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A comparative study on Real Life Application of Fuzzy Time Cost Trade-off problems

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ABSTRACT
This research paper deals out with the real life application of fuzzy time cost trade-off problems using triangular fuzzy number. A house construction project of 1020 square feet has been taken as the network, quotations form three builders is taken into account as triangular fuzzy variable. A comparative study has also been carried out between the defuzzication or ranking method and decomposition -aggregated method.

Keywords: Fuzzy theory, fuzzy numbers, fuzzy time cost trade-off problems, fuzzy linear programming problem, decomposition techniques, aggregation of m-LPPs, ranking method of triangular fuzzy number.

1. Literature review

The tradeoff between the project cost and the project completion time and the uncertainty of the environment are considerable issues for all real life project decision makers. Project management is most important fields in business and industry, a special aspect of project management is to schedule the time accurately. In the literature, there are various approaches proposed over the past years to find the optimum duration with minimum cost. Zadeh [14] introduced the concept of fuzzy sets and nowadays all research areas have depended on the development of the minimal cost with optimal duration. James E. Kelley[9] was the first to introduce the critical path scheduling and planning and followed by that Ghazanfari et al.[8] introduced the new optimal method for fuzzy time cost trade off problem using goal programming problem. P. Pandian et al. [11] proposed a new method called decomposition method to solve integer linear programming problems by using triangular fuzzy numbers and also a new approach to fuzzy network crashing in a project network whose activity durations are fuzzy finding an optimal duration without converting the fuzzy activity duration to classical number was proposed by Shakeela sathish et al. [12] , Evangeline Jebaseeli et al.[6] proposed a new method for time cost trade off problems in which both times and costs are fuzzy numbers in the same era. Evangeline Jebaseeli et al.[5] introduced an algorithm to solve fully fuzzy time cost trade off models through multi objective linear programming technique. Aggregated techniques of m-LPPs was proposed by Antony raj et al[4].

In this research article we give out a comparative study for fuzzy time cost trade off problem using decomposition and aggregated techniques and graded mean integration ranking method to obtain the optimal solution of the project with numerical illustration is carried out.

2. Preliminaries

Definition 1
The characteristic function μ_A in a crisp set A⊆S assigns a value either 0 or 1 for each member in S. The function is generalised to a function $\mu_{\tilde{A}}$ such that the value assigned with the element of S lies within a specified range i.e. $\mu_{\tilde{A}} : S \to [0,1]$. The assigned values $\mu_{\tilde{A}}(s)$for each s∈S denote the membership grade of the element in the set A. The set $\tilde{A} = \{A, \mu_A(x) : x \in X\}$ is called Fuzzy Set.

Definition 2
Triangular fuzzy number is a fuzzy number represented with three points as follows:
$\tilde{A} = (g_1, g_2, g_3)$ This representation is interpreted as membership functions:
We use F(R) to denote the set of all triangular fuzzy numbers.

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$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & if\ \ x<g_1 \ \ and\ \ g_3>x \\ \dfrac{x-g_1}{g_2-g_1} & if\ \ g_1\le x<g_2 \\ \dfrac{g_3-x}{g_3-g_2} & if\ \ g_2\le x\le g_3 \end{cases}$$

Definition 3

Let (g_1,g_2,g_3) and $(h_1,\mathrm{h}_2,\mathrm{h}_3)$ be two triangular fuzzy numbers. Then

$$(g_1,g_2,g_3)\oplus\ (h_1,h_2,h_3)\ =\ (g_1+h_3,g_2+h_2,g_3+h_1)$$

$$(g_1,g_2,g_3)-(h_1,h_2,h_3)=(g_1-h_3,g_2-h_2,g_3-h_1)$$

$$(g_1,g_2,g_3)=(cg_1,cg_2,cg_3)\ ,\ \ for\, c\ge 0.$$

$$c(g_1,g_2,g_3)=(cg_3,cg_2,cg_1)\ ,\ \ for\ \ c<0.$$

$$\frac{(g_1,g_2,g_3)}{(h_1,h_2,h_3)}=\left(\frac{g_1}{h_3},\frac{g_2}{h_2},\frac{g_3}{h_1}\right)$$

