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El-Algebra in Soft Sets

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ABSTRACT

A soft set can be calculated by a set-valued mapping that assigns precisely one crisp subset of the universe to each parameter. In 1998, X. Liu gave Axiomatic Fuzzy set structure and El-algebra. They pointed out that ordinary fuzzy concepts or human concepts can be represented through any molecular or atomic fuzzy concepts in El-algebra over any finite set of fuzzy concepts. However, this isn't the only way to represent transitive human ideas. The definition of soft sets was extended to El-algebra in this article by taking El-algebra as universe of discourse in soft sets, and certain properties of soft El-algebras were investigated. We also introduced homomorphism between two El-algebras.

Keywords: Soft set, El-algebra, Soft El-algebra, Homomorphism etc.

Introduction

Some fields like economics, engineering and environment have high degree of uncertainties. For these complex problems, we cannot effectively utilize classical approaches. As a mathematical tool to deal with complexity in mathematics, there are three concepts that we can accept: the theory of probability, Fuzzy sets and interval mathematics. But all these concepts, as pointed out by Molodtsov [2], have their own difficulties. It was proposed by Molodtsov [2] and Maji with others in [7] that one explanation for these problems might be the deficiency of the parameterization approach. In order to resolve these challenges, Molodtsov presented the soft set concept as a revolutionary mathematical tool for interacting with ambiguity. Soft sets are free of the challenges that have associated with normal scientific procedures. Molodtsov has identified several approaches for the objectives of the soft set. At the moment, attempts based on the soft set theory are gaining momentum. Maji with others [7] identified the classification of the soft set theory to a problem of decision-making. Also, Maji with others [8], investigated multiple operations of the soft sets. Many researchers have looked at the algebraic framework of set theories that deal with ambiguity. The fuzzy sets theory is the most suitable theory for working with uncertainties, established [3] by Zadeh.

The author Liu Xiaodong [1] defined an infinite distributive molecular lattice and called it El-algebra and Ell-algebra. They also gave a new system "AFS Structure" of fuzzy sets and systems, which is more appropriate than the classical mathematical opinions. This paper applied soft sets to El-algebra and proposed Soft El-algebra with its basic properties.

1. Basic Results on soft sets:

Definition 1.1 ([2]): Let E and U be the sets of parameters/attributes and essential universe respectively. P(U) be the power set of U, and $A \subseteq E$. A couple (F, A) or F_A is a *soft set* on U, here F is a mapping defined as:

$$F: A \rightarrow P(U).$$

Soft set F_A is simply not a classical set, it is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A). Molodtsov produced a lot of details for the illustration in [2].

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Definition 1.2 ([8]): Let F_A and G_B are two soft sets on a common universe set U, then *intersection* is specified as a soft set H_C, that meets the following requirements:

(i) $C = A \cap B$,

(ii) $\forall\ c \in C$, $H(c) = F(c)$ or $H(c) = G(c)$, (due to the fact that both sets are similar).

In this context, we're writing $F_A \tilde{\cap} G_B = H_C$.

Definition 1.3 ([8]): Let F_A and G_B are two soft sets on a common universe set U, then *union* is specified as a soft set H_C satisfying the following conditions:

(i) $C = A \cup B$,

(ii) for all $c \in C$,

$$H(c) = \begin{cases} F(c) & , \text{if } c \in A \setminus B, \\ G(c) & , \text{if } c \in B \setminus A, \\ F(c) \cup G(c) & , \text{if } c \in A \cap B. \end{cases}$$

In this case, we're writing $F_A \tilde{\cup} G_B = H_C$.

Definition 1.4 ([8]): "*AND*" of two soft sets F_A and G_B can be written as $F_A \tilde{\wedge} G_B$ and characterized by $F_A \tilde{\wedge} G_B = H_{A\times B}$, where $H(a, b) = F(a) \cap G(b)\ \forall\ (a, b) \in A \times B$.

Definition 1.5 ([8]): "*OR*" of two soft sets F_A and G_B can be written as $F_A \tilde{\vee} G_B$ and characterized by $F_A \tilde{\vee} G_B = H_{A\times B}$, where $H(a, b) = F(a) \cup G(b)\ \forall\ (a, b) \in A \times B$.

