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Use L_{24c} Transform to Solve Three-Dimensional Convection Heat Transfer Problem in Cylindrical Coordinate

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ABSTRACT

Convection heat transfer problem solved by L_{24c} transform which used to solve energy equation and navier-stokes equations in cylindrical coordinate and that explain distribution of temperature and fluid flow with zero incline angle. Results plotted by using Matlab .

Keywords: L_{24c} Transform, Convection Heat Transfer, Navier-Stokes Equations, Cylindrical Coordinate.

1. Introduction:

The enhancement of heat transfer has gained a lot of attention in the current times, because of it numerous uses in engineering and diligence. In utmost operations, heat transfer fluids Used as cooling fluids, similar as water, paraffin oil painting, thermal oil painting, ethylene glycol, Naphthalic mineral oil painting, vegetable oil painting ,etc., still, these common fluids have poor heat conductivity, which reduces the quantum of heat transferred[11]. Many times, studies of the heat transfer parcels of fine fluid inflow(two- phase fluid)- in terms of understanding colorful real- world problems, in particular In the fields of atmospheric, physiology and engineering- it has captured the interest of many experimenters. For illustration, the operation of dust patches can be seen in Petroleum assiduity, soil corrosion by natural winds, crude oil painting sanctification, aerosols and Paint scattering, lacing, enmeshing dust through a nuclear explosion in a pall and Sewage water treatment[9]. V. S. Kulkarni, K. C. Deshmukh, and P. H. Munjankar utilized transform of finite Hankel to get the solution of the cylinder's steady state temperature, which satisfies equation of heat conduction (r,z,t coordinate) [12]. Transform of Mellin was also used to solve equation of steady state heat (r,θ,t coordinate)[3]. Furthermore, Laplace transform solved heat equation(one-dimension) in a cylindrical coordinate [7].To solve three-dimensional heat conduction equation in cylindrical coordinate, Ahmed S.Jalal and Ahmed M.J.Jassim used L_{24c} transform [2]. Najma Ahmed, Nehad Ali Shah, and Dumitru Vieru used integral transforms (Laplace transform and finite Hankel transform) to solve natural convection with damped thermal flux in a vertical circular cylinder[8]. Joao N.N. Quaresma, Carlos C.S. da Cruz, Neil Cagney, Renato M. Cotta, and Stavroula Balabani used technique of generalized integral transform to solve blended convection on laminar vortex breakdown in a cylindrical enclosure with a revolving bottom plate[5].

2. The model and mathematical method to solve convection heat transfer problem :

The convection heat transfer problem consist of the following equations[1][10][6]:

1- Continuity equation :

$$\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0 \quad \dots\dots(1)$$

2- Navier-Stokes equations:

(r-direction)

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$$\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z} =\frac{\mu}{\rho}\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}-\frac{v_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right)-\frac{\partial P}{\rho\,\partial r}+\beta g\sin(\alpha)\,(T-T_o) \quad \dots\dots(2)$$

(θ-direction)

$$\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}-\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z} =\frac{\mu}{\rho}\left(\frac{\partial^2 v_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial v_\theta}{\partial r}-\frac{v_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right)-\frac{1}{r}\frac{\partial P}{\rho\,\partial \theta}-\beta g\cos(\alpha)\,(T-T_o) \quad \dots\dots(3)$$

(z-direction)

$$\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}=\frac{\mu}{\rho}\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)-\frac{\partial P}{\rho\,\partial z} \quad \dots\dots(4)$$

3- Energy equation:

$$\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}=\frac{k}{\rho\, c_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right) \quad \dots\dots(5)$$

L_{24c} transform is defined as[2]:

$$L_{24c}\big(f(r,\theta,z,t)\big)=\int_0^\infty\int_0^\infty\int_0^{\pi/2}\int_0^\infty (r^2 sin\theta\, cos\theta\, z\, t)e^{-r^2(p^2cos^2\theta+q^2sin^2\theta)}e^{-v^2z^2-s^2t^2}f(r,\theta,z,t)\, rdr\, d\theta\, dz\, dt$$

firstly we will solve energy equation(5) by L_{24c} transform as follows:

$$\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}=\frac{k}{\rho\, c_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right) \quad \dots\dots(5)$$

With boundary and initial conditions:

$T(a,\theta,z,t)=0$

$T(r,\theta,0,t)=0 \quad , \quad T(r,\theta,b,t)=0$

$T(r,0,z,t)=0 \quad , \quad T\left(r,\frac{\pi}{2},z,t\right)=T_1$

$T(r,\theta,z,0)=T_a$

$|T(0,\theta,z,t)|\ bounded\ at\ r=0$

Now apply L_{24c} transform to energy equation(5):

$$L_{24c}\left[\frac{\partial T}{\partial t}\right]+L_{24c}\left[v_r\frac{\partial T}{\partial r}\right]+L_{24c}\left[\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}\right]+L_{24c}\left[v_z\frac{\partial T}{\partial z}\right]=\frac{k}{\rho\, c_p}L_{24c}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

