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Ant Colony Optimization Used in Backward Production Scheduling-Single Stage Process

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ABSTRACT

Scheduling is the process of arranging, controlling and optimizing work and workloads in a production process or manufacturing process. Scheduling is used to allocate plant and machinery resources, plan human resources, plan production processes and purchase materials. It is an important tool for manufacturing and engineering, where it can have a major impact on the productivity of a process. In manufacturing, the purpose of scheduling is to minimize the production time and costs, by telling a production facility when to make, with which staff, and on which equipment. Production scheduling aims to maximize the efficiency of the operation and reduce costs. In some situations, scheduling can involve random attributes, such as random processing times, random due dates, random weights, and stochastic machine breakdowns. In this case, the scheduling problems are referred to as "stochastic scheduling." The goal of the scheduling framework is to distribute assignments (orders or "occupations") to assets and organize them as efficiently and financially as possible . In various cases, a good solution that is quickly found is more likely to prefer. In situations like mentioned above, the use of metaheuristics is an appropriate strategy. In recent years, such tools have been used to build some off-the-shelf systems. Here we investigate the development of a task shop booking work that utilizes hidden Ant Colony Optimization in a regressive planning issue in an assembling situation with single-stage preparing, equal assets, and adaptable paths. This situation was found in an enormous grocery industry. This work shows the necessity of this computerized reasoning method. Truth be told, Ant Colony Optimization has ended up being as efficient as branch-and-bound, be that as it may, executing a lot quicker.

1. Production Scheduling

The globalized world's economic scenario makes competitiveness unavoidable and being competitive has become an indispensable prerequisite to organizations that look into for success. Going ahead with the context, manufacturing activities become important, for they decisively influence performance, directly affecting (and being affected) by forecast, planning, scheduling decisions.

(i) Ant Colony Optimization (Metaheuristic), was used in an efficient method to solve a specific production scheduling

(ii) The tools with different output capacities are used in parallel.

(iii) Backward scheduling (based on dispatch dates) and

(iv) Things with all supply (JSS) routing options.

This manufacturing method was discovered in a massive grocery industry that is found in many countries around the world. According to a literature review, the ACO Metaheuristic is something which solves complex output scheduling problems. For example, an ACO algorithm for smart manufacturing was proposed by C. J. Liao and C. C. Liao [2] Shyu et al. [3] suggested using ACO to solve a work shop scheduling problem with two computers. Two ACO scheduling processes for flow-shops were studied by Rajendran and Ziegler [4]. Bauer et al. [5] used ACO to solve a single-machine production scheduling problem. Lin et al. [6] conducted a study for production scheduling using Ant Colony Optimization and suggested Production schedules are evaluated according to the performance measures stated below for reviewing the implemented Ant Colony Optimization algorithm.:

(a) The maximum time it takes to complete a task, also known as the makespan.

(b) The time it takes a machine to create a production plan (effort).

2. ACO METAHEURISTIC

In this paper, we will address on a brief note the application of the Ant Colony Optimization metaheuristic to the travelling salesman problem (TSP) and to the output scheduling.

2.1. Travelling Salesman Problem

In Ant Colony Optimization, in need of food, a certain number of ants leaving their nest and there are several different paths they can take to get there. Ants secrete a substance called pheromone during their walk, which attracts other ants to take food. Ants can make a certain amount of journeys away from the nest to the food source and back to the nest. Each trip leaves a certain amount of pheromone behind. If the Ant's path does not provide any improvements over the best previous trail, there will be a regular measure of pheromone; otherwise, if the ant's path is shorter than the previous best path, there will be a greater quantity of pheromone. In the meantime, evaporation continues to reduce the amount of pheromone available. The direction is chosen solely on the basis of chance, which is dependent on the amount of pheromone on each arc and its distance. It is important for the small business to have a website. Meanwhile, there's a Traveling Salesman on the loose. The problem consists of a collection of areas that must be visited only once by any agent who must return to the origin location after completing a loop. The aim of this issue is to figure out how to frame a visit that takes you through all of the city's neighborhoods. This distance is represented as

$$d_{mn} = \sqrt{(X_m - X_n)^2 + (Y_m - Y_n)^2}$$

We assume that there are m ants in the system, having the below mentioned characteristics.

(i) Determines the next city to be visited based on the distance between cities and the amount of pheromone in the arcs that

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connect them.