Definition 4

Let F(R) represents the set of triangular fuzzy numbers. Define a ranking function $\mathcal{R}$: F (R)→R maps triangular fuzzy numbers into R. Let $\tilde{A}=(f,g,h)$ be a triangular fuzzy number, and then Graded Mean Integration Representation (GMIR) method to defuzzify the number is noted as $R(\tilde{A})=\left(\dfrac{f+4g+h}{6}\right)$.

Definition 5

Linear programming problem is one among the most habitually applied operations research technique by assuming that all variables and parameters are real numbers. But in real life circumstance we do not have proper data. So, the fuzzy variables and fuzzy numbers are used in Linear programming problem. The standard form fully fuzzy linear programming problems with n fuzzy variables and m fuzzy constants are given below:

$$Maximimize\ or\ (Minimize)\ (\tilde{\mathrm{A}}^T\otimes\tilde{Y})$$

Subject to $\tilde{B}\tilde{Y}=\tilde{d}$

$\tilde{Y}$ is a non-negative fuzzy number.

$$\tilde{A}^T=\tilde{a}_{j_{1xn}}\tilde{Y}=\tilde{y}_{i_{nx1}},\tilde{\mathrm{B}}=[\tilde{\mathrm{b}}_{ij}]_{mxn},\tilde{d}=[\tilde{d}_i]_{mx1}and$$

Where $\tilde{c}_j,\tilde{y}_j,\tilde{b}_{ij},\tilde{d}_i\in F(R)$

$$wherei=1,2,\ldots\mathrm{m}\&\ j=1,2,\ldots\mathrm{n}$$

Definition 6

A fuzzy project network can be defined by an activity-on-activity arc network P=(N,L) where N={1,2,...,m} is the set of nodes(points) and A is the set of arcs(oriented lines) represents the activities. In the fuzzy project network, node 1 and n denotes the initial and terminal of the project respectively. The complete fuzzy Mathematical model for fully fuzzy time cost trade-off problems is given as follows:

$$\mathrm{Min}\tilde{\mathrm{Z}}=\sum_k\sum_l A_{kl}$$

$$subject\,to$$

$$\tilde{D}_1=0,\tilde{\mathrm{D}}_l-\tilde{D}_k-\tilde{y}_{kl}\ge 0,\tilde{D}_m\le\tilde{D};\tilde{a}_{kl}=\tilde{s}*(N\tilde{D}_{kl}-\tilde{y}_{kl}),A\tilde{D}_{kl}\le\tilde{y}_{kl}\le N\tilde{D}_{kl}$$

$$\forall(\mathrm{k},\mathrm{l})\in P,\tilde{A}_{kl}=\sum_k\sum_l\tilde{a}_{kl}+\tilde{I}*(\tilde{D}_m-\tilde{D}_1)+\sum m\tilde{K}_m;\quad Where\,a=(1,2,\ldots m)\ and\ b=(1,2,\ldots m).$$

Theorem 1

A triangular fuzzy number $\tilde{y} = (\tilde{y}_1, \tilde{y}_2, \tilde{y}_3)$ is an optimal result of the problem (Q) if and only if $\tilde{y}_1, \tilde{y}_2$ *and* $\tilde{y}_3$ are optimal results of the prescribed crisp linear programming problems (Q2), (Q1) and (Q3) respectively where:

(Q) Maximize $\tilde{Z} = Ay$ Subject to $\tilde{B}y \le \tilde{d}, y \ge 0$

(Q2) Maximize $Z_2 = Ay_2$ Subject to $By_2 \le d_2, y_2 \ge 0$

(Q1) Maximize $Z_1 = Ay_1$ Subject to $By_1 \le d_1, y_1 \ge 0, y_1 \le y_2$

(Q3) Maximize $Z_3 = Ay_3$ Subject to $By_3 \le d_3, y_3 \ge 0, y_3 \ge y_2$

Aggregation of m-LPPs :