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$$
\begin{aligned}
&2s^2\, L_{24c}[t\,T] - L_{24c}\left[\frac{1}{t}T\right] + L_{24c}[T\, v_r (2r)((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)] - L_{24c}\left[T\, v_r\, \left(\frac{3}{r}\right)\right] + L_{24c}\left[\left(\frac{-v_\theta}{r}\right) T (2r^2) cos\theta\; sin\theta\; (p^2 - q^2\right.\\
&\quad + L_{24c}\left[\left(\frac{v_\theta}{r}\right) T\; \frac{sin\theta}{cos\theta}\right] + L_{24c}\left[\left(\frac{-v_\theta}{r}\right) T\; \frac{cos\theta}{sin\theta}\right] + L_{24c}[v_z\,(2T) z\, v^2] - L_{24c}\left[T\, v_z\, \frac{1}{z}\right]\\
&\quad = \frac{k}{\rho\, c_p}[L_{24c}[(4r^2)(((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)^2)T] - L_{24c}[8((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)T]\\
&\quad - L_{24c}[6((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)T] + L_{24c}\left[\frac{6}{r^2}\, T\right] + L_{24c}[2((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)T] - L_{24c}\left[\frac{2}{r^2}\, T\right]\\
&\quad + L_{24c}[4\,(sin\theta)^2\,(cos\theta)^2\,(p^2 - q^2)^2\, T] - 3L_{24c}[2(p^2 - q^2)(sin\theta)^2 T] + 3L_{24c}[2(p^2 - q^2)(cos\theta)^2 T]\\
&\quad + \left(\left(\frac{1}{2q^2}\right)\left(\frac{1}{2v^2}\right)\left(\frac{1}{2s^2}\right) T \left(\frac{\pi}{2}\right)\right) - 2L_{24c}\left[\frac{2}{r^2}\, T\right] + L_{24c}[4\, z^2\, v^4\, T] - L_{24c}[4\, v^2\, T] - L_{24c}[2\, v^2\, T]]
\end{aligned}
$$

After simplicity:

$$
\begin{aligned}
&L_{24c}\left[\frac{2s^2\, t\, T}{2p^2}\right] - L_{24c}\left[\frac{T}{2p^2\, t}\right] + L_{24c}\left[\frac{T\, v_r\,(2r)((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)}{2p^2}\right] - L_{24c}\left[\frac{3\, T\, v_r}{2p^2\, r}\right] + L_{24c}\left[\frac{(-v_\theta\, T\,(2r^2) cos\theta\; sin\theta\;(p^2 - q^2}{2p^2\, r}\right.\\
&\quad + L_{24c}\left[\frac{v_\theta\, T\, sin\theta}{2p^2\, r\;\; cos\theta}\right] + L_{24c}\left[\frac{(-v_\theta) T\, cos\theta}{2p^2\, r\, sin\theta}\right] + L_{24c}\left[\frac{v_z\,(2\,T) z\, v^2}{2p^2}\right] - L_{24c}\left[\frac{T\, v_z}{2p^2\, z}\right]\\
&\quad - L_{24c}\left[\frac{k\,(4r^2)((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)T}{\rho\, c_p\,(2p^2)}\right] + L_{24c}\left[\frac{8k((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)\, T}{\rho\, c_p\,(2p^2)}\right]\\
&\quad + L_{24c}\left[\frac{6k((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)\, T}{\rho\, c_p\,(2p^2)}\right] - L_{24c}\left[\frac{6k\, T}{\rho\, c_p\,(2p^2)\, r^2}\right] - L_{24c}\left[\frac{2k((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)\, T}{\rho\, c_p\,(2p^2)}\right]\\
&\quad + L_{24c}\left[\frac{k\,(2\,T)}{\rho\, c_p\,(2p^2)\, r^2}\right] - L_{24c}\left[\frac{4k\,(sin\theta)^2\,(cos\theta)^2\,(p^2 - q^2)^2\, T}{\rho\, c_p\,(2p^2)}\right] + L_{24c}\left[\frac{6k(p^2 - q^2)(sin\theta)^2 T}{\rho\, c_p\,(2p^2)}\right]\\
&\quad - L_{24c}\left[\frac{6k(p^2 - q^2)(cos\theta)^2\, T}{\rho\, c_p\,(2p^2)}\right] + L_{24c}\left[\frac{4k\, T}{\rho\, c_p\,(2p^2)}\right] - L_{24c}\left[\frac{4k\, z^2\, v^4\, T}{\rho\, c_p\,(2p^2)}\right] + L_{24c}\left[\frac{4k\, v^2\, T}{\rho\, c_p\,(2p^2)}\right] + L_{24c}\left[\frac{2k\, v^2\, T}{\rho\, c_p\,(2p^2)}\right]\\
&\quad = \frac{\left(\frac{k}{\rho\, c_p}\right) T (\frac{\pi}{2})}{(2p^2)(2q^2)(2v^2)(2s^2)}
\end{aligned}
$$

