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A New One-Step Method Inverse Polynomial Method for Solving Stiff and Non-Stiff Delay Differential Equations

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ABSTRACT
This paper presents the one-step fourth order inverse method to solve the Delay differential equations (DDEs) by using interpolating function which consists of inverse function. The delay argument is approximated using Lagrange interpolation. The stability polynomial of this method and the corresponding stability region are obtained. The applicability of this method has been demonstrated by numerical examples of stiff and non-stiff DDEs with constant delay, time dependent delay and state dependent delays and the results are compared with the existing method. The numerical results are compared with the theoretical solution.

Keywords: Lagrange interpolation, One-Step Method, Inverse function, Stability polynomial and Stability region, Stiff and Non-Stiff Delay differential equations.

1 Introduction

Delay differential equations arise in chemical kinetics [1], population dynamics [2], and management systems [3] and in several areas of science and engineering. Recently there has been a growing interest in the numerical solutions of stiff and non-stiff DDEs. Some of the significant numerical methods are Runge Kutta Method [4], Homotopy Perturbation method [5], Adomian Decomposition Method [6], Block method [7], Differential Transform Method [8] and Parallel two-step ROW method [9].

Most of the models in differential equations are 'stiff' in nature. For solving stiff equations, the step size is taken to be extremely small. Also, many problems may be stiff in some intervals and non-stiff in others. Therefore, we need an efficient technique to be suitable for stiff and non-stiff problems. Several one-step numerical techniques have been developed for the solution of first order differential equations by means of interpolating functions. These works can be referred in [10-14].

In this paper we present a new one-step inverse method for solving stiff and non-stiff DDEs with constant, time and state dependent delays. In this method, the solution is locally represented as an inverse function. The organization of this paper is as follows: In section 2, the derivation of the new method is described. In section 3, the analysis of the new one-step method is investigated. Stability region is also determined. In section 4, numerical illustrations of DDEs are provided to demonstrate the efficiency of this method.

2 The Derivation of the New One-Step Inverse Polynomial Method (ISM4)

Consider the first order DDEs with delay τ,

$$y'(t) = f\left(t, y(t), y\left(t - \tau\big(t, y(t)\big)\right)\right), \qquad t > t_0$$
$$y(t) = \Phi(t), \qquad t \le t_0 \tag{1}$$

where $\Phi(t)$ is the initial function.

Let us assume that the analytical solution $y(t_{n+1})$ to the initial value problem (1) can be locally represented in the interval $[t_n, t_{n+1}]$, $n \ge 0$ by inverse function as

$$y_{n+1} = y_n\left[\sum_{j=0}^{k} b_j t_n^j\right]^{-1} \tag{2}$$

where $b_j's$ are undetermined coefficients.
By taking k = 4 and $b_0 = 1$,

$$y_{n+1} = y_n[1 + (b_1t_n + b_2t_n^2 + b_3t_n^3 + b_4t_n^4)]^{-1} \quad (3)$$

Using Binomial expansion in (3), we get

$$y_{n+1} = y_n(1 + (-1)[b_1t_n + b_2t_n^2 + b_3t_n^3 + b_4t_n^4] + \frac{(-1)(-2)}{2!}[b_1t_n + b_2t_n^2 + b_3t_n^3 + b_4t_n^4]^2 + \frac{(-1)(-2)(-3)}{3!}[b_1t_n + b_2t_n^2 + b_3t_n^3 + b_4t_n^4]^3 + \frac{(-1)(-2)(-3)(-4)}{4!}[b_1t_n + b_2t_n^2 + b_3t_n^3 + b_4t_n^4]^3)$$

$$y_{n+1} = y_n(1 - b_1t_n - b_2t_n^2 - b_3t_n^3 - b_4t_n^4 + {b_1}^2t_n^2 + {b_2}^2t_n^4 + 2b_1b_2t_n^3 + 2b_1b_3t_n^4 - {b_1}^3t_n^3 - 3{b_1}^2b_2t_n^4 + {b_1}^4t_n^4)$$

$$y_{n+1} = y_n - b_1t_ny_n + t_n^2(-b_2 + {b_1}^2)y_n + t_n^3(-b_3 + 2b_1b_2 - {b_1}^3)y_n + t_n^4(-b_4 + {b_2}^2 + 2b_1b_3 - 3{b_1}^2b_2 + {b_1}^4)y_n \quad (4)$$