Definition 1.6 ([8]): Let F_A and G_B are two soft sets. Then F_A is a *soft subset* of G_B, represented by $F_A \tilde{\subseteq} G_B$, if it satisfies the following requirements:

(i) $A \subset B$, (ii) For each $a \in A$, $F(a)$ and $G(a)$ are approximations, that are similar.

2. Basic Definitions on El-algebra:

Definition 2.1 ([1]): Let T be any set and P(T) be non-empty power set of T. Define $ET = \{\sum_{i\in I} T_i \mid T_i \in P(T), i \in I, I \text{ is an index set}\}$, where $\sum_{i\in I} T_i$ is written in sum form. When T_i ($i \in I$) are summed by different orders, $\sum_{i\in I} T_i$ indicates the same element of ET. For example $\sum_{i\in\{1,2\}} T_i$, $T_1 + T_2$ and $T_2 + T_1$ are the same element of ET, $T_1, T_2 \in P(T)$.

Let R be a binary relation on ET expressed as: $\sum_{i\in I} T_i, \sum_{j\in J} L_j \in ET$, $\sum_{i\in I} T_i\ R\ \sum_{j\in J} L_j \Leftrightarrow \forall\ T_i\ (i \in I), \exists\ L_h\ (h \in J)$ such that $T_i \supseteq L_h$ and $\forall\ L_j\ (j \in J), \exists\ T_u\ (u \in I)$ such that $L_j \supseteq T_u$. It can be seen that R is an equivalence relation.

Then $(ET, \vee, \wedge)$ is called El-algebra over T, if $\vee$ and $\wedge$ are operations on ET defined as:

$\sum_{i\in I} T_i \vee \sum_{j\in J} L_j = \sum_{k\in I\cup J} P_k$; $I \cup J$ is the disjoin union of I and J, $P_k = T_k$ if $k \in I$; $P_k = L_k$ if $k \in J$, and $\sum_{i\in I} T_i \wedge \sum_{j\in J} L_j = \sum_{i\in I, j\in J} T_i \cup L_j$.

$(ET, \vee, \wedge)$ has the following common properties:

(1) $(\sum_{i\in I} T_i) \wedge (\sum_{i\in I} T_i) = \sum_{i\in I} T_i$,

(2) $(\sum_{i\in I} T_i) \vee (\sum_{i\in I} T_i) = \sum_{i\in I} T_i$,

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(3) $\phi \vee \sum_{i\in I} T_i = \phi$,

(4) $T \vee \sum_{i\in I} T_i = \sum_{i\in I} T_i$,

(5) $\phi \wedge \sum_{i\in I} T_i = \sum_{i\in I} T_i$,

(6) $T \wedge \sum_{i\in I} T_i = T$,

(7) $[(\sum_{i\in I} T_i) \vee (\sum_{k\in U} P_k)] \wedge [(\sum_{j\in J} L_j) \vee (\sum_{k\in U} P_k)] = [(\sum_{i\in I} T_i) \wedge (\sum_{j\in J} L_j)] \vee (\sum_{k\in U} P_k)$.

Since, T and ϕ are the units of (ET, $\vee$) and (ET, $\wedge$) respectively.

Definition 2.2 (see [6]): If $S \subseteq ET$, then (S, $\wedge$, $\vee$) is called an El sub-algebra of (ET, $\wedge$, $\vee$) if for any $s_1, s_2 \in S$, (1). $s_1 \vee s_2 \in S$, and (2). $s_1 \wedge s_2 \in S$.

Proposition 2.3 (see [1]): Let $\sum_{u\in I} T_u \in ET$. If there exist $j, k \in I$, $j \neq k$ such that $T_j \subseteq T_k$, then $\sum_{u\in I} T_u = \sum_{u\in I, u\neq k} T_u$.

3. Soft El algebra:

Let T and E be any non-empty sets and (ET, $\wedge$, $\vee$) be an El-algebra defined on T. Then a function F: E $\rightarrow$ P(ET) is being characterized as:

$$F(e) = \{\alpha_j \in ET \mid e\ R\ \alpha_j\},\ e \in E,$$

Where, R is a binary relation between E and ET, that is $R \subseteq E \times ET$. Then the pair (F, E) or F_E is known as a soft set over El-algebra ET.