By apply boundary and initial conditions and take L_{24c}^{-1} :

$$
\begin{aligned}
&, z, t) = \left[\left(\frac{k}{\rho\, c_p}\right) T_1\right] / \left[\frac{2s^2\, t}{2p^2} - \frac{1}{2p^2\, t} + \frac{v_r\,(2r)\big((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2\big)}{2p^2} - \frac{3v_r}{2r\, p^2} - \frac{v_\theta\,(2r) cos\theta\; sin\theta\;(p^2 - q^2)}{2p^2} + \frac{v_\theta\, sin\theta}{2r\, p^2\, cos\theta} - \frac{v_\theta\, cos\theta}{2r\, p^2\, sin\theta} + \frac{v_z\,(2z) v^2}{2p^2} - \frac{v_z}{2z\, p^2} - \right.\\
&\frac{(cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)}{\rho\, c_p\,(2p^2)} + \frac{14k\,\big((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2\big)}{\rho\, c_p\,(2p^2)} - \frac{4k}{r^2\, \rho\, c_p\,(2p^2)} - \frac{2k\,\big((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2\big)}{\rho\, c_p\,(2p^2)} - \frac{4k\,(sin\theta)^2\,(cos\theta)^2\,\big(p^2 - q^2\big)^2}{\rho\, c_p\,(2p^2)} +\\
&\left.\frac{-q^2)\big((sin\theta)^2 - (cos\theta)^2\big)}{\rho\, c_p\,(2p^2)} + \frac{4k}{\rho\, c_p\,(2p^2)} - \frac{4k\, z^2\, v^4}{\rho\, c_p\,(2p^2)} + \frac{6k\, v^2}{\rho\, c_p\,(2p^2)}\right] \qquad \ldots\ldots(6)
\end{aligned}
$$

Now after combine navier-stokes equations(2),(3)and(4) and simplicity we get[4]:

$$
\begin{aligned}
&(cos\theta - sin\theta)\frac{\partial^2\zeta}{\partial r\,\partial t} - \left(\frac{sin\theta}{r} + \frac{cos\theta}{r}\right)\frac{\partial^2\zeta}{\partial\theta\,\partial t}\\
&\quad = \left(\frac{-3\mu p}{\rho} cos\theta + \frac{3\mu q}{\rho} sin\theta\right)\frac{\partial\zeta}{\partial r} + \left(\frac{3\mu p}{\rho}\left(\frac{sin\theta}{r}\right) + \frac{3\mu q}{\rho}\left(\frac{cos\theta}{r}\right)\right)\frac{\partial\zeta}{\partial\theta} + \left(\frac{-1}{3}\beta g\; sin\alpha\; sin\theta\; + \frac{1}{3}\beta g\; cos\alpha\; cos\theta\right)\frac{\partial^2 T}{\partial r\,\partial z}\\
&\quad - \left(\frac{1}{3}\beta g\; sin\alpha\; \left(\frac{cos\theta}{r}\right) + \frac{1}{3}\beta g\; cos\alpha\; \left(\frac{sin\theta}{r}\right)\right)\frac{\partial^2 T}{\partial\theta\,\partial z} \quad \ldots\ldots(7)
\end{aligned}
$$

With boundary and initial conditions:

$\zeta(a, \theta, z, t) = 0$

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$\zeta(r,\theta,0,t)=0 \quad , \quad \zeta(r,\theta,b,t)=0$

$\zeta(r,0,z,t)=0 \quad , \quad \zeta\left(r,\frac{\pi}{2},z,t\right)=0$

$\zeta(r,\theta,z,0)=V_o$

$|\zeta(0,\theta,z,t)|\ bounded\ at\ r=0$

Apply L_{24c} transform to equation(7):

$$L_{24c}\left[(cos\theta-sin\theta)\frac{\partial^2\zeta}{\partial r\,\partial t}\right]-L_{24c}\left[\left(\frac{sin\theta}{r}+\frac{cos\theta}{r}\right)\frac{\partial^2\zeta}{\partial\theta\,\partial t}\right]$$
$$=L_{24c}\left[\left(\frac{-3\mu p}{\rho}cos\theta+\frac{3\mu q}{\rho}sin\theta\right)\frac{\partial\zeta}{\partial r}\right]+L_{24c}\left[\left(\frac{3\mu p}{\rho}\left(\frac{sin\theta}{r}\right)+\frac{3\mu q}{\rho}\left(\frac{cos\theta}{r}\right)\right)\frac{\partial\zeta}{\partial\theta}\right]$$
$$+L_{24c}\left[\left(\frac{-1}{3}\beta g\ sin\alpha\ sin\theta\ +\frac{1}{3}\beta g\ cos\alpha\ cos\theta\right)\frac{\partial^2 T}{\partial r\,\partial z}\right]-L_{24c}\left[\left(\frac{1}{3}\beta g\ sin\alpha\ \left(\frac{cos\theta}{r}\right)+\frac{1}{3}\beta g\ cos\alpha\ \left(\frac{sin\theta}{r}\right)\right)\frac{\partial^2 T}{\partial\theta\,\partial z}\right]$$