Notations

k	:	*k^{th} problem (k=1,2,…m)*
l	:	*l^{th} problem (l=1,2,…n_k)*
y_{kl}	:	*l^{th} variable of the k^{th} problem*
a_{kl}	:	*constant coefficient of the l^{th} variable of the k^{th} problem*
n_k	:	*Number of variables in the k^{th} problem*
r_k	:	*Number of constraints in the k^{th} problem*
d_{kr_k}	:	*RHS value of the r_k^{th} constraints of the k^{th} problem*

General LPP structure of the k^{th}- problem (k=1,2,...m) can be given as:

$$Max\,Z_k = a_{k1}\,y_{k1} + a_{k2}y_{k2} + \ldots + a_{kn_k}\,y_{kn_k}$$

Subject to the constraints:

$$b_{k11}y_{k1} + b_{k12}y_{k2} + \ldots + b_{k1n_k}\,y_{kn_k}\,\{\le,=,\ge\}\,d_{k1}$$
$$b_{k21}y_{k1} + b_{k22}y_{k2} + \ldots + b_{k2n_k}\,y_{kn_k}\,\{\le,=,\ge\}\,d_{k2}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$b_{ki_k1}y_{k1} + b_{ki_k2}y_{k2} + \ldots + b_{ki_kn_k}\,y_{kn_k}\,\{\le,=,\ge\}\,d_{ki_k}$$
$$y_{kl} \ge 0, \{k = 1,\ldots.m\ ,\ l = 1,2,\ldots,n_k\}$$

Aggregated structure of m-LPPs together

$$Max\,Z = \sum_{k=1}^{m}\sum_{l=1}^{n_k} a_{kl}y_{kl}$$

Subject to the constraints:

$$b_{11}y_{11} + b_{12}y_{12} + \ldots + b_{11n1}y_{1n1}\{\le,=,\ge\}d_{11}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$b_{1i_{11}}y_{11} + b_{1i_{12}}y_{12} + \ldots + b_{1i_{1n1}}\ y_{1n1}\{\le,=,\ge\}d_{1k_1}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$b_{m_{11}}y_{11} + b_{m_{12}}y_{12} + \ldots + b_{m_{1n1}}y_{1n1}\{\le,=,\ge\}d_{m_1}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$b_{m_{km1}}y_{11} + b_{m_{km2}}y_{12} + \ldots + b_{m_{kmn1}}y_{mn1}\{\le,=,\ge\}d_{m_{km}}$$

$$x_{kl} \ge 0,\ \{k = 1,\ldots.m,\ \boldsymbol{l} = 1,2,\ldots,n_k\}.$$

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3. Algorithm

Step 1 Find the direct cost and the cost slope of the fuzzy time cost trade-off problem using triangular fuzzy number

Step 2 Convert the fuzzy project network into fuzzy linear programming problem with the use of fully fuzzy mathematical structure.

Step 4 Use graded mean ranking method to convert fuzzy linear programming problem into crisp linear programming problem.

Step 5 The optimum result of the crash cost and crash duration for all the activities can be found in the respective variables.

Step 6 Split up fuzzy linear programming problem into crisp linear programming problems by using decomposition techniques.

Step 7 Aggregation of m-LPPs is used to aggregate the crisp linear programming problems into single linear programming problem.

Step 8 The optimum result of the crash cost and crash duration for all the activities can be found in the respective variables.

Step 9 Compare the result obtained in Step 5 and step 8.

4. Numerical Template

List of activities for house construction on 1020 square feet is given below with the relevant data details. Table 1 elaborates the details of the project. Quotation of the project has been got from three builders and considered as triangular fuzzy variables. Indirect cost of the project per day is (250,250,250). The project manager wishes to complete within (140,141,142) days (ie) around 4 to 5 months. Activities information is given in table 2.

