (2012) have also studied linear programming problems under intuitionistic fuzzy environment". "Paramasivam et al. (2010) developed a piecewise-uniform Shishkin mesh for a linear second-order Singular Perturbation Problem (SPP), which was utilised to develop numerical approaches for the problem". "Robinson et al. (2019) discussed MAGDM problems using SPPs". In this paper, "method to solve Intuitionistic Fuzzy Linear Programming Problem (IFLPP) with Triangular Intuitionistic fuzzy numbers (TIFN) is proposed where the weights are derived from singular perturbation problems, normalized and utilized" and a numerical example is given to prove the efficiency of the proposed method and it is compared with the existing approach.

2. PRELIMINARIES

In this section, "basic definitions of fuzzy theory which are needed for this paper are given".

Definition 1: An Intuitionistic Fuzzy subset $\tilde{F}^I = \left\{\left(\upsilon, \mu_{\tilde{F}^I}(\upsilon),\ \nu_{\tilde{F}^I}(\upsilon)\right)/\upsilon \in U\right\}$, of the real line $\boldsymbol{R}$ is called an IFN if
(i) $\mathscr{F}\upsilon \in \boldsymbol{R}$ such that $\mu_{\tilde{F}^I}(\upsilon) = 1\ and\ \nu_{\tilde{F}^I}(\upsilon) = 0$
(ii) " $\mu_{\tilde{F}^I}$ and $\nu_{\tilde{F}^I}$ are piecewise continuous mappings from R to the closed interval [0,1]" and the relation $0 \le \mu_{\tilde{F}^I}(\upsilon) + \nu_{\tilde{F}^I}(\upsilon) \le 1\ \upsilon \in U$ holds .

Definition 2: A Triangular Intuitionistic Fuzzy Number (TIFN) $\tilde{F}^I$ with membership function $\mu_{\tilde{F}^I}$ and non-member ship function $\nu_{\tilde{F}^I}$ is defined as an IFS in R where

$$\mu_{\tilde{F}^I}(\upsilon) = \begin{cases} \dfrac{\upsilon - \rho_1}{\rho_2 - \rho_1}, & if\ \rho_1 < \upsilon \le \rho_2 \\ \dfrac{\rho_3 - \upsilon}{\rho_3 - \rho_2}, & if\ \rho_2 \le \upsilon < \rho_3 \\ 0, & \text{otherwise} \end{cases}$$

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$$\nu_{\tilde{F}^I}(\upsilon) = \begin{cases} \dfrac{\rho_2 - \upsilon}{\rho_2 - \rho_1'}, & if\, \rho_1' < \upsilon \le \rho_2 \\ \dfrac{\upsilon - \rho_2}{\rho_3' - \rho_2}, & if\ \rho_2 \le \upsilon < \rho_3' \\ 1, & \text{otherwise} \end{cases}$$

where $\rho_1' \le \rho_1 < \rho_2 < \rho_3 \le \rho_3'$. This TIFN is denoted by $\tilde{F}^I = (\rho_1, \rho_2, \rho_3; \rho_1', \rho_2, \rho_3')$

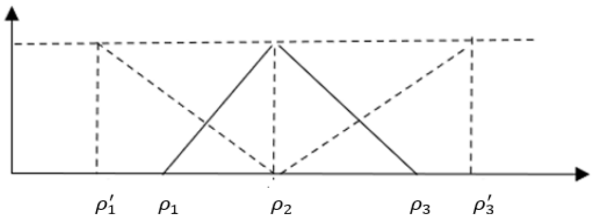

FIGURE 1. Membership and non-membership functions of TIFN

Definition 3:
Let $\tilde{\mathcal{F}}^I{}_1 = (\tau_1, \tau_2, \tau_3; \tau_1', \tau_2, \tau_3')$ and $\tilde{\mathcal{F}}^I{}_2 = (\upsilon_1, \upsilon_2, \upsilon_3; \upsilon_1', \upsilon_2, \upsilon_3')$ be two TIFNs. Then the following operations can be done.