Then we get:

$$L_{24c}[(2r)(cos\theta-sin\theta)((cos\theta)^2\,p^2+(sin\theta)^2\,q^2)(2t\,s^2)\,\zeta]-L_{24c}\left[(2r)(cos\theta-sin\theta)((cos\theta)^2\,p^2+(sin\theta)^2\,q^2)\frac{1}{t}\,\zeta\right]$$
$$-L_{24c}\left[(cos\theta-sin\theta)\frac{6t\,s^2}{r}\,\zeta\right]+L_{24c}\left[(cos\theta-sin\theta)\frac{3}{rt}\,\zeta\right]+L_{24c}\left[(2ts^2)\left((sin\theta)^2\,cos\theta\,(2r)(p^2-q^2)\right)\zeta\right]$$
$$-L_{24c}\left[\frac{1}{t}\,\left((sin\theta)^2\,cos\theta\,(2r)(p^2-q^2)\right)\zeta\right]-L_{24c}\left[(2ts^2)\left(\frac{(sin\theta)^2}{cos\theta}\right)\frac{1}{r}\,\zeta\right]+L_{24c}\left[\frac{1}{t}\,\left(\frac{(sin\theta)^2}{cos\theta}\right)\frac{1}{r}\,\zeta\right]$$
$$+L_{24c}\left[(2ts^2)(2cos\theta)\frac{1}{r}\,\zeta\right]-L_{24c}\left[\frac{1}{t}(2cos\theta)\frac{1}{r}\,\zeta\right]+L_{24c}[(2ts^2)sin\theta\,(cos\theta)^2\,(p^2-q^2)(2r)\,\zeta]$$
$$-L_{24c}\left[\frac{1}{t}\,sin\theta\,(cos\theta)^2\,(p^2-q^2)(2r)\,\zeta\right]-L_{24c}\left[(2ts^2)(2sin\theta)\frac{1}{r}\,\zeta\right]+L_{24c}\left[\frac{1}{t}(2sin\theta)\frac{1}{r}\,\zeta\right]$$
$$+L_{24c}\left[(2ts^2)\left(\frac{(cos\theta)^2}{sin\theta}\right)\frac{1}{r}\,\zeta\right]-L_{24c}\left[\frac{1}{t}\left(\frac{(cos\theta)^2}{sin\theta}\right)\frac{1}{r}\,\zeta\right]$$
$$-L_{24c}\left[(2r)((cos\theta)^2\,p^2+(sin\theta)^2\,q^2)\left(\frac{(-3\mu p)}{\rho}cos\theta+\frac{3\mu q}{\rho}sin\theta\right)\,\zeta\right]+L_{24c}\left[\left(\frac{3}{r}\right)\left(\frac{(-3\mu p)}{\rho}cos\theta+\frac{3\mu q}{\rho}sin\theta\right)\,\zeta\right.$$
$$+L_{24c}\left[(sin\theta)^2\,cos\theta\,(2r)(p^2-q^2)\frac{3\mu p}{\rho}\,\zeta\right]-L_{24c}\left[\left(\frac{(sin\theta)^2}{cos\theta}\right)\left(\frac{3\mu p}{\rho}\right)\frac{1}{r}\,\zeta\right]+L_{24c}\left[(2cos\theta)\frac{3\mu p}{\rho}\left(\frac{1}{r}\right)\,\zeta\right]$$
$$+L_{24c}\left[sin\theta\,(cos\theta)^2\,(p^2-q^2)(2r)\frac{3\mu q}{\rho}\,\zeta\right]-L_{24c}\left[(2sin\theta)\frac{3\mu q}{\rho}\left(\frac{1}{r}\right)\,\zeta\right]+L_{24c}\left[\left(\frac{(cos\theta)^2}{sin\theta}\right)\frac{3\mu q}{\rho}\left(\frac{1}{r}\right)\,\zeta\right]$$