Definition 3.01: A soft set F_E over El-algebra (ET, $\wedge$, $\vee$) is known as a *soft El-algebra* over ET, if for all $e \in E$, F(e) is an El-subalgebra of an El-algebra ET, that is for every $\alpha_i, \alpha_j \in F(e)$ $(i, j \in I)$, $\alpha_i \wedge \alpha_j \in F(e)$ and $\alpha_i \vee \alpha_j \in F(e)$.

Example 3.02: Let T = {1, 2}, E = {e, x} and ET = $\{\alpha_1, \alpha_2\}$ be an El-algebra. Let F: E $\rightarrow$ P(ET) be defined as:

$F(e) = \{\alpha_1\}$
$F(x) = \{\alpha_1, \alpha_2\}$
Where, $\alpha_1 = \{1\}$ and $\alpha_2 = \{1\} \vee \{2\}$.

Now, $\{1\} \vee \{1\} = \{1\}$, $\{1\} \wedge \{1\} = \{1\}$, $[\{1\} \vee \{2\}] \vee [\{1\} \vee \{2\}] = \{1\} \vee \{2\}$, $[\{1\} \vee \{2\}] \wedge [\{1\} \vee \{2\}] = \{1\} \vee \{2\}$.

So, $\{\alpha_1\}$ and $\{\alpha_1, \alpha_2\}$ are El-subalgebras of El-algebra ET, written as $(\alpha_1)_{El}$ and $(\alpha_1, \alpha_2)_{El}$ respectively. Hence α_1, α_2 is the base of ET. Similarly by the definition of Soft El-algebra, F_E is then called Soft El-algebra over ET.

Example 3.03: Let $T = \{t_1, t_2, t_3, t_4\}$ and $ET = \{\sum_{i\in I} T_i \mid i \in I, T_i \subseteq T\}$ or $ET = \{\alpha_1, \alpha_2, ..., \alpha_{13}\}$ be an El-algebra, where
$\alpha_1 = \{t_1\}$, $\alpha_2 = \{t_1\} \vee \{t_2\}$, $\alpha_3 = \{t_3\} \vee \{t_4\}$, $\alpha_4 = \{t_2\} \vee \{t_3\} \vee \{t_4\}$, $\alpha_5 = \{t_1\} \vee \{t_2\} \vee \{t_3\} \vee \{t_4\}$, $\alpha_6 = \{t_1\} \vee \{t_3\} \vee \{t_4\}$, $\alpha_7 = \{t_1, t_3\} \vee \{t_1, t_4\}$, $\alpha_8 = \{t_1, t_2\} \vee \{t_1, t_3\} \vee \{t_1, t_4\}$, $\alpha_9 = \{t_1, t_3\} \vee \{t_1, t_4\} \vee \{t_2, t_3\} \vee \{t_2, t_4\}$, $\alpha_{10} = \{t_3\} \vee \{t_4\} \vee \{t_1, t_2\}$, $\alpha_{11} = \{t_1, t_3\} \vee \{t_1, t_4\} \vee \{t_2\}$, $\alpha_{12} = \{t_1\} \vee \{t_2, t_3\} \vee \{t_2, t_4\}$ and $\alpha_{13} = \{t_1, t_2\} \vee \{t_1, t_3\} \vee \{t_1, t_4\} \vee \{t_2, t_3\} \vee \{t_2, t_4\}$.

Let $E = \{e_1, e_2, e_3, e_4\}$ and G: E $\rightarrow$ P(ET) be defined as:
$G(e_1) = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)_{El}$,
$G(e_2) = \{\alpha_1, \alpha_3, \alpha_{11}\} = (\alpha_1, \alpha_3, \alpha_{11})_{El}$,

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$G(e_3) = \{\alpha_1, \alpha_7\} = (\alpha_1, \alpha_7)_{El}$,
$G(e_4) = \{\alpha_1, \alpha_3, \alpha_6, \alpha_7\} = (\alpha_1, \alpha_3, \alpha_6, \alpha_7)_{El}$.

Then, (G, E) or G_E is called Soft El-algebra over ET.