Expressing the left hand side of (4) in terms of Taylor's series expansion,

$$y_{n+1} = y_n + hy_n' + \frac{h^2}{2!}y_n'' + \frac{h^3}{3!}y_n''' + \frac{h^4}{4!}y_n^{(iv)} \quad (5)$$

By making the above expression to agree term by term for each parameter, we get

$$b_1 = \frac{-hy_n'}{t_ny_n} \quad (6)$$

$$b_2 = \frac{2h^2(y_n')^2 - h^2y_n''y_n}{2t_n^2y_n^2} \quad (7)$$

$$b_3 = \frac{6h^3y_ny_n'y_n'' - 6h^3(y_n')^3 - h^3y_n'''y_n^2}{6t_n^3y_n^3} \quad (8)$$

$$b_4 = \frac{8h^4y_n^2y_n'y_n''' - h^4y_n^3y_n^{(iv)} + 6h^4(y_n'')^2y_n^2}{24t_n^4y_n^4} \quad (9)$$

Substituting (6), (7), (8) and (9) in (3), we get

$$y_{n+1} = \left[\frac{24y_n^5}{24y_n^4 - 24hy_n^3y_n' + h^2(24y_n^2(y_n')^2 - 12y_n^3y_n'') + h^3(-4y_n^3y_n''' + 24y_n^2y_n'y_n'' - 24y_n(y_n')^3) + h^4(-y_n^3y_n^{(iv)} + 8y_n^2y_n'y_n''' + 6y_n^2(y_n'')^2)}\right] \quad (10)$$

Eq. (10) is the new one-step inverse numerical technique.
This one-step method can also be extended to solve DDEs with multiple delays. In this paper, Lagrange interpolation is used to approximate the delay argument.

3 Analysis of the One-Step Inverse Polynomial Method (ISM4)

A general one-step method is given in form

$$y_{n+1} = y_n + \phi(t_n, y_n; h),$$

where $\phi(t_n, y_n; h)$ is called as incremental function of the method.

3.1 Order and Consistency of the Method (ISM4)

Definition:

A numerical scheme with an incremental function $\phi(t_n, y_n; h)$ is said to be consistent with the initial value problem (1) if

$$\phi(t_n, y_n; 0) = f(t_n, y_n).$$

Substituting (6), (7), (8) and (9) in (4) and applying Binomial expansion, we get

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$$
\begin{aligned}
y_{n+1} = y_n \Bigg[1 &- \left(\left(\frac{-hy'_n}{t_n y_n}\right) t_n + \left(\frac{2h^2 (y'_n)^2 - h^2 y''_n y_n}{2t_n^2 y_n^2}\right) t_n^2 + \left(\frac{6h^3 y_n y'_n y''_n - 6h^3 (y'_n)^3 - h^3 y'''_n y_n^2}{6t_n^3 y_n^3}\right) t_n^3 + \left(\frac{8h^4 y_n^2 y'_n y'''_n - h^4 y_n^3 y_n^{(iv)} + 6h^4 (y''_n)^2 y_n^2}{24 t_n^4 y_n^4}\right) t_n^4 \right) \\
&+ \left(\left(\frac{-hy'_n}{t_n y_n}\right) t_n + \left(\frac{2h^2 (y'_n)^2 - h^2 y''_n y_n}{2t_n^2 y_n^2}\right) t_n^2 + \left(\frac{6h^3 y_n y'_n y''_n - 6h^3 (y'_n)^3 - h^3 y'''_n y_n^2}{6t_n^3 y_n^3}\right) t_n^3 + \left(\frac{8h^4 y_n^2 y'_n y'''_n - h^4 y_n^3 y_n^{(iv)} + 6h^4 (y''_n)^2 y_n^2}{24 t_n^4 y_n^4}\right) t_n^4 \right)^2 \\
&- \left(\left(\frac{-hy'_n}{t_n y_n}\right) t_n + \left(\frac{2h^2 (y'_n)^2 - h^2 y''_n y_n}{2t_n^2 y_n^2}\right) t_n^2 + \left(\frac{6h^3 y_n y'_n y''_n - 6h^3 (y'_n)^3 - h^3 y'''_n y_n^2}{6t_n^3 y_n^3}\right) t_n^3 + \left(\frac{8h^4 y_n^2 y'_n y'''_n - h^4 y_n^3 y_n^{(iv)} + 6h^4 (y''_n)^2 y_n^2}{24 t_n^4 y_n^4}\right) t_n^4 \right)^3 \\
&+ \left(\left(\frac{-hy'_n}{t_n y_n}\right) t_n + \left(\frac{2h^2 (y'_n)^2 - h^2 y''_n y_n}{2t_n^2 y_n^2}\right) t_n^2 + \left(\frac{6h^3 y_n y'_n y''_n - 6h^3 (y'_n)^3 - h^3 y'''_n y_n^2}{6t_n^3 y_n^3}\right) t_n^3 + \left(\frac{8h^4 y_n^2 y'_n y'''_n - h^4 y_n^3 y_n^{(iv)} + 6h^4 (y''_n)^2 y_n^2}{24 t_n^4 y_n^4}\right) t_n^4 \right)^4 \Bigg]
\end{aligned}
$$