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$$= L_{24c}\left[(2r)((cos\theta)^2\,p^2 + (sin\theta)^2\,q^2)\left(\frac{-1}{3}\right)\beta g\ sin\alpha\ sin\theta + \frac{1}{3}\beta g\ cos\alpha\ cos\theta\ (2zv^2)T\ \right]$$
$$- L_{24c}\left[(2r)((cos\theta)^2\,p^2 + (sin\theta)^2\,q^2)\left(\left(\frac{-1}{3}\right)\beta g\ sin\alpha\ sin\theta + \frac{1}{3}\beta g\ cos\alpha\ cos\theta\right)\frac{1}{z}T\ \right]$$
$$- L_{24c}\left[\frac{3}{r}(2zv^2)\left(\left(\frac{-1}{3}\right)\beta g\ sin\alpha\ sin\theta + \frac{1}{3}\beta g\ cos\alpha\ cos\theta\right)T\right] + L_{24c}\left[\frac{3}{r}\left(\frac{1}{z}\right)\left(\left(\frac{-1}{3}\right)\beta g\ sin\alpha\ sin\theta + \frac{1}{3}\beta g\ cos\alpha\ cos\right.\right.$$
$$+ L_{24c}\left[(2zv^2)\big(sin\theta\ (cos\theta)^2\ (p^2 - q^2)(2r)\big)\left(\frac{1}{3}\right)\beta g\ sin\alpha\ T\right] - L_{24c}\left[\frac{1}{z}\big(sin\theta\ (cos\theta)^2(p^2 - q^2)(2r)\big)\left(\frac{1}{3}\right)\beta g\ sin\alpha\right.$$
$$- L_{24c}\left[(2zv^2)(2sin\theta)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)sin\alpha\ T\ \right] + L_{24c}\left[\left(\frac{1}{z}\right)(2sin\theta)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)sin\alpha\ T\ \right]$$
$$+ L_{24c}\left[(2zv^2)\left(\frac{(cos\theta)^2}{sin\theta}\right)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)sin\alpha\ T\ \right] - L_{24c}\left[\left(\frac{1}{z}\right)\left(\frac{(cos\theta)^2}{sin\theta}\right)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)sin\alpha\ T\ \right]$$
$$+ L_{24c}\left[(2zv^2)\big((sin\theta)^2\ cos\theta\ (2r)(p^2 - q^2)\big)\left(\frac{1}{3}\right)\beta g\ cos\alpha\ T\ \right] - L_{24c}\left[\left(\frac{1}{z}\right)\big((sin\theta)^2\cos\theta\ \ (2r)(p^2 - q^2)\big)\left(\frac{1}{3}\right)\beta g\right.$$
$$- L_{24c}\left[(2zv^2)\left(\frac{(sin\theta)^2}{cos\theta}\right)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)cos\alpha\ T\ \right] + L_{24c}\left[\left(\frac{1}{z}\right)\left(\frac{(sin\theta)^2}{cos\theta}\right)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)cos\alpha\ T\ \right]$$
$$+ L_{24c}\left[(2zv^2)(2cos\theta)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)cos\alpha\ T\ \right] - L_{24c}\left[\left(\frac{1}{z}\right)(2cos\theta)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)cos\alpha\ T\ \right]$$