(ii) Transfers to those already visited places are discarded, before a tour is completed.

(iii) An ant deposits a certain level of pheromone when a loop is completed on the arc *(i, j)*

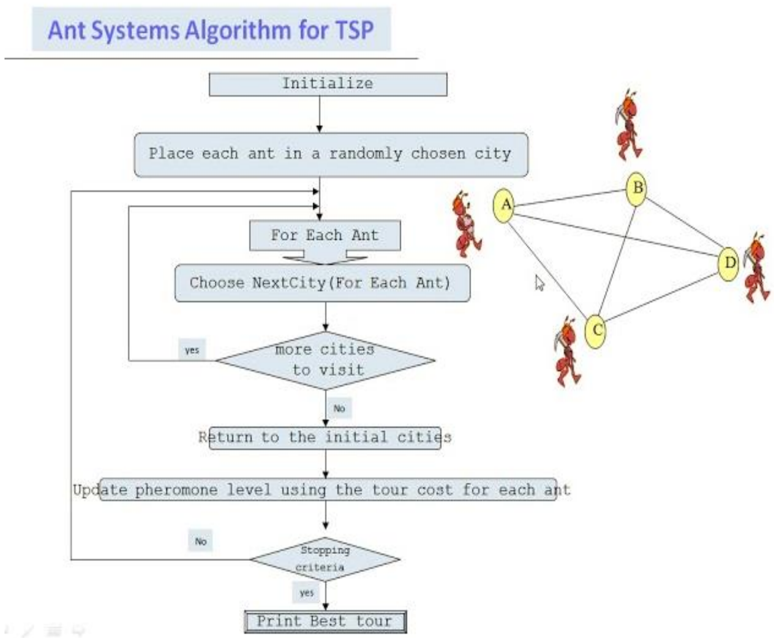

Figure 1 : An example of an Ant System algorithm for Travelling Salesman Problem.

Let $\tau_{ij}(t)$ be the intensity of pheromone in arc (*i*, *j*) in time *t*. Each ant in time *t* chooses the next city to which it will go in time $(t+1)$. Defining one ACO iteration as the *n* movements realised by *m* ants in the interval (*t*, *t*+1), then the *n* iterations of each ant form a loop, that is, each ant realises a tour passing by all the cities. On every one, the intensity of the pheromone is updated by

$$\tau_{ij}(t+n) = \ell\,\tau_{ij}(t) + \Delta\tau_{ij} + \Delta\tau_{ij}, \qquad (2)$$

where ρ is a coefficient (constant) with (1-ρ) representing the pheromone fading between the times *t* and (*t* + *n*) of the arc (*i*, *j*) and $\Delta\tau_{ij} = \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$, where $\Delta\tau_{ij}^{k}$ is the quantity of the pheromone deposited in the arc (*i*, *j*) by the *k*-th ant between the times *t*

.

The law in order to satisfy the limitation that each ant visits *n* different cities is to associate to each ant a list, called tabu list, which supplies the cities which have already been visited and prohibits the ant visit them over before the tour has already been concluded. When a tour is completed, the tabu list is used to calculate the present solution of the ant (i.e., the distance travelled in the path). It is defined tabu as for the vector which grows dynamically and contains the tabu list of *k*-th and and tabu, the *s*-th city visited by ant *k* at the present tour.

The probability of transaction from one city to another city by the *k*-th ant is given as

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$$P_{ij}{}^{k}(t) = \begin{cases} \dfrac{(\tau_{ij}(t))^{\alpha}.(\eta_{ij})^{\beta}}{\sum_{k \in allowed_k} (\tau_{ik}(t))^{\alpha}.(\eta_{ik})^{\beta}}, & \text{if} \quad j \in permitted_k, \\ 0 \end{cases}$$

Where permitted k=N-tabu k, N denotes the number of cities in the problem, and denote the variables that govern the relative importance of pheromone strength versus appealingness. As a result, the business deal likelihood is a function of the pheromone's appealingness and strength at time t.

There are many ways to calculate, according to Dorigo et al. [8].

$$\Delta_{ij}{}^{k}(t) = \begin{cases} \dfrac{Q_1}{L_{k_1}}, & \text{if the k}_1\text{-th ant goes through the arc (i,j)} \\ 0 \end{cases}$$

Where Q_1 is called the constant and L_k is the length of the path taken by the k_1-th ant .