After simplifying this, we get

$$y_{n+1} = y_n + hy'_n + \frac{h^2}{2} y''_n + \frac{h^3}{6} y'''_n + \frac{h^4}{24} y_n^{(iv)} + o(h^5)$$

$$\frac{y_{n+1} - y_n}{h} = y'_n + \frac{h}{2} y''_n + \frac{h^2}{6} y'''_n + \frac{h^3}{24} y_n^{(iv)}$$

Taking the limit $h \to 0$, we get

$$\frac{y_{n+1} - y_n}{h} = y'_n = f(t_n, y_n).$$

Using the above definition, we see that our one-step inverse method is consistent. Also, by virtue of Taylor series it is found that this one-step inverse method given by (10) is of order four.

3.2 Stability Polynomial of the Method (ISM4)

Here we consider a commonly used linear test equation with a constant delay $\tau = mh$, where m is a positive integer,

$$y'(t) = \lambda y(t) + \mu y(t - \tau), \quad t > t_0$$
$$y(t) = \phi(t), \qquad t \le t_0$$

where $\lambda, \mu \in C$, $\tau > 0$ and ϕ is continuous.

A slight arrangement of (10), we obtain

$$y_{n+1} = y_n + hy'_n + \frac{h^2}{2} y''_n + \frac{h^3}{6} y'''_n + \frac{h^4}{24} y_n^{(iv)} \qquad (11)$$

This implies,

$$
\begin{aligned}
y_{n+1} = y_n &+ h\big(\lambda y_n + \mu y(t_n - \tau)\big) + \frac{h^2}{2}\left(\lambda y'_n + \mu y'(t_n - \tau)\right) \\
&+ \frac{h^3}{6}\left(\lambda y''_n + \mu y''(t_n - \tau)\right) + \frac{h^4}{24}\left(\lambda y'''_n + \mu y'''(t_n - \tau)\right) \qquad (12)
\end{aligned}
$$

Here Lagrange interpolation is used to approximate the delay term.

$$y(t_n - mh) = y(t_{n-m}) = \sum_{l=-r_1}^{s_1} L_l(c_i) y_{n-m+l}$$

with $L_l(c_i) = \prod_{j_1=-r_1}^{s_1} \frac{c_i - j_1}{l - j_1}$, $\quad j_1 \ne l \quad and\ r_1, s_1 > 0$

The new one-step method is applied to DDE with (1), with constant delay $\tau = 1$.

Now $\quad y(t_n - \tau) = \sum_{l=-r_1}^{s_1} L_l(c) y_{n-m+l}$,

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$$y'(t_n-\tau)=\lambda\sum_{l=-r_1}^{s_1}L_l(c)y_{n-m+l}+\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}$$
$$y''(t_n-\tau)=\lambda\left(\lambda\sum_{l=-r_1}^{s_1}L_l(c)y_{n-m+l}+\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}\right)$$
$$+\mu\left(\lambda\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}+\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-3m+l}\right)$$

and

$$y'''(t_n-\tau)=\lambda\begin{pmatrix}\lambda\left(\lambda\sum_{l=-r_1}^{s_1}L_l(c)y_{n-m+l}+\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}\right)\\+\mu\left(\lambda\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}+\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-3m+l}\right)\end{pmatrix}+$$
$$\mu\begin{pmatrix}\lambda\left(\lambda\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}+\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-3m+l}\right)\\+\mu\left(\lambda\sum_{l=-r_1}^{s_1}L_l(c)y_{n-3m+l}+\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-4m+l}\right)\end{pmatrix}$$
(13)