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$$
\begin{aligned}
\zeta(r,\theta,z,t) = \Bigg[\Bigg[\Bigg[& (2r)((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)\left(\frac{-1}{3}\right)\beta g\ sin\alpha\ sin\theta + \frac{1}{3}\beta g\ cos\alpha\ cos\theta\ (2zv^2)\Bigg] \\
& - \left[(2r)((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)\left(\left(\frac{-1}{3}\right)\beta g\ sin\alpha\ sin\theta + \frac{1}{3}\beta g\ cos\alpha\ cos\theta\right)\frac{1}{z}\right] \\
& - \left[\frac{3}{r}(2zv^2)\left(\left(\frac{-1}{3}\right)\beta g\ sin\alpha\ sin\theta + \frac{1}{3}\beta g\ cos\alpha\ cos\theta\right)\right] + \left[\frac{3}{r}\left(\frac{1}{z}\right)\left(\left(\frac{-1}{3}\right)\beta g\ sin\alpha\ sin\theta + \frac{1}{3}\beta g\ cos\alpha\ cos\theta\right)\right] \\
& + \left[(2zv^2)\big(sin\theta\ (cos\theta)^2\ (p^2-q^2)(2r)\big)\left(\frac{1}{3}\right)\beta g\ sin\alpha\right] - \left[\frac{1}{z}\big(sin\theta\ (cos\theta)^2(p^2-q^2)(2r)\big)\left(\frac{1}{3}\right)\beta g\ sin\alpha\right] \\
& - \left[(2zv^2)(2sin\theta)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)sin\alpha\right] + \left[\left(\frac{1}{z}\right)(2sin\theta)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)sin\alpha\right] + \left[(2zv^2)\left(\frac{(cos\theta)^2}{sin\theta}\right)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)sin\alpha\right. \\
& - \left[\left(\frac{1}{z}\right)\left(\frac{(cos\theta)^2}{sin\theta}\right)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)sin\alpha\right] + \left[(2zv^2)\big((sin\theta)^2\ cos\theta\ (2r)(p^2-q^2)\big)\left(\frac{1}{3}\right)\beta g\ cos\alpha\right] \\
& - \left[\left(\frac{1}{z}\right)\big((sin\theta)^2 \cos\theta\ \ (2r)(p^2-q^2)\big)\left(\frac{1}{3}\right)\beta g\ cos\alpha\right] - \left[(2zv^2)\left(\frac{(sin\theta)^2}{cos\theta}\right)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)cos\alpha\right] \\
& + \left[\left(\frac{1}{z}\right)\left(\frac{(sin\theta)^2}{cos\theta}\right)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)cos\alpha\right] + \left[(2zv^2)(2cos\theta)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)cos\alpha\right] - \left[\left(\frac{1}{z}\right)(2cos\theta)\left(\frac{1}{3}\right)\beta g\ \left(\frac{1}{r}\right)cos\alpha\right]\Bigg] \\
& / \Bigg[[(2r)(cos\theta - sin\theta)((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)(2t\, s^2)] - \left[(2r)(cos\theta - sin\theta)((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)\frac{1}{t}\right] \\
& - \left[(cos\theta - sin\theta)\frac{6t\, s^2}{r}\right] + \left[(cos\theta - sin\theta)\frac{3}{rt}\right] + \big[(2ts^2)\big((sin\theta)^2\ cos\theta\ (2r)(p^2-q^2)\big)\big] \\
& - \left[\frac{1}{t}\ \big((sin\theta)^2\ cos\theta\ (2r)(p^2-q^2)\big)\right] - \left[(2ts^2)\left(\frac{(sin\theta)^2}{cos\theta}\right)\frac{1}{r}\right] + \left[\frac{1}{t}\ \left(\frac{(sin\theta)^2}{cos\theta}\right)\frac{1}{r}\right] + \left[(2ts^2)(2cos\theta)\frac{1}{r}\right] \\
& - \left[\frac{1}{t}(2cos\theta)\frac{1}{r}\right] + [(2ts^2)sin\theta\ (cos\theta)^2\ (p^2-q^2)(2r)] - \left[\frac{1}{t}\ sin\theta\ (cos\theta)^2\ (p^2-q^2)(2r)\right] - \left[(2ts^2)(2sin\theta)\frac{1}{r}\right] \\
& + \left[\frac{1}{t}(2sin\theta)\frac{1}{r}\right] + \left[(2ts^2)\left(\frac{(cos\theta)^2}{sin\theta}\right)\frac{1}{r}\right] - \left[\frac{1}{t}\left(\frac{(cos\theta)^2}{sin\theta}\right)\frac{1}{r}\right] \\
& - \left[(2r)((cos\theta)^2\, p^2 + (sin\theta)^2\, q^2)\left(\frac{(-3\mu p)}{\rho}cos\theta + \frac{3\mu q}{\rho}sin\theta\right)\right] + \left[\left(\frac{3}{r}\right)\left(\frac{(-3\mu p)}{\rho}cos\theta + \frac{3\mu q}{\rho}sin\theta\right)\right] \\
& + \left[(sin\theta)^2\ cos\theta\ (2r)(p^2-q^2)\frac{3\mu p}{\rho}\right] - \left[\left(\frac{(sin\theta)^2}{cos\theta}\right)\left(\frac{3\mu p}{\rho}\right)\frac{1}{r}\right] + \left[(2cos\theta)\frac{3\mu p}{\rho}\left(\frac{1}{r}\right)\right] \\
& + \left[sin\theta\ (cos\theta)^2\ (p^2-q^2)(2r)\frac{3\mu q}{\rho}\right] - \left[(2sin\theta)\frac{3\mu q}{\rho}\left(\frac{1}{r}\right)\right] + \left[\left(\frac{(cos\theta)^2}{sin\theta}\right)\frac{3\mu q}{\rho}\left(\frac{1}{r}\right)\right]\Bigg]
\end{aligned}
$$

$T = T(r,\theta,z,t)$ is solution of energy equation(6) .

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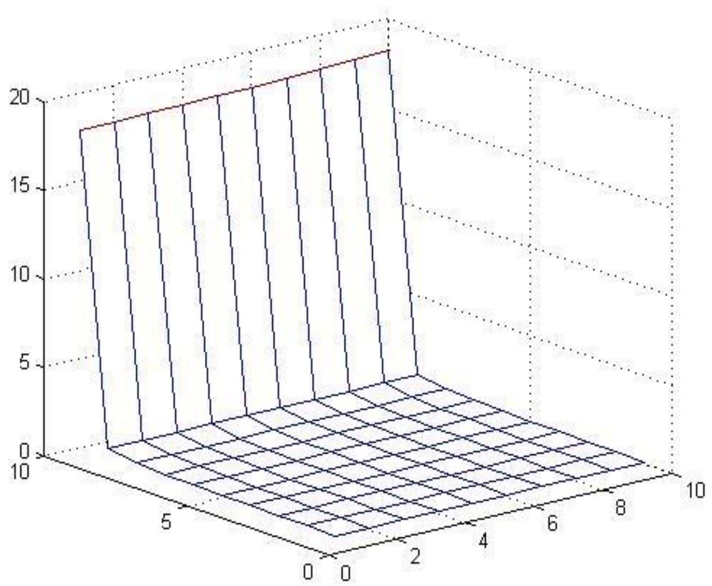
3. Results:

By use matlab we get results represented by the following illustrations



(1-1) Figure shows distribution of temperature for T(r,θ,z,t) , r=1:10 , $\theta=9^{o}$, z=1:10 , t=1 .

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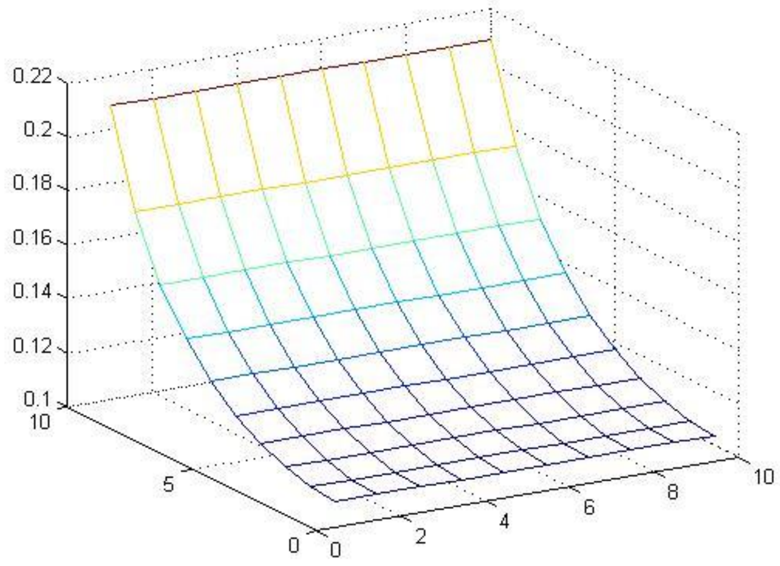
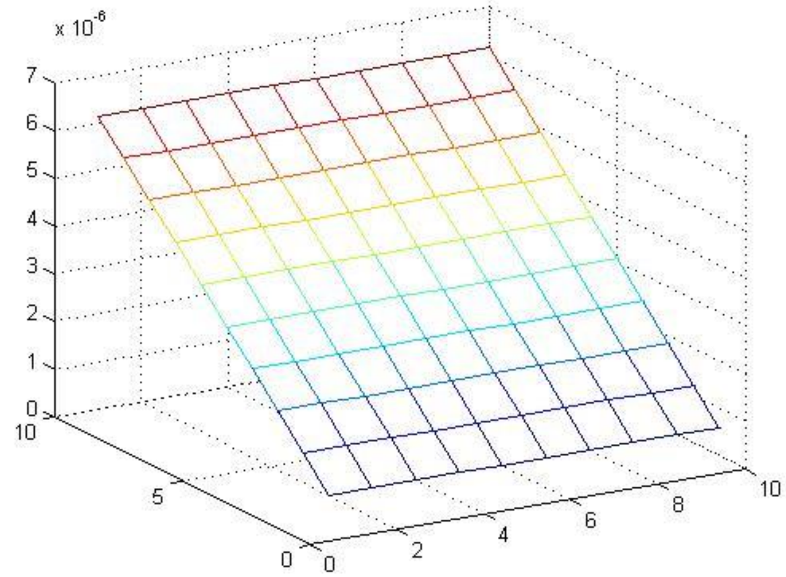


(1-2) Figure shows distribution of temperature for T(r,θ,z,t) , r=1:10 , θ=45^o , z=1:10 , t=1 .



(1-3) Figure shows distribution of temperature for T(r,θ,z,t) , r=1:10 , θ=90^o , z=1:10 , t=1 .