2.1 Role of ACO Metaheuristic in Production Scheduling

It was decided that a disjunctive graph by Dorigo et al. [9], Ventresca and Ombuki [10], and Mazzucco Jr. [11] would be used in ant systems to demonstrate the production scheduling problem This graph has the formula $Q_1 = (V_1, A_1, E_1)$, where V denotes the graph's set of vertices, which corresponds to a collection of procedures to be scheduled, and O denotes the number of operations to be scheduled.. Two fictitious operations, described as "0" and *"N + 1,"* are also added to the set V_1, that is, $V_1 = \{0, O, N_1 + 1\}$, representing the origin (nest) and the terminus (food) nodes.

'A' will be a set of curves that connect continuous cycles from a previous project and J3 individually. The underlying and final activities are indicated by the circular segments that connect activity 0 to each work's primary genuine activity and activity (N_1 + 1) to each work's final activity. '*E'* is a set of the edges connecting two operations to be achieved by the same resource (a machine) and it may be given as $E = \{(v,w)/M_v = M_w\}$. Each curve has a pair of numbers $\{\tau_{ij}, \eta_{ij}\}$ which is a pair of absorption of pheromone and reflectivity.

3. The manufacturing consequence considered in this work

Enhancing creation planning frameworks with a single regulation level, equal properties, and adaptable routings is a part of the project. Every product has a single operation that may necessitate the use of more than 1 resource or above that. As a result, each job J only has one process. All machines in the shops are capable of processing at most 1 job at a time as told by J. Vilma Roseline and D. Saravanan [12]. Within a minimum makespan, P = $J_1, J_2, ..., J_n$, the considered productive system is characterized by having parallel resources, that is, an operation can be formed by more than one machine or dynamic resources . As a result, there are M machines available, where M = $\{M_1, M_2, .., M_m\}$. It is worth noting that such

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machines vary in capability and efficiency, resulting in the device having different processing capacities for the same product. Another essential feature of the considered efficient method is that goods can be produced in a variety of ways. This implies that there could be several process plans for a given product. It is assumed in this work that each job J can be processed by any of the M machines, that is, each job J has a flexible routing . Schematically, $J_1 = \{RF_1\}$, $J_2 = \{RF_2\}$, and $J_S = \{RF_S\}$, where RF_1 is the flexible routing of job 1, RF_2 is the flexible routing of job 2, and $\{RF_S\}$ is the flexible routing of job S. Each flexible routing is formed by a set of similar operations.

A scheme of arrangement says that, $RF_1 \models \{O_{11m1}, O_{11m2}, \ldots, O_{12mM}\}$, $RF_2 = \{O_{12m1}, O_{12m2}, \ldots, O_{12mM}\}$, and $RF_S = \{O_{1Sm1}, O_{1Sm2}, \ldots, O_{1SmM}\}$, where O_{11m1} represents operation 1 of job 1 processed by machine 1, O_{12m1} represents operation 1 of job 2 processed by machine 1, and O_{1Sm1} represents operation 1 of job S processed by machine 1. The system implemented also uses backward scheduling.

There is a deadline for each work j. Thus, the issue entails arranging all operations in order to bring down the overall time required for their execution (Makespan), while keeping in mind all products' give away deadlines. In this work, there are also other limitations.

(a) Contractor lead times: if the required material(s) is (are) not available, the system does not schedule an order.

(b) Time to setup: this is an algorithm (scheduling) that is setup-dependent.

The grocery store considers to bring down total making time to be the most important optimization goal, so this is the act with a goal used by the ACO system. The duration (total time) of the production schedule is represented by this performance metric. Otherwise, it's the time difference between the last scheduled job's end time and the first scheduled job's start time. Makespan can also be described as the time difference between the end of the last operation to be processed and the beginning of the first machine's operation. Since the ACO system's goal is to reduce the makespan, this study also looked at total machine time (effort) as a performance metric to compare ACO to another optimisation technique (BB), which was used in this paper as a yardstick to assess the proposed ACO's efficiency.