Substituting (13) in (12), we get

$$y_{n+1}=y_n+h\left(\lambda y_n+\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-m+l}\right)+\frac{h^2}{2}\begin{pmatrix}\lambda^2y_n+2\lambda\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-m+l}\\+\mu^2\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}\end{pmatrix}$$
$$+\frac{h^3}{6}\begin{pmatrix}\lambda^3y_n+3\lambda^2\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-m+l}\\+3\lambda\mu^2\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}\\+\mu^3\sum_{l=-r_1}^{s_1}L_l(c)y_{n-3m+l}\end{pmatrix}+\frac{h^4}{24}\begin{pmatrix}\lambda^4y_n+4\lambda^3\mu\sum_{l=-r_1}^{s_1}L_l(c)y_{n-m+l}\\+6\lambda^2\mu^2\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}\\+4\lambda\mu^3\sum_{l=-r_1}^{s_1}L_l(c)y_{n-3m+l}\\+\mu^4\sum_{l=-r_1}^{s_1}L_l(c)y_{n-4m+l}\end{pmatrix}$$
$$y_{n+1}=y_n+\lambda hy_n+\frac{\lambda^2h^2}{2}y_n+\frac{\lambda^3h^3}{6}y_n+\frac{\lambda^4h^4}{24}y_n+\sum_{l=-r_1}^{s_1}L_l(c)y_{n-m+l}\left(\mu h+\mu\lambda h^2+\frac{\lambda^2\mu h^3}{2}+\frac{\lambda^3\mu h^4}{6}\right)$$
$$+\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}\left(\frac{\mu^2h^2}{2}+\frac{\lambda\mu^2h^3}{2}+\frac{\lambda^2\mu^2h^4}{4}\right)+\sum_{l=-r_1}^{s_1}L_l(c)y_{n-3m+l}\left(\frac{\mu^3h^3}{6}+\frac{\lambda\mu^3h^4}{6}\right)$$
$$+\sum_{l=-r_1}^{s_1}L_l(c)y_{n-4m+l}\left(\frac{\mu^4h^4}{24}\right)$$

Let $\alpha=\lambda h$ and $\beta=\mu h$. Then the above equation becomes,

$$y_{n+1}=y_n\left(1+\alpha+\frac{\alpha^2}{2!}+\frac{\alpha^3}{3!}+\frac{\alpha^4}{4!}\right)+\left(\beta\left(1+\alpha+\frac{\alpha^2}{2}+\frac{\alpha^3}{6}\right)\right)\sum_{l=-r_1}^{s_1}L_l(c)y_{n-m+l}$$
$$+\left(\beta^2\left(\frac{1}{2}+\frac{\alpha}{2}+\frac{\alpha^2}{4}\right)\right)\sum_{l=-r_1}^{s_1}L_l(c)y_{n-2m+l}+\left(\beta^3\left(\frac{1}{6}+\frac{\alpha}{6}\right)\right)\sum_{l=-r_1}^{s_1}L_l(c)y_{n-3m+l}$$
$$+\left(\frac{\beta^4}{24}\right)\sum_{l=-r_1}^{s_1}L_l(c)y_{n-4m+l}$$

To obtain the stability region of the method, the delay term is approximated using five points Lagrange interpolation. By putting $n-m+l=0$, $n-2m+l=0$, $n-3m+l=0$, $n-4m+l=0$ and by taking $l=-1,0,1,2,3$, the stability polynomial will be in the standard form. The recurrence is stable if the zeros ζ_i of the stability polynomial

$$S(\alpha,\beta:\zeta)=\zeta^{n+1}-\left(1+1+\alpha+\frac{\alpha^2}{2!}+\frac{\alpha^3}{3!}+\frac{\alpha^4}{4!}\right)\zeta^n$$
$$-\left(\beta\left(1+\alpha+\frac{\alpha^2}{2}+\frac{\alpha^3}{6}\right)\right)(L_{-1}(c)+L_0(c)\zeta+L_1(c)\zeta^2+L_3(c)\zeta^4)$$