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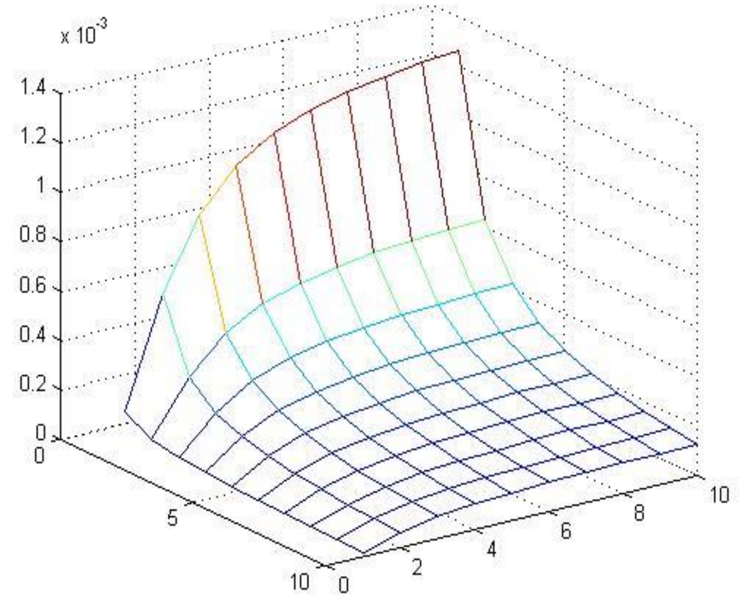
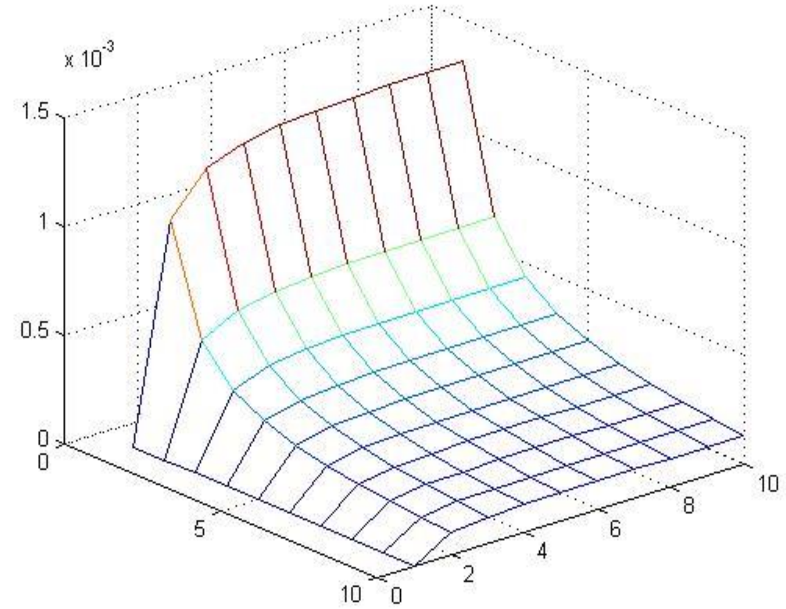


(1-4) Figure shows distribution of fluid movement for ζ(r,θ,z,t) , r=1:10 , θ=9^{o} , z=1:10 , t=0.1 , $\alpha = 0$.



(1-5) Figure shows distribution of fluid movement for ζ(r,θ,z,t) , r=1:10 , θ=45^{o} , z=1:10 , t=0.1 , $\alpha = 0$.

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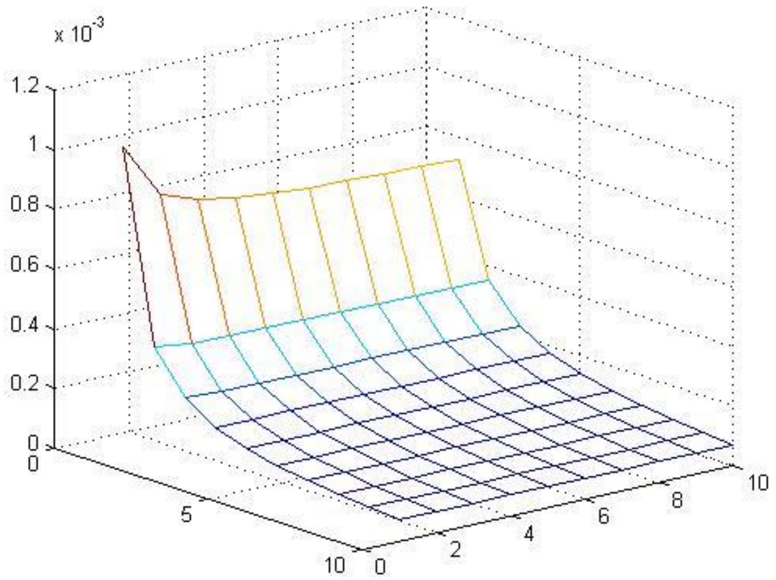

(1-6) Figure shows distribution of fluid movement for ζ(r,θ,z,t) , r=1:10 , θ=90^o , z=1:10 , t=0.1 , $\alpha = 0$.

4. Conclusions:

From the illustrations(1-1),(1-2)and(1-3) which appears distribution of temperature and illustrations (1-4),(1-5)and(1-6) explain distribution of fluid flow it's clear that the temperature decreases when the value of θ increases. Also fluid flow increases then decreases gradually and slightly when the value of θ increases, all that when incline angle (α) equal to zero .

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