Definition 3.04: Let G_E be a soft El-algebra defined on ET. Then,
(i). if $G(e) = \phi$ for all $e \in E$, then G_E is referred to *trivial soft El-algebra* over ET.
(ii). if $G(e) = ET$ for all $e \in E$, then G_E is referred to *whole soft El-algebra* over ET.

Theorem 3.05: If F_E and G_H be two soft El-algebras defined on ET, then $F_E \tilde{\wedge} G_H$ is also a soft El-algebra over ET.

Proof: From 1.4, we have $F_E \tilde{\wedge} G_H = K_{E\times H}$, where $K(e, h) = F(e) \cap G(h)\ \forall\ (e, h) \in E \times H$. But F(e) and G(h) are El-subalgebras of El-algebra ET and so their intersection $F(e) \cap G(h)$ is also an El-subalgebra of ET. Therefore K(e, h) is an El-subalgebra of ET for all $(e, h) \in E \times H$. Hence $F_E \tilde{\wedge} G_H = K_{E\times H}$ is a soft El-algebra over ET.

Theorem 3.06: Let F_E and G_H are two non-empty soft El-algebras defined on ET. Then $F_E \tilde{\cap} G_H$ is also soft El-algebra over ET, if $E \cap H \neq \phi$.

Proof: From 1.2, we can write $F_E \tilde{\cap} G_H = K_D$, where $D = E \cap H \neq \phi$ and K(d) = F(d) or G(d) for all $d \in D$. Here, $K: D \rightarrow P(ET)$ is a mapping, and so K_D is a soft set defined on ET. Now, F_E and G_H are soft El-algebras over ET, therefore K(d) = F(d) is an El-subalgebra of ET, or K(d) = G(d) is also an El-subalgebra of ET for all $d \in D$. Therefore, $F_E \tilde{\cap} G_H$ is a soft El-algebra over ET.

Theorem 3.07: Let F_E and G_H are non-empty soft El-algebras defined on ET. If $E \cap H = \phi$, then their soft union i.e., $F_E \tilde{\cup} G_H$ is also a soft El-algebra defined on ET.

Proof: From 1.3, we can write $F_E \tilde{\cup} G_H = K_C$, where $C = E \cup H$ and for every $c \in C$,

$$K(c) = \begin{cases} F(c) & \text{if } c \in E \setminus H, \\ G(c) & \text{if } c \in H \setminus E, \\ F(c) \cup G(c) & \text{if } c \in E \cap H. \end{cases}$$

Since $E \cap H = \phi$, then for all $c \in C$ either $c \in E \setminus H$ or $c \in H \setminus E$. If $c \in E \setminus H$, then K(c) = F(c) is a soft El-subalgebra, and if $c \in H \setminus E$ then K(c) = G(c) is also a soft El-subalgebra, since F_E and G_H are soft El-algebras over ET. Hence $K_C = F_E \tilde{\cup} G_H$ is a soft El-algebra over ET.

Remark: If $E \cap H \neq \phi$ and for every $\alpha_i \in F(e)$ ($i \in I$, $e \in E$), $\beta_j \in G(h)$ ($j \in J$, $h \in H$), $\alpha_i \vee \beta_j \in F(e) \cup G(h)$ and $\alpha_i \wedge \beta_j \in F(e) \cup G(h)$, then $F_E \tilde{\cup} G_H$ is a soft El-algebra defined on ET.

Example 3.08: Let $T = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the set of six persons and E = {A = Age, We = Weight, H = Height, S = Salary, W = Woman, M = Man} be the set of attributes.