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$$-\left(\beta^2\left(\frac{1}{2}+\frac{\alpha}{2}+\frac{\alpha^2}{4}\right)\right)(L_{-1}(c)+L_0(c)\zeta+L_1(c)\zeta^2+L_2(c)\zeta^3+L_3(c)\zeta^4)$$
$$-\left(\beta^3\left(\frac{1}{6}+\frac{\alpha}{6}\right)\right)(L_{-1}(c)+L_0(c)\zeta+L_1(c)\zeta^2+L_2(c)\zeta^3+L_3(c)\zeta^4)$$
$$-\left(\frac{\beta^4}{24}\right)(L_{-1}(c)+L_0(c)\zeta+L_1(c)\zeta^2+L_2(c)\zeta^3+L_3(c)\zeta^4)$$

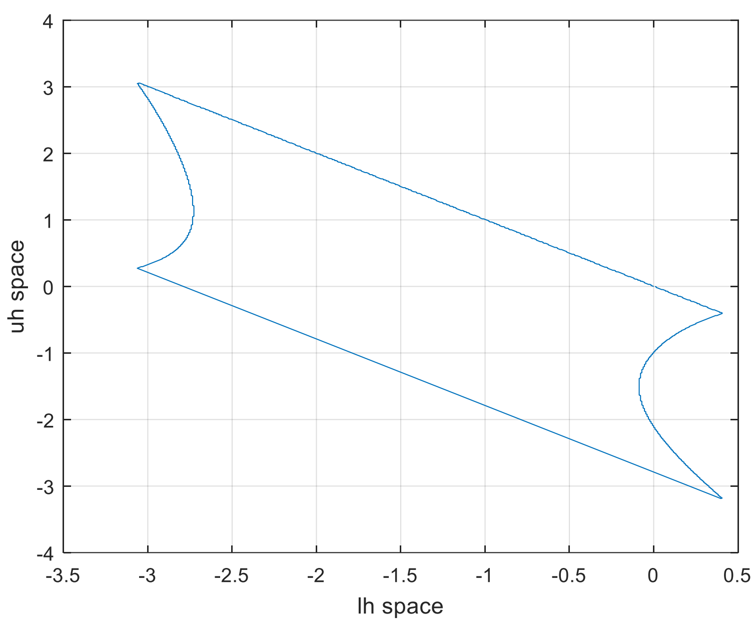
satisfies the root condition $|\zeta_i| \le 1$.

Then the stability polynomial for this method is

$$S(\alpha,\beta:\zeta)=\zeta^{n+1}-\left(1+1+\alpha+\frac{\alpha^2}{2!}+\frac{\alpha^3}{3!}+\frac{\alpha^4}{4!}\right)\zeta^n$$
$$-\left(\beta+\frac{\beta^2}{2}+\frac{\beta^3}{6}+\frac{\beta^4}{24}+\frac{\alpha\beta^2}{2}+\frac{\alpha^2\beta}{2}+\frac{\alpha\beta^3}{6}+\frac{\alpha^3\beta}{6}+\frac{\alpha^2\beta^2}{4}+\alpha\beta\right)\zeta$$

The corresponding stability region is given in Figure 1.

Figure 1. Stability region of ISM4



4 Numerical Examples

Problem 1: (Stiff linear system with multiple delays)

$$y_1'(t)=-\frac{1}{2}y_1(t)-\frac{1}{2}y_2(t-1)+f_1(t),$$
$$y_2'(t)=-y_2(t)-\frac{1}{2}y_1\left(t-\frac{1}{2}\right)+f_2(t),\quad 0\le t\le 1$$

with initial conditions,

$$y_1(t)=e^{-t/2},\qquad \frac{-1}{2}\le t\le 0,$$
$$y_2(t)=e^{-t},\qquad -1\le t\le 0$$

and $f_1(t)=\frac{1}{2}e^{-(t-1)},\quad f_2(t)=\frac{1}{2}e^{-(t-1/2)/2}$

The exact solution is,

$$y_1(t)=e^{-t/2},\qquad y_2(t)=e^{-t}$$

By taking the step-size h = 0.01 and comparing this method with RSM4 [14], the approximate value and the absolute error are given in Tables 1-4.

