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Transformations And Solutions of Integral Equation Involving Bessel Maitland Function

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ABSTRACT

In this paper, we have solved integral equation involving Bessel Maitland function. In case of single kernel the equations have been transform by using Erdélyi-Kober operators to one having Fox's *H*-function while the equations having summation of two or more Bessel Maitland functions have been transformed into a summation of two or more *H*-functions as kernel. In first case the solutions are expressed in terms of *H*-function while in second, the solutions are in terms of Saxena's *I*-function. The particular case is also obtained.

Keywords: Bessel Maitland function, Erdélyi-Kober fractional integral operators, Fox's *H*- function, Saxena's *I*-function.

1 Introduction

The fractional integrals have played an important role in the definitions and development of theory of special function. The theory of fractional integral operators is very useful in the solution of various integral equations. Various definitions of fractional integrations have been given by many authors viz., Kober [6], Erdélyi [2], Saxena [8] etc.

In this paper we have obtained solution of an integral equation in which Bessel Maitland function occurs as a part of kernel. Erdélyi-Kober fractional integral operators have been applied to transform the kernel in desired form, so that the conditions of unsymmetrical fourier kernels are satisfied. These results may be useful in many applications of integral equations.

Throughout this paper, we shall follow the following notations and definitions.

- **Fractional Integrals**

Fox [9] has used following generalized Erdélyi- Kober operators

$$(1.1)\qquad \Im[\gamma,\varepsilon{:}\,m]\;g(t)=\frac{m}{\Gamma\gamma}t^{-\varepsilon-\gamma m+m-1}\int_0^t (t^m-u^m)^{\gamma-1}u^{\varepsilon}\,g(u)du$$

and

$$(1.2)\qquad \Re[\gamma,\varepsilon{:}\,m]\;g(t)=\frac{m}{\Gamma\gamma}t^{\varepsilon}\int_t^\infty (u^m-t^m)^{\gamma-1}u^{\varepsilon-\gamma m+m-1}\,g(u)du\quad,$$

where $\Im$ exists if $g(t)\,\epsilon\,L_p(0,\infty), p\geq 0, \gamma>0. \varepsilon>\frac{1-p}{p}$. If in addition $g(t)$ can be differentiated sufficiently often, then the operator $\Im$ exists for both negative and positive values of γ.

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$\Re$ exists if $g(t) \epsilon L_p(0,\infty), p \geq 1$. If $g(t)$ can be differentiated sufficiently often, then $\Re$ exists if $m > 0, \varepsilon > -\frac{1}{p}$ while γ can take any negative or positive values.

The Beta function in the integral forms are most useful for the application of the above mentioned $\Im$ and $\Re$ operators. These integrals are given as

$$(1.3) \qquad \int_0^t (t^{\frac{1}{g_1}} - u^{\frac{1}{g_1}})^{a_1-\tau_1-1} u^{\frac{\tau_1}{g_1}-1-\xi}\, du = \frac{g_1\Gamma(a_1-\tau_1)\Gamma(\tau_1-g_1\xi)}{\Gamma(a_1-g_1\xi)} t^{\frac{a_1}{g_1}-\frac{1}{g_1}-\xi} \quad ,$$

provided $a_1 > \tau_1$ and $\frac{\tau_1}{g_1} > c, (\xi = c + it)$.

$$(1.4) \qquad \int_t^\infty (u^{\frac{1}{f_1}} - t^{\frac{1}{f_1}})^{b_1-\mu_1-1} u^{\frac{1}{f_1}-\frac{b_1}{f_1}-\xi-1}\, du = \frac{f_1\Gamma(b_1-\mu_1)\Gamma(\mu_1+f_1\xi)}{\Gamma(b_1+f_1\xi)} t^{-\frac{\mu_1}{f_1}-\xi}$$

provided $b_1 > \mu_1$ at the lower limit and $f_1 Re\,(\xi) + \mu_1 > 0$ at the upper limit.

- **Mellin Transform**

Let $g(x)$ be a function on the positive real axis $0 < x < \infty$. The Mellin transformation M is the operation mapping the function g into the function G defined on the complex plane by the relation [1]

$$(1.5) \qquad M\{g(x):p\} = G(p) = \int_0^\infty g(x)\, x^{p-1} dx \quad .$$

The function $G(p)$ is called the Mellin transform of $g(x)$.

- **Inversion Formula**

The inversion formula of Mellin transform [1] is given as

$$(1.6) \qquad M^{-1}\{G(p):x\} = g(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} G(p)\, x^{-p} dp \quad ,$$

where, the integration is along a vertical line through $Re(p) = c$.

- **Parseval's Theorem (Convolution theorem)**

Parseval's theorem for the Mellin transform is given as follows [9]

$$(1.7) \qquad \int_0^\infty f(tx)\; h(t)\; dt = \lim_{T\to\infty} \frac{1}{2\pi i}\int_{c-iT}^{c+iT} x^{-p} F(p)\; H(1-p) dp \quad ,$$

Additional conditions, for the validity of (1.7), are that

$H(p)\, \epsilon\, L_n(c-i\infty, c+i\infty)\ and\ x^{1-c} f(x)\, \epsilon\, L_n(0,\infty)\,; n \geq 1,$

where we denote by L_n the class of functions $f(x)$ such that $\int_0^\infty |f(x)|^n \frac{dx}{x} < \infty$.