Let $ET = \{\sum_{i\in I} T_i \mid T_i \subseteq T, i \in I\}$ and $(ET, \vee, \wedge)$ is an El-algebra. Let B = {A, We, H, S} and C = {A, We, W, M} are two subsets of E. (F, B) and (G, C) are two soft sets over El-algebra ET, defined as:

$F: B \rightarrow P(ET)$

$F(A) = \{\{x_3\}\}$,
$F(We) = \{\{x_2\}, \{x_2\} \vee \{x_5\}\}$,

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$F(H) = \{\{x_3\}, \{x_3\} \vee \{x_2\}, \{x_6\} \vee \{x_5\}, \{x_2\} \vee \{x_6\} \vee \{x_5\}\}$,
$F(S) = \{\{x_1\}, \{x_1, x_3\} \vee \{x_1, x_4\} \vee \{x_2\}, \{x_3\} \vee \{x_4\}\}$.

and, $G: C \rightarrow P(ET)$, be defined as:

$G(A) = \{\{x_3\}\}$,
$G(We) = \{\{x_2\}, \{x_2\} \vee \{x_5\}\}$,
$G(W) = \{\{x_6\}, \{x_6, x_4\} \vee \{x_6, x_3\} \vee \{x_5\}, \{x_4, x_3\}\}$,
$G(M) = \{\{x_1\}\}$.

Now,
(1). From definition 1.2 $F_B \tilde{\cap} G_C = H_D$, where $D = \{A, We\} \neq \phi$, and $H(A) = \{\{x_3\}\}$, $H(We) = \{\{x_2\}, \{x_2\} \vee \{x_5\}\}$ both are El-subalgebra. Hence, H_D is a soft El-algebra.

(2). From definition 1.3 $F_B \tilde{\cup} G_C = H_D$, where $D = B \cup C = \{A, We, H, S, W, M\}$ and

$H(A) = F(A) \cup G(A) = (\{x_3\})_{El}$,
$H(We) = F(We) \cup G(We) = (\{x_2\}, \{x_2\} \vee \{x_5\})_{El}$,
$H(H) = F(H) = (\{x_3\}, \{x_3\} \vee \{x_2\}, \{x_6\} \vee \{x_5\}, \{x_2\} \vee \{x_6\} \vee \{x_5\})_{El}$,
$H(S) = (\{x_1\}, \{x_1, x_3\} \vee \{x_1, x_4\} \vee \{x_2\}, \{x_3\} \vee \{x_4\})_{El}$,
$H(W) = (\{x_6\}, \{x_6, x_4\} \vee \{x_6, x_3\} \vee \{x_5\}, \{x_4, x_3\})_{El}$,
$H(M) = (\{x_1\})_{El}$.

Hence, H_D is a soft El-algebra.

(3). From definition 1.4 $F_B \tilde{\wedge} G_C = H_D$, where $D = B \times C = \{(b, c) \mid b \in B \text{ and } c \in C\}$ and $H(b, c) = F(b) \cap G(c)$ for all $(b, c) \in D$.
So, $H(A, M) = F(A) \cap G(M) = \{\{x_1\}\} = (\{x_1\})_{El} = H(S, M) = H(We, M)$,
$H(We, A) = \{x_3\} = (\{x_3\})_{El} = H(H, A)$,
$H(b, c) = \phi$, $\forall$ $(b, c) \in D-\{(A, M), (S, M), (We, M), (We, A), (H, A)\}$.

Hence, H_D is a soft El-algebra.

Proposition 3.09: Let F_E and G_H are two soft El-algebras over ET. Then $F_E \tilde{\vee} G_H$ need not be a soft El-algebra over ET (see example 3.10).

Example 3.10: From definition 1.5 and Example 3.08, let $F_B \tilde{\vee} G_C = H_D$, where $D = B \times C$ and $H(b, c) = F(b) \cup G(c)$ $\forall$ $(b, c) \in D$.

So, if we take $(A, A) \in D$, then $H(A, A) = \{\{x_3\}\} = (\{x_3\})_{El}$, but if we take $(A, We) \in D$, then $H(A, We) = \{\{x_3\}, \{x_2\}, \{x_2\} \vee \{x_5\}\}$ is not an El-subalgebra of a soft El-algebra $(ET, \vee, \wedge)$. Hence H_D is not a soft El-algebra over ET.

Theorem 3.11: Let F_E be a soft El-algebra defined on ET. If $H \subset E$, then F_H is a soft El-algebra over ET.

Proof: Follow definitions 1.6 and 3.01.

We give following example in which a soft set F_E defined on ET is not a soft El-algebra over ET but there exists $H \subset E$, such that F_H is a soft El-algebra over ET.

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Example 3.12: Consider T = {u, v, w} be any set and ET = {α_1, α_2, α_3, α_4, α_5, α_6, α_7, α_8} be an El-algebra where, α_1 = {u}, α_2 = {u} $\vee$ {v, w}, α_3 = {v} $\vee$ {w}, α_4 = {u, v} $\vee$ {v, w}, α_5 = {u} $\vee$ {v} $\vee$ {w}, α_6 = {u, v} $\vee$ {u, w}, α_7 = {u, v}, α_8 = {u, v} $\vee$ {u, w} $\vee$ {v, w}. Let G_E be a soft set over El-algebra ET, that is G: E $\rightarrow$ P(ET) and E = {e_1, e_2, e_3, e_4, e_5}, such that

$G(e_1) = \{\alpha_1, \alpha_2\}$,
$G(e_2) = \{\alpha_4, \alpha_6\}$,
$G(e_3) = \{\alpha_4, \alpha_6, \alpha_7, \alpha_8\}$,
$G(e_4) = \{\alpha_1, \alpha_3, \alpha_7\}$,
$G(e_5) = \{\alpha_2, \alpha_5\}$.

Since $G(e_2)$ and $G(e_4)$ are not an El-subalgebras of ET, so G_E is not a soft El-algebra. But, when H = {e_1, e_3, e_5} $\subset$ E, then G_H is a soft El-algebra defined on ET.

4. Soft El-subalgebra:

Definition 4.1: Let F_E and G_H are two soft El-algebras over El-algebra ET. Then G_H is said to be a *soft subalgebra* of F_E, if it meets the following criteria:
(i) H $\subset$ E,
(ii) G(h) is an El-subalgebra of F(h) for all h $\in$ H.
It can be written as $G_H \lesssim F_E$.

Example 4.2: Let ET = {α_1, α_2, α_3, α_4, α_5, α_6, α_7, α_8} be an El-algebra defined in Example 3.12 and F_E be a soft El-algebra defined as: F: E $\rightarrow$ P(ET), as

$F(e_1) = \{\alpha_1, \alpha_2\}$,
$F(e_2) = \{\alpha_4, \alpha_6, \alpha_7, \alpha_8\}$,
$F(e_3) = \{\alpha_4, \alpha_6, \alpha_7, \alpha_8\}$,
$F(e_4) = \{\alpha_1, \alpha_3, \alpha_5, \alpha_6, \alpha_7\}$,
$F(e_5) = \{\alpha_2, \alpha_5\}$.

Now, we take H = {e_1, e_4, e_5} as a subset of E and G_H be a soft set defined as: G: H $\rightarrow$ P(ET), such that

$G(e_1) = \{\alpha_1\}$,
$G(e_4) = \{\alpha_1, \alpha_3, \alpha_5, \alpha_6\}$,
$G(e_5) = \{\alpha_5\}$.

Note that $G(e_1)$, $G(e_4)$ and $G(e_5)$ are El-subalgebra of $F(e_1)$, $F(e_4)$ and $F(e_5)$ respectively. Hence G_H is a soft El-subalgebra of F_E.

Theorem 4.3: Let F_E be a soft El-algebra defined on ET and $G_H \lesssim F_E$, $K_D \lesssim F_E$. Then
(i) $G_H \tilde{\cap} K_D \lesssim F_E$,
(ii) If $H \cap D = \phi$, then $G_H \tilde{\cup} K_D \lesssim F_E$.

Proof: (i) From Definition 1.2, we can write

$$G_H \tilde{\cap} K_D = R_S$$

Where, $S = H \cap D$ and R(s) = G(s) or K(s), $\forall$ s $\in$ S. Obviously, S $\subset$ E. Let s $\in$ S. Then s $\in$ H and s $\in$ D. If s $\in$ H, then R(s) = H(s) and if s $\in$ D, then R(s) = K(s). Here, both G(s) and K(s) are El-subalgebras of F(s) since $G_H \lesssim F_E$ and $K_D \lesssim F_E$. Hence, $G_H \tilde{\cap} K_D = R_s \lesssim F_E$.
(ii) Assume that $H \cap D = \phi$. We can write $G_H \tilde{\cup} K_D = R_S$ where, $S = H \cup D$ and

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$$R(s) = \begin{cases} G(s) & \text{if } s \in H \setminus D, \\ K(s) & \text{if } s \in D \setminus H, \\ G(s) \cup K(s) & \text{if } s \in H \cap D. \end{cases} \quad \forall\, s \in S.$$

Since $G_H \tilde{<} F_E$, $K_D \tilde{<} F_E$, $S = H \cup D \subset E$, and G(s) and K(s) are El-subalgebras of F(s) for all $s \in H$ or $s \in D$. Since $H \cap D = \phi$, so G(s) is an El-subalgebra of F(s), $\forall\, s \in S$. Hence, $G_H \tilde{\cup} K_D = R_S \tilde{<} F_E$.

5. Homomorphism on Soft El-algebras:

Let ET_1 and ET_2 are two soft El-algebras, and $g: ET_1 \to ET_2$ be a map. For a soft set H_E over ET_1, $g(H)_E$ is a soft set defined on ET_2. Here, $g(H): E \to P(ET_2)$ be a mapping described by $g(H)(e) = g(H(e))$ for all $e \in E$.

Lemma 5.1: Let $g: ET_1 \to ET_2$ be a homomorphism between El-algebras ET_1 and ET_2. If H_E is a soft El-algebra defined on ET_1, then $g(H)_E$ is also a soft El-algebra defined on ET_2.

Proof: Although H(e), for all $e \in E$ is an El-subalgebra of an El-algebra ET_1 and $g(H)(e) = g(H(e))$. Now, g be a homomorphism between El-algebras ET_1 and ET_2. Also, we know that homomorphic image of an El-subalgebra must be an El-subalgebra. Therefore, $g(H)_E$ is a soft El-algebra defined on ET_2.

Theorem 5.2: Let $g: ET_1 \to ET_2$ be a homomorphism between El-algebras ET_1 and ET_2 and G_E be a soft El-algebra defined on ET_1.
(i) if G(e) = ker(g) for all $e \in E$, then $g(G)_E$ is the trivial soft El-algebra over ET_2.
(ii) if g is onto homomorphism and G_E is a whole soft El-algebra defined on ET_1, then $g(G)_E$ is also a whole soft El-algebra defined on ET_2.

Proof: Let ϕ_1 and ϕ_2 are the identities of El-algebras ET_1 and ET_2 respectively, and $\ker(g) = \{\alpha \in ET_1 \mid g(\alpha) = \phi_2\}$.
(i) Consider that G(e) = ker(g) for all $e \in E$. But g is a homomorphism, and so $\ker(g) = \{\phi_1\}$. Therefore $g(G)(e) = g(G(e)) = g(\{\phi_1\}) = \{\phi_2\}$ for all $e \in E$. Hence $g(G)_E$ is the trivial soft El-algebra defined on ET_2 from Lemma 5.1 and Definition 3.04.
(ii) Assume that g is an onto homomorphism and G_E is a whole soft El-algebra over ET. Therefore, $G(e) = ET_1$ for all $e \in E$, and so $g(G)(e) = g(G(e)) = g(ET_1) = ET_2$ for all $e \in E$. Hence from lemma 5.1 and Definition 3.04, $g(G)_E$ is also a whole soft El-algebra defined on ET_2.

Theorem 5.3: Let $g: ET_1 \to ET_2$ be a homomorphism between El-algebras ET_1 and ET_2. Let F_E and G_H are two soft El-algebras over ET_1. Then

$$F_E \tilde{<} G_H \Rightarrow g(F)_E \tilde{<} g(G)_H.$$

Proof: Consider that $F_E \tilde{<} G_H$. Let $e \in E$. Then $E \subset H$ and F(e) is an El-subalgebra of G(e). Now, g is a homomorphism, so $g(F)(e) = g(F(e))$ is an El-subalgebra of $g(G)(e) = g(G(e))$. Hence, $g(F)_E \tilde{<} g(G)_H$.

CONCLUSION

The present paper gives some essential and compulsory propositions which provides the base to the investigation of El-algebras in soft set theory. These results can be used to study the algebraic structure of El-algebras. El-algebras has expected applications in data mining and fuzzy clustering analysis. We had examined our results through examples at length, which will be helpful in additional studies.

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