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Table 1 Solution of y_1 in Example 1

t	y_1 (ISM4)	y_1 (RSM4)	Exact
0.1	0.9512294215	0.9512296234	0.9512294245
0.2	0.9048374124	0.9048377958	0.9048374180
0.3	0.8607079684	0.8607085130	0.8607079764
0.4	0.8187307429	0.8187314275	0.8187307531
0.5	0.7788007710	0.7788015730	0.7788007831
0.6	0.7408182069	0.7408191019	0.7408182207
0.7	0.7046880744	0.7046890359	0.7046880897
0.8	0.6703200294	0.6703210286	0.6703200460
0.9	0.6376281338	0.6376291398	0.6376281516
1.0	0.6065306409	0.6065316206	0.6065306597

Table 2 Absolute errors of y_1 in Example 1

t	ISM4	RSM4
0.1	2.957519e-09	1.988753e-07
0.2	5.626559e-09	3.777464e-07
0.3	8.028223e-09	5.365609e-07
0.4	1.018224e-08	6.744531e-07
0.5	1.210706e-08	7.899672e-07
0.6	1.381991e-08	8.812456e-07
0.7	1.533689e-08	9.461861e-07
0.8	1.667303e-08	9.825730e-07
0.9	1.784236e-08	9.881841e-07
1.0	1.885797e-08	9.608773e-07

Table 3 Solution of y_2 in Example 1

t	y_2(ISM4)	y_2 (RSM4)	Exact
0.1	0.9048368913	0.9048384551	0.9048374180
0.2	0.8187297774	0.8187326031	0.8187307531
0.3	0.7408168651	0.7408206835	0.7408182207
0.4	0.6703183714	0.6703229384	0.6703200460
0.5	0.6065287198	0.6065338108	0.6065306597
0.6	0.5488094782	0.5488148852	0.5488116361
0.7	0.4965829696	0.4965884992	0.4965853038
0.8	0.4493264901	0.4493319631	0.4493289641
0.9	0.4065670780	0.4065723290	0.4065696597
1.0	0.3679085406	0.3790854026	0.3679074412

Table 4 Absolute errors of y_2 in Example 1

t	ISM4	RSM4
0.1	5.267804e-07	1.037073e-06
0.2	9.756611e-07	1.850012e-06
0.3	1.355611e-06	2.462844e-06
0.4	1.674646e-06	2.892396e-06
0.5	1.939944e-06	3.151105e-06

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0.6	2.157888e-06	3.249069e-06
0.7	2.334192e-06	3.195370e-06
0.8	2.473976e-06	2.998959e-06
0.9	2.581763e-06	2.669257e-06
1.0	1.120596e-05	1.120596e-05

Figure 2: The plot of the error generated via ISM4 and RSM4 by using Table 2

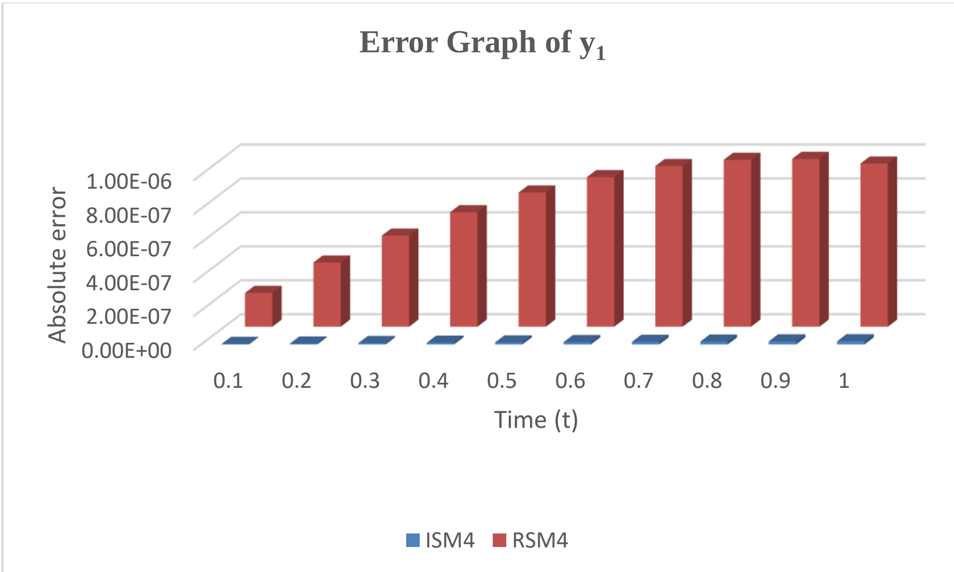
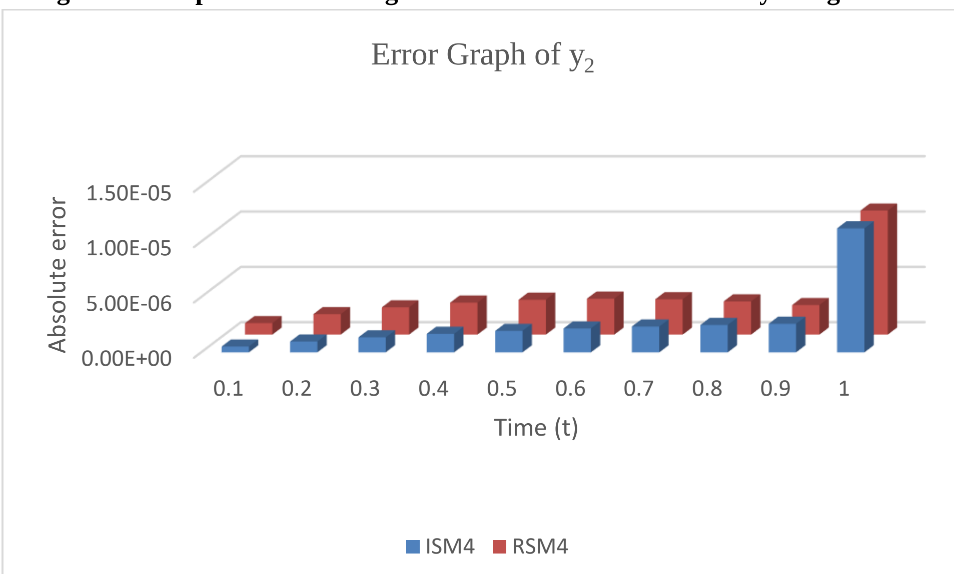