Addition:
$\tilde{\mathcal{F}}^I{}_1 \oplus \tilde{\mathcal{F}}^I{}_2 = (\tau_1 + \upsilon_1, \tau_2 + \upsilon_2, \tau_3 + \upsilon_3; \tau_1' + \upsilon_1', \tau_2 + \upsilon_2, \tau_3' + \upsilon_3')$

Subtraction:
$\tilde{\mathcal{F}}^I{}_1 \ominus \tilde{\mathcal{F}}^I{}_2 = (\tau_1 - \upsilon_3, \tau_2 - \upsilon_2, \tau_3 - \upsilon_1; \tau_1' - \upsilon_3', \tau_2 - \upsilon_2, \tau_3' - \upsilon_1')$

Multiplication:
$\tilde{\mathcal{F}}^I{}_1 \otimes \tilde{\mathcal{F}}^I{}_2 = (\mathfrak{f}_1, \mathfrak{f}_2, \mathfrak{f}_3; \mathfrak{f}_1', \mathfrak{f}_2, \mathfrak{f}_3')$, where $\mathfrak{f}_1 = min\{\tau_1\upsilon_1, \tau_1\upsilon_3, \tau_3\upsilon_1, \tau_3\upsilon_3\}$
$\mathfrak{f}_2 = \tau_2\upsilon_2$; $\mathfrak{f}_3 = max\{\tau_1\upsilon_1, \tau_1\upsilon_3, \tau_3\upsilon_1, \tau_3\upsilon_3\}$
$\mathfrak{f}_1' = min\{\tau_1'\upsilon_1', \tau_1'\upsilon_3', \tau_3'\upsilon_1', \tau_3'\upsilon_3'\}$
$\mathfrak{f}_3' = max\,\{\tau_1'\upsilon_1', \tau_1'\upsilon_3', \tau_3'\upsilon_1', \tau_3'\upsilon_3'\}$

Scalar Multiplication:
$k\tilde{\mathcal{F}}^I{}_1 = (k\tau_1, k\tau_2, k\tau_3; k\tau_1', k\tau_2, k\tau_3') : k > 0$
$k\tilde{\mathcal{F}}^I{}_1 = (k\tau_3, k\tau_2, k\tau_1; k\tau_3', k\tau_2, k\tau_1') : k < 0$

3. INTUITIONISTIC FUZZY LINEAR PROGRAMMING PROBLEM

Intuitionistic Fuzzy Linear Programming Problem (IFLPP) is defined as follows

$Max\ \tilde{Z}^I = \sum_{j=1}^{n} \tilde{c}_j^I \otimes \tilde{x}_j^I$

s.to $\sum_{j=1}^{n} \tilde{a}_{ij}^I \otimes \tilde{x}_j^I \preccurlyeq \approx \succcurlyeq \tilde{b}_i^I$ for all i =1,2,…,m

where $\tilde{a}_{ij}^I$, $\tilde{c}_j^I$, $\tilde{x}_j^I$ and $\tilde{b}_i^I$ are Intuitionistic fuzzy numbers and

$\tilde{x}_j^I \succcurlyeq \tilde{0}$ for all j =1, 2, …, n

(1)

4. WEIGHTS DETERMINATION FOR SOLVING IFLPP USING SINGULARLY PERTURBED DIFFERENTIAL EQUATION OF REACTION-DIFUSSION TYPE

The manufacturer represents weighting vector in the form of the following singularly perturbed problem $-\varepsilon u''(x) + u(x) = 1 + x$, $x \in (0,1)$ with $u(0) = 2; u(1) = 1$.

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which occurs in solving the Intuitionistic fuzzy linear programming problem.

The exact solution of the above problem is $u(x)\left(\frac{1+e^{\frac{1}{\sqrt{\varepsilon}}}}{e^{\frac{-1}{\sqrt{\varepsilon}}}-e^{\frac{1}{\sqrt{\varepsilon}}}}\right)e^{\frac{x}{\sqrt{\varepsilon}}}+\left(\frac{1+e^{\frac{1}{\sqrt{\varepsilon}}}}{e^{\frac{1}{\sqrt{\varepsilon}}}-e^{\frac{-1}{\sqrt{\varepsilon}}}}\right)e^{\frac{-x}{\sqrt{\varepsilon}}}+1+x$

By normalizing the exact solution for $\varepsilon = 0.01$ the weight vectors can be obtained and is given in Table1

Table 1 : Exact solution for $-\varepsilon u''(x) + u(x) = 1 + x$

x	*u(x)*	Normalization
0.2	8.4861	0.40866
0.4	6.1682	0.29704
0.6	4.0537	0.19521
0.8	2.0578	0.09910

5. METHOD TO SOLVE IFLPP USING SPP

The steps of the proposed method to solve the IFLPP using SPP are as follows:

Step1: If all the parameters $\tilde{c}_j^I$, $\tilde{x}_j^I$, $\tilde{a}_{ij}^I$, $\tilde{b}_i^I$ are represented by Triangular Intuitionistic fuzzy numbers $(c_j^1, c_j^2, c_j^3; c_j^4, c_j^2, c_j^5), (x_j^1, x_j^2, x_j^3; x_j^4, x_j^2, x_j^5), (a_{ij}^1, a_{ij}^2, a_{ij}^3; a_{ij}^4, a_{ij}^2, a_{ij}^5)$, $(b_j^1, b_j^2, b_j^3; b_j^4, b_j^2, b_j^5)$ then the IFLPP (1) can be written as

$Max\ \tilde{Z}^I = \sum_{j=1}^{n}(c_j^1, c_j^2, c_j^3; c_j^4, c_j^2, c_j^5) \otimes (x_j^1, x_j^2, x_j^3; x_j^4, x_j^2, x_j^5)$

s.to

$\sum_{j=1}^{n} (a_{ij}^1, a_{ij}^2, a_{ij}^3; a_{ij}^4, a_{ij}^2, a_{ij}^5) \otimes (x_j^1, x_j^2, x_j^3; x_j^4, x_j^2, x_j^5) \preccurlyeq (b_j^1, b_j^2, b_j^3; b_j^4, b_j^2, b_j^5)$ for all i = 1, 2,…,m

$\tilde{x}_j^I \succcurlyeq \tilde{0}^I$, j =1, 2,…,n (2)

Step2: The IFLPP in step 1can be written as

$Max\ \tilde{Z}^I = \sum_{j=1}^{n}(min(c_j^1\ x_j^1,\ c_j^1\ x_j^3,\ c_j^3 x_j^1, c_j^3 x_j^3), c_j^2 x_j^2, max(c_j^1\ x_j^1, c_j^1\ x_j^3,\ c_j^3 x_j^1, c_j^3 x_j^3), min(c_j^4\ x_j^4,$
$c_j^4\ x_j^5,\ c_j^5 x_j^4, c_j^5 x_j^5), c_j^2 x_j^2, max(c_j^4\ x_j^4,\ c_j^4\ x_j^5,\ c_j^5 x_j^4, c_j^5 x_j^5))$

s.to $\sum_{j=1}^{n}(min(a_{ij}^1\ x_j^1,\ a_{ij}^1\ x_j^3,\ a_{ij}^3 x_j^1, a_{ij}^3 x_j^3), a_{ij}^2 x_j^2,\ max(a_{ij}^1\ x_j^1,$
$a_{ij}^1\ x_j^3,\ a_{ij}^3 x_j^1, a_{ij}^3 x_j^3), min(a_{ij}^4\ x_j^4,\ a_{ij}^4\ x_j^5,\ a_{ij}^5 x_j^4, a_{ij}^5 x_j^5), a_{ij}^2 x_j^2, max(a_{ij}^4\ x_j^4,\ a_{ij}^4\ x_j^5,\ a_{ij}^5 x_j^4, a_{ij}^5 x_j^5)) \preccurlyeq$
$(b_j^1, b_j^2, b_j^3; b_j^4, b_j^2, b_j^5)$

$x_j^1 - x_j^4 \geq 0$

$x_j^2 - x_j^1 \geq 0$

$x_j^3 - x_j^2 \geq 0$

$x_j^5 - x_j^3 \geq 0$

$x_j^4 \geq 0$, for all i = 1, 2,…,m and for all $j = 1,2,\ldots,n.$ (3)

by calculating $(c_j^1, c_j^2, c_j^3; c_j^4, c_j^2, c_j^5) \otimes (x_j^1, x_j^2, x_j^3; x_j^4, x_j^2, x_j^5)$
and $(a_{ij}^1, a_{ij}^2, a_{ij}^3; a_{ij}^4, a_{ij}^2, a_{ij}^5) \otimes (x_j^1, x_j^2, x_j^3; x_j^4, x_j^2, x_j^5)$ respectively and using the order relation.

Step 3: Convert the IFLPP in step 2 into the following multi-objective linear programming problem with Intuitionistic fuzzy coefficients:

$Max\ (f_1(x), f_2(x), f_3(x), \ldots, f_k(x))$

s.t. (3)

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where $f_i: R^n \to R^i$, where R is the set of all real numbers and R^n is the n-dimensional Euclidean space.

By considering the weighting factor, the multi-objective linear programming problem is defined as
$Max\ w(x) = \sum_{m=1}^{k} w_m\, f_m(x)$
s.t. (3)
Step4: Convert the "MOLPP with Intuitionistic fuzzy coefficients" obtained from step 3 to the "crisp LPP", by giving weights achieved from "Singular Perturbation Problem" given in section 4 to obtain the optimal solution.