4. ACO- Production Scheduling

Previously, each product procedure plan consisted of just one task. As a result, each versatile way has a collection of similar unique operations that are discarded after selecting one of these operations. These are “virtual” models at first, available for an ant to move through. The execution of this job has been scheduled until an ant selects and passes through a node. The ACO graph has two branches: the nest and the final node food. All other nodes refer to operations that can be planned, while these nodules are "fictitious." The edges correspond to the operation's length, and there are no edges connecting procedures from the same party. That is from the same task, since they will be excluded if they are planned. Non-oriented edges, on the other hand, link service groups and show that all jobs must be scheduled, not taking into consideration, their order. The Ants are starting from their "Nest" to collect their "Food," but before they can get to the food node, all jobs must be scheduled (the complete path between nest and food nodes comprises a feasible schedule). The Ants initially seek out various food sources, and as the Algorithm advances, the number of food sources gets closer to the “best food sources.”

5. Planning andAnalyzing

(a) Experiments with Factorials (2^k) have been used to substantiate the guidance of every single ACO configuration

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constraint in the objective function (makespan + computing time); (b) to experiment with confirmation of ACO efficiency in study of variance has been performed in relation to branch-and-bound are the 2 methods of tryouts made in this work

5.1. Impact of ACO Alignment Performance Parameters

This first form of study has a two-fold goal:

(a) to capture how the input (or configuration) limitations affect the quantity of solution and the amount of time it takes for the machine to execute it
(b) these 2^k We'll also use factorial experiments to figure out what the best values are for the configuration parameters of the ACO systems.

In the 2^k factorial examination , two levels for the 6 parameters (BRP, NA, QIP, QAP, NT, and EP) are taken into consideration heading to 64 (2^6) Various ACO configurations will be tested.. The low and high levels are shown in Table 1.

Considering the 2^k experiments, TWO WAY ANOVA was performed. For this, each one of the 64 configuration circumstances was run sixteen times (sample size was chosen arbitrarily). Table 2 shows the ANOVA experiment results obtained. Based on the interpretation of Table 2 and the theory of ANOVA, the parameters BRP, EP, NT QIP, and QUAP had a major impact on the problem performance . The number of ants (NA) was the only parameter that did not appear to affect results, according to a 95% confidence level. It's possible that the difference in the NA parameter's low and high values was insufficient to have a major impact on the system's efficiency, and that the number of travels obscured some of the parameter's effect.

Parameter ER (evaporation rate) regulates the pheromone amount evaporation as time moves in the paths. The reality that there is a greater quantity of material evaporates in a limited period of time can be considered important, so that the cheapest local alternative is not chosen.. Parameter NT (number of travels that each ant must perform) clearly affected the solution quality. As said before, this parameter probably hid the NA effect on performance. The importance of parameter QIP (quantity of initial pheromone in the system) is noticed when compared to the quantity of pheromone added after a travel is complete. Setting QIP too high and using low QAP may not influence ants to take the best paths. But both parameters suggestively affect the excellence of response. Finally, as explained previously, parameter NA (number of ants in the system) had no significant stimulus in the quality of the problem comebacks.

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TABLE 1: Data for an analysis of the considered 2^6 factorial experiments.

Scenarios		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
BRP	low	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
	High	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
NA	Low	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
	High	60	60	60	60	20	20	20	20	20	20	20	20	20	20	20	20
QIP	Low	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	High	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
QAP	Low	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
	High	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
NT	Low	70	70	70	70	70	70	70	70	70	70	70	70	70	70	70	70
	High	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
EP	Low	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	High	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
Number of machines		3	3	3	3	6	6	6	6	10	10	10	10	10	10	10	10
Number of orders		15	20	30	45	15	20	30	45	15	20	30	45	15	20	30	45

TABLE 3: Summary of the 2^k experiment results.

TABLE 3: Summary of the 2^k experiment results

Parameter	F_0	F critic	Evaluation
BRP	96.51	4	Significant
EP	6.04	4	Significant
NT	22.04	4	Significant
MIP	10.79	4	Significant
MAP	5.32	4	Significant
NA	2.76	4	Not significant

TABLE 4: Scenarios for the execution of ACO efficiency tests

Scenario	No.of possible Machines to execute the the operation	jobs OPs
1	5	20
2	5	40
3	5	60
4	10	20
5	10	40
6	10	60

This outcome is due to the truth that in the employed ACO, the pheromone is only added to the paths after an ant gets close to the food node not during the search. This, on the other hand, is more in line with ant behavior, which involves bringing

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food back to their nest, scratch their stomachs on the field, “leaving” the pheromone. However, it's reasonable to believe that the number of ants has an effect on the outcome. For the two levels taken into account in this ACO scheme (20 and 50 ants) and the strong influence of the number of travels, this did not come true. Table 3 brings a summary of this analysis, demonstrating which variables are important and are not important in the 2^k executed experiment.