Figure 3: The plot of the error generated via ISM4 and RSM4 by using Table 4



Problem 2: (Time-dependent delay)

$$y'(t) = \frac{t-1}{t} y(\ln(t) - 1)y(t), \qquad t \geq 1$$

with initial condition

$$y(t) = 1, \qquad t \leq 1$$

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and the exact solution is,
$y(t) = \exp(t - \ln(t) - 1), \qquad t \geq 1$
By taking the step-size $h = 0.01$ and comparing this method with RSM4, the approximate value and the absolute error are given in Tables 5-6.

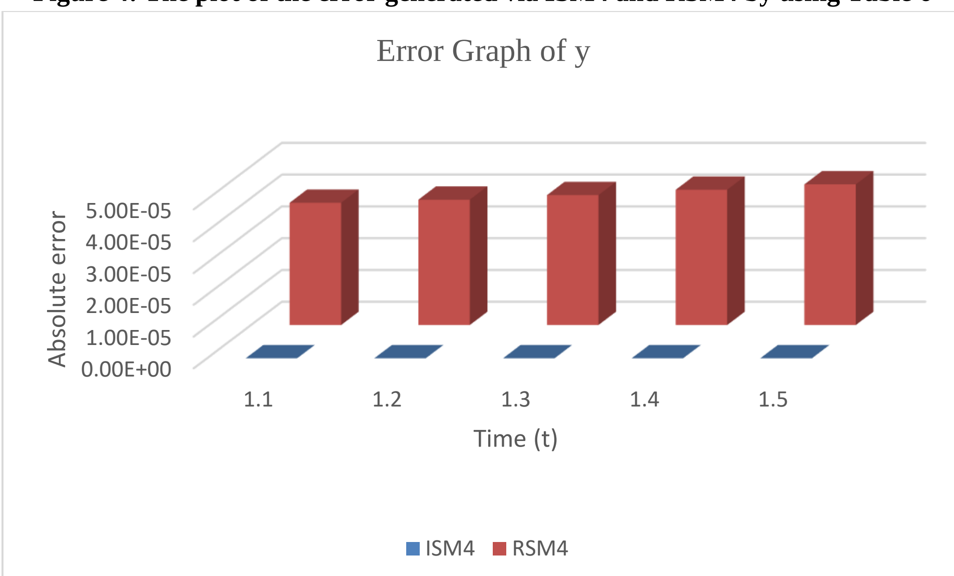
Table 5 Numerical results of y in Example 2

t	y (ISM4)	y (RSM4)	Exact
1.1	1.0047008340	1.0046623512	1.0047008346
1.2	1.0178356287	1.0177962004	1.0178356318
1.3	1.0383529211	1.0383120404	1.0383529289
1.4	1.0655890552	1.0655465006	1.0655890697
1.5	1.0991474904	1.0991032513	1.0991475138