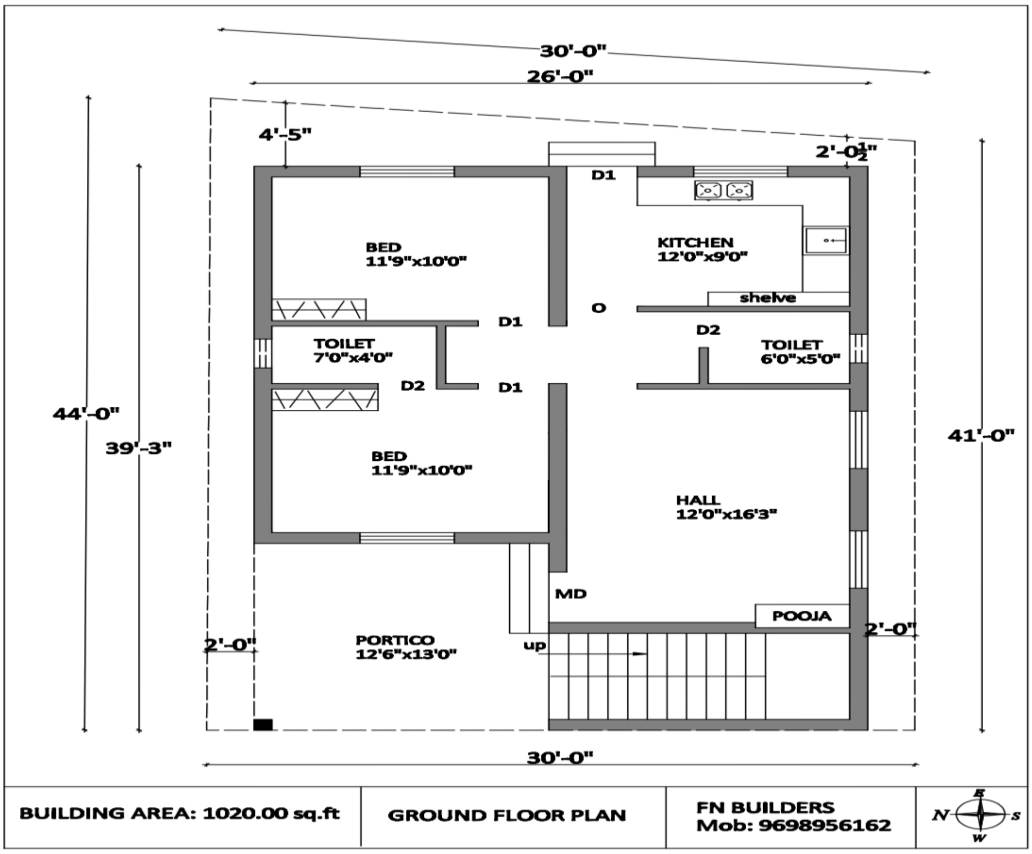

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Table 1

Activities	Description
1-2(a)	Site preparation
1-3(b)	Levelling work
2-4(c)	Excavation and PPC
3-4(d)	Barpending work
4-5(e)	Foundation plinth beam
5-6(f)	Super structure column construction
6-7(g)	Brick masonry work
7-8(h)	Lintel over door window gaps
8-9(i)	Roof structure
9-10(j)	Electrical and plumbing work
9-11(k)	Fall ceiling
10-12(l)	Plastering
11-12(m)	Modular kitchen
12-13(n)	Door windows framing and fixation
12-14(o)	Painting

Quotation 1:

F.N. Builders

Work Description		**Cost**
House Area [1020 x 2200]		
a) Ground area as per drawing	22,44,000	
b) Basement 3 ½ feet		
Sump		50,000
Head room		90,000
Electricity board	7,500	
Motor and accessories (Crompton) 150 ft	34,000	
Elevation and gate		90,000
Fall ceiling hall only		65,000
Modular kitchen	65,000	
PVC cub board	30,000	
3 cub board door	40,000	
Total		**27,15,500**