The main inference in the first phase of this study is that most of the configuration parameters considered, which affect directly or indirectly the quantity of pheromone in a path, significantly impact the response quality.

In the next phase, the configuration parameters used for proportional tests are set, according to the approach proposed by Montgomery and Runger [12], which basically consists in picking the variable level in which the sum of the response averages is higher. This way, the parameters chosen were

BRP = 5; NA = 50; MIP =10;

MAP = 50; NT = 100; EP = 5.

As referenced before, the boundary referring to the quantity of buried insects in the framework didn't give off an impression of being critical in plan quality. As indicated by Montgomery and Runger [13], in such cases, a particular boundary level should be picked to enhance spending plan, activity or whatever other solid procedural factor while accomplishing the calculation.

5.2.1 Applying F-Test for the two Variables

The F- test is to verify the difference in variance between the two samples, about both makespan and computer time needed to achieve the best results. This step ahead is to define the test that serves to compare averages. This test was conducted using Microsoft Excel's Data Analysis kit, specifically the F-Test feature "two samples for variances," with a significance level of 5%. Table 6 gives us the results got.

Since $F_t > F_c$ (0.83 > 0.29), the null hypothesis must be rejected once the variances of the two samples are assumed to be equal. As a result, there is statistical evidence that the variation in variances is important, indicating that the variances in makespan obtained with ACO differ from those obtained with BB within a 95 % confidence interval. Table 7 shows the F-test result for the computer variance analysis used to obtain the results for each technique.

Since $F_o > F_c$ (100.69 > 3.44), H_0 (null hypothesis) will not be accepted. The variances of the two samples are equal, the difference between the variances is significant concerning the computer time (effort).Summing up, F-tests showed that the variance regarding the minimal of maximum makespan was highly better using BB compared to ACO.

The intention to verify whether the Ant Colony Optimization Metaheuristics was a feasible technique for solving backward production scheduling optimisation in a stage productive systems, with parallel manufacturing resources, different production capacities, and flexible routings and the learning also intended to evaluate the efficiency of ACO compared to the branch-and-bound technique were the two main objectives that took this work into a conclusive state.

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	Makespan [Hrs]		Computer time [s]	
Scenario	BB	ACO	BB	ACO
1	13.99	13.12	1	14
2	26.01	25.99	3	56
3	38.86	37.95	14	93
4	10.91	10.93	547	28
5	19.20	19.08	1298	112
6	25.14	28.90	3254	186
7	36.62	38.36	8620	357
8	41.31	43.75	19865	800
9	57.45	62.39	23838	2867
	F_o	F_c	F_o	F_c
F test	0.84	0.28	100.69	3.44
For the variance:	Statistically different Value		Statistically different Value	

TABLE 8: Description of the ACO efficiency study'

The ACO metaheuristics efficiency was proven using statistical F-test, taking the consistency of the produced production plan (in terms of makespan) and the computer time needed to construct production schedules. While there wasn't any statistical difference in makespan between BB and ACO, if one believes that BB provides a good response, ACO would look similar. In terms of machine time, however, the ant colony outperformed the branch-and-bound process. It is important to keep in mind that these figures are based on the types of scenarios that were considered.

6. CONCLUSION

The aim of this study was to see if ACO could be used to solve production scheduling problem in some types of grocery industries that use backward scheduling and truly reflect single stage process improvements, parallel resources, and elastic routings. The effect of heuristic search arrangement variables upon response variable variations and means was examined in this work.

Future studies may be extended based on

(i) It is essential to understand complex systems with multiple processing stages.
(ii) Incorporation into a forward-scheduling environment
(iii) Various Ant Colony Optimization execution qualities could be tested, such as encouraging dissipation to occur in each move of the insects rather than just when they spread a food source) and various metaheuristics pattern variables.
(iv) Other powerful executions might consider a multi-objective skill with various goals, such as restricting delay, daily growth time, arrangements, and benefits slowness.

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