Table 6 Absolute errors of y in Example 2

t	ISM4	RSM4
1.1	5.816350e-10	3.848338e-05
1.2	3.086207e-09	3.943139e-05
1.3	7.775001e-09	4.088848e-05
1.4	1.458287e-08	4.256916e-05
1.5	2.340341e-08	4.426253e-05

Figure 4: The plot of the error generated via ISM4 and RSM4 by using Table 6



Problem 3: (State-dependent delay)
$y'(t) = \cos(t) y(y(t) - 2), \qquad t \geq 0$
with initial condition,
$y(t) = 1, \qquad t \leq 0$
and the exact solution is,
$$y(t) = \sin(t) + 1, \qquad 0 \leq t \leq 1$$

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By taking the step-size $h = 0.01$ and comparing this method with RSM4, the approximate value and the absolute error are given in Tables 7-8.

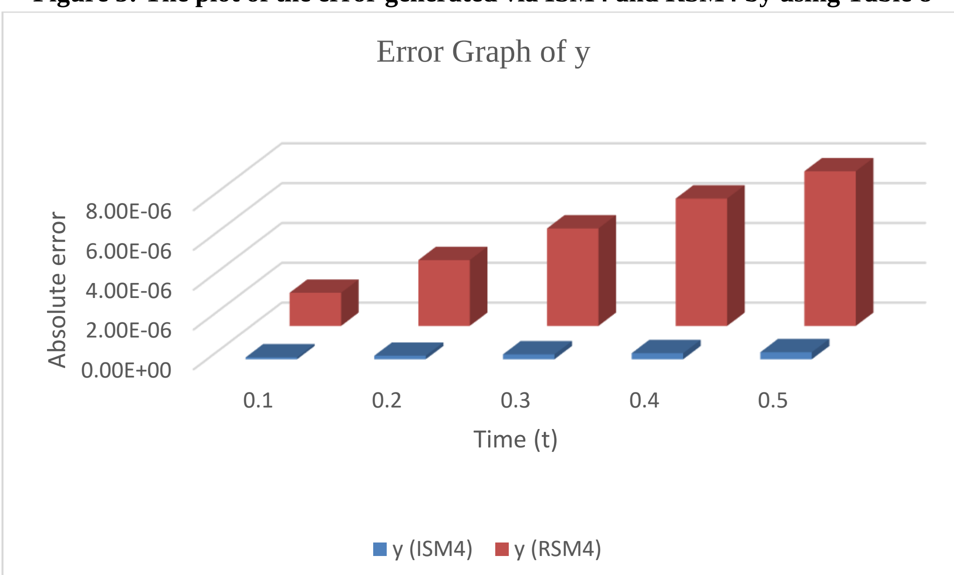
Table 7 Numerical results of y in Example 3

t	y (ISM4)	y (RSM4)	Exact
0.1	1.0998335116	1.0998350817	1.0998334166
0.2	1.1986695084	1.1986726385	1.1986693308
0.3	1.2955204523	1.2955251047	1.2955202067
0.4	1.3894186471	1.3894247418	1.3894183423
0.5	1.4794258925	1.4794333077	1.4794255386

Table 8 Absolute errors of y in Example 3

t	y (ISM4)	y (RSM4)
0.1	9.492799e-08	1.665101e-06
0.2	1.775677e-07	3.307686e-06
0.3	2.455909e-07	4.898041e-06
0.4	3.047433e-07	6.399451e-06
0.5	3.538607e-07	7.769094e-06

Figure 5: The plot of the error generated via ISM4 and RSM4 by using Table 8



5 Conclusion

In this paper, a new one-step method has been developed to solve the Delay differential equations (DDEs) by means of interpolating function which consists of inverse functions. The delay argument is approximated using Lagrange interpolation. The stability polynomial of this method and the corresponding stability region have been determined. Numerical examples of stiff and non-stiff DDEs with constant delay, time dependent delay and state dependent delays have been considered to demonstrate the efficiency of this method. The numerical results of ISM4 and RSM4 are shown in Tables 1 – 8. From Figures 1 - 4, it is obvious that the absolute errors in ISM4 are lesser than RSM4 method. This proves that the new method has an edge over RK5 scheme in terms of accuracy. Hence, it is concluded that the proposed scheme is consistent, highly stable, convergent and very much applicable for solving stiff linear and non-linear stiff IVPs in DDEs.

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