Quotation 2:

Santhiya Builders

Work Description		**Cost**
House Area [1020 x 2100]		
c) Ground area as per drawing	21,42,000	

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d) Basement 3 ½ feet

Septic tank 45,000

Electricity board 10,000

Motor and accessories (Crompton) 150 ft 38,000

Elevation and gate 1,00,000

Fall ceiling hall only 70,000

Modular kitchen 70,000

PVC cub board 45,000

3 cub board door 45,000

Total **25,65,000**

Arun Builders

Work Description **Cost**

House Area [1020 x 1900]

e) Ground area as per drawing 19,38,000
f) Basement 3 ½ feet

Sump 50,000

Head room 1,10,000

Electricity board 7,500

Motor and accessories (Crompton) 150 ft 34,000

Elevation and gate 1,50,000

Fall ceiling hall only 75,000

Modular kitchen 75,000

PVC cub board 25,000

3 cub board door 50,000

Total **25,14,500**

By using the above quotations cost, duration and cost slope for each activity is given as below

Table 2

Activities	Normal Duration	Normal Cost	Cost Slope($\frac{\Delta c}{\Delta t}$)
1-2(a)	(2,2,2)	(26600,28000,30000)	-
1-3(b)	(3,3,3)	(18000,19000,20000)	-
2-4(c)	(4,4,4)	(20000,25000,30000)	(5000,10000,15000)
3-4(d)	(3,3,3)	(18000,19000,20000)	-
4-5(e)	(25,26,30)	(200000,220000,300000)	(0,9000,12500)

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5-6(f)	(18,21,25)	(300000,350000,400000)	(15000,16667,50000)
6-7(g)	(19,20,25)	(550000,570000,600000)	(0,6000,12500)
7-8(h)	(18,19,20)	(450000,490000,500000)	(3333,10000,20000)
8-9(i)	(18,19,20)	(270000,290000,300000)	(0,6667,6667)
9-10(j)	(2,2,2)	(48000,49000,50000)	(0,20000,30000)
9-11(k)	(2,2,2)	(36000,38000,40000)	-
10-12(l)	(3,4,5)	(90000,95000,100000)	-
11-12(m)	(8,9,10)	(180000,190000,200000)	(0,10000,13333)
12-13(n)	(8,9,10)	(150000,155000,200000)	(0,75000,95000)
12-14(o)	(10,11,12)	(80000,85000,100000)	(5000,25000,40000)

Total duration of the proect is (132,142,160)

Direct cost of the project is (2262600,2441000,2650000)

Total cost of the project is (2295600,2476500,2690000)

Graded Mean Integration ranking method is used to defuzzify the duration cost and slope cost of the activities

Table 3

Activities	Normal Duration	Normal Cost	Cost Slope($\frac{\Delta c}{\Delta t}$)
1-2(a)	2	28100	-
1-3(b)	3	19000	-
2-4(c)	4	25000	10000
3-4(d)	3	19000	-
4-5(e)	26.5	221667	8083.33
5-6(f)	20.5	35000	21944.67
6-7(g)	20.67	571667	6083.33
7-8(h)	19	485000	10555.5
8-9(i)	19	288333	7222.5
9-10(j)	2	49000	18333.33
9-11(k)	2	38000	-
10-12(l)	4	95000	-
11-12(m)	9	190000	8888.83
12-13(n)	9	161667	65833.33
12-14(o)	11	86667	24166.67

Total duration of the project is 142.67

Direct of the project is 24,46,100.01

Total cost of the project is 24,81,850

5. Conclusion

In this research article we made a comparative study between graded mean integration method and decomposition - aggregated techniques. Hence it is found that optimal solution obtained by both the methods is approximate to each other. We can able to get a crisp solution by using both the methods for fuzzy time cost trade-off problems.

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