- **Bessel Maitland function**

The special function of the form defined by the series representation [12]

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(1.8) $$J_a^b(z) = \sum_{r=0}^{\infty} \frac{(-1)^r z^r}{\Gamma(1+a+br)} \quad ,$$

is known as Wright generalized Bessel function or Bessel Maitland function. It has a wide application in the problem of physics, chemistry, biology, engineering and applied sciences.

- **The *I*- Function**

The *I*-function defined by V.P.Saxena is the latest and most general form of hyper geometric functions. This function emerged by itself while solving a class of dual integral equations involving Fox's *H*- function as kernel. The *I*- function [12] is defined in terms of following Mellin - Barnes type integral

(1.9) $$I(z) = I_{p_l,q_l;r}^{m,n}[z] = I_{p_l,q_l;r}^{m,n}\left[z \left| \begin{matrix} (a_j,\alpha_j)_{1,n}\ ; & (a_{jl},\alpha_{jl})_{n+1,p_l} \\ (b_j,\beta_j)_{1,m}; & (b_{jl},\beta_{jl})_{m+1,q_l} \end{matrix} \right.\right]$$
$$= \frac{1}{2\pi i}\int_L \Omega(\xi) z^{\xi} d\xi$$

where

$$\Omega(\xi) = \frac{\prod_{j=1}^m \Gamma(b_j-\beta_j\xi)\ \prod_{j=1}^n \Gamma(1-a_j+\alpha_j\xi)}{\sum_{l=1}^r \left[\prod_{j=m+1}^{q_l}\Gamma(1-b_{jl}+\beta_{jl}\xi)\ \prod_{j=n+1}^{p_l}\Gamma(a_{jl}-\alpha_{jl}\xi)\right]} \quad ,$$

p_l, q_l $(l = 1,2,\dots,r)$, *m* and *n* are integers satisfying $0 \le n \le p_l$, $0 \le m \le q_l$ $(l = 1,2,\dots,r)$, r is finite, α_j, β_j, α_{jl}, β_{jl} are real and positive and a_j, b_j, a_{jl}, b_{jl} are numbers such that

$$\alpha_k(b_h+u) \ne \beta_h(a_k-1-k)\ for\ k,u = 0,1,2\dots;\ h = 1,2,\dots,m;\ l = 1,2,..,r \quad .$$

L is contour running form $\sigma - i\infty$ to $\sigma + i\infty$,where σ is real in the complex μ-plane such that the points

$$\mu = \frac{(a_j-1-u)}{\alpha_j}, \qquad j = 1,2,\dots n\ ;\ u = 0,1,2,\cdots$$
$$\mu = \frac{(b_j+u)}{\beta_j}, \qquad j = 1,2\cdots,m\ ;\ u = 0,1,2,\cdots$$

lie to the left- and right-hand sides of *L* respectively.

The *I*-function converges absolutely in ξ-plane if

(i) $A > 0, |argz| < \frac{\pi}{2} A$

(ii) $A \ge 0, |argz| \ge \frac{\pi}{2} A$, Re $(B+1) < 0$,

where

(1.10) $$A = \sum_{j=1}^{n}\alpha_j + \sum_{j=1}^{m}\beta_j - \max_{1\le l\le r}\left[\sum_{j=n+1}^{p_l}\alpha_{jl} + \sum_{j=m+1}^{q_l}\beta_{jl}\right]$$

and

(1.11) $$B = \sum_{j=1}^{m} b_j + \sum_{j=1}^{n} a_j - \max_{1\le l\le r}\left[\sum_{j=n+1}^{p_l} a_j - \sum_{j=m+1}^{q_l} b_j + \frac{p_l}{2} - \frac{q_l}{2}\right].$$

If we put $r = 1$ in (1.9), it reduces Fox’s *H*-function (see [4], [7], [10] and [11]).

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2. Problem and solution.

We consider the integral equation of the type

$$(2.1) \int_0^\infty u^\alpha J_\mu^\lambda(ux)\, f(u)du = g(x), \qquad x > 1 \qquad ,$$

Here α, λ and μ are arbitrary real numbers and the kernel does not satisfy condition for direct inversion. Hence we transform the same to Fox's H-function which makes the equation eligible to find the solution. Here g(x) is known and f(x) is to be determined. $J_\mu^\lambda(u)$ is Bessel Maitland function defined in (1.8).

We have [3]

$$(2.2) \qquad M\left\{u^\alpha J_\mu^\lambda(u)\right\} = \frac{\Gamma(\alpha+\xi)}{\Gamma(1+\mu-\lambda\alpha-\lambda\xi)} \quad .$$

Applying Parseval's theorem of the Mellin transforms in (2.1) from (1.7), we obtain

$$(2.3) \qquad \lim_{t\to\infty} \frac{1}{2\pi i}\int_{\sigma_0-it}^{\sigma_0+it} \frac{\Gamma(\alpha+\xi)}{\Gamma(1+\mu-\lambda\alpha-\lambda\xi)} x^{-\xi} F(1-\xi)d\xi = g(x), x > 1 \quad ,$$

where $\xi = \sigma_0 + it