6. NUMERICAL EXAMPLE

In this section, we present a numerical example given in Jayalakshmi et al (2019) to illustrate the method proposed in Section 5. We aim to find the optimal solution and corresponding objective value of the following IFLPP using the weights (0.40866, 0.29704, 0.19521, 0.09910) obtained from SPP proposed by the manufacturer in section 4.
Maximize

$$\tilde{Z}^I = (1,2,3; 0.5,2,3.5) \otimes \tilde{x}_1^I \oplus (2,3,4; 1.5,3,4.5) \otimes \tilde{x}_2^I$$

Subject to $(0,1,2; 0,1,2.5) \otimes \tilde{x}_1^I \oplus (1,2,3; 0.5,2,3.5) \otimes \tilde{x}_2^I \leq (1,10,27; 0.5,10,36)$

$(1,2,3; 0.5,2,3.5) \otimes \tilde{x}_1^I \oplus (0,1,2; 0,1,2.5) \otimes \tilde{x}_2^I \leq (2,11,28; 1.5,11,38)$

$\tilde{x}_1^I,\ \tilde{x}_2^I \succcurlyeq \tilde{0}^I$

where $\tilde{x}_1^I = (x_1^1, x_1^2, x_1^3; x_1^4, x_1^2, x_1^5)$ and $\tilde{x}_2^I = (x_2^1, x_2^2, x_2^3; x_2^4, x_2^2, x_2^5)$

Using the steps of the proposed method we obtain the following linear programming problem

Maximize 0.40866 x_1^1 + 0.81732 x_2^1 + 1.18816 x_1^2 + 1.78224 x_2^2 + 0.58563 x_1^3 + 0.78084 x_2^3 + 0.20433 x_1^4 + 0.61299 x_2^4 + 0.34685 x_1^5 + 0.44595 x_2^5

Subject to
$x_2^1 \leq 1;\ x_1^2 + 2x_2^2 \leq 10\ ;\ 2x_1^3 + 3x_2^3 \leq 27\ ;$
$0.5x_2^4 \leq 0.5\ ;\ 2.5x_1^5 + 3.5x_2^5 \leq 36\ ;$
$x_1^1 \leq 2;\ 2x_1^2 + x_2^2 \leq 11\ ;\ 3x_1^3 + 2x_2^3 \leq 28;$
$0.5x_1^4 \leq 1.5\ ;\ 3.5x_1^5 + 2.5x_2^5 \leq 38\ ;$
$x_1^1 - x_1^4 \geq 0\ ;\ x_1^2 - x_1^1 \geq 0\ ;\ x_1^3 - x_1^2 \geq 0\ ;$
$x_1^5 - x_1^3 \geq 0\ ;\ x_2^1 - x_2^4 \geq 0\ ;\ x_2^2 - x_2^1 \geq 0\ ;$
$x_2^3 - x_2^2 \geq 0\ ;\ x_2^5 - x_2^3 \geq 0\ ;\ x_1^4 \geq 0;\ x_2^4 \geq 0$
Using LINGO fuzzy optimal solution $\tilde{x}_1^I = (2,4,6; 2,4,7.166667)$; $\tilde{x}_2^I = (1,3,5; 1,3,5.166667)$ and the fuzzy objective value (4,17,38;2.5,17,48.3333360) is obtained.

7. COMPARATIVE STUDY

In the table below, the solution obtained from the numerical example is compared to the solution obtained from Jayalakshmi et al (2019).

Table 2: Comparison of the numerical solution of the proposed approach with the existing approach

Sl. No.	Approach	$\tilde{x}_1^I$	$\tilde{x}_2^I$	Max $\tilde{Z}^I$
1.	The Proposed Approach	(2,4,6; 2,4,7.166667)	(1,3,5; 1,3,5.166667)	(4,17,38;2.5,17,48.3333360)
2.	Jayalakshmi etal (2019)	(2,4,6; 2,4,7.17)	(1,3,5; 1,3,5.17)	(4,17,38;2.5,17,48.33)

The table shows that the solution of the above example obtained by the proposed method is same as those obtained by the existing method but the proposed method, on the other hand, is simpler and provides the optimal solution faster than the existing approach.

8. CONCLUSION

Here, IFLPP with TIFN is examined and a method to solve IFLPP is proposed. The proposed approach converts the IFLPP to MOLPP with Intuitionistic fuzzy coefficients and the weights are calculated using singularly perturbed differential equation of reaction diffusion type problem according to the manufacturer decisions. Finally, the proposed approach is illustrated by the numerical example and the solution is compared with the existing approach and it is found that the proposed method produces results faster than the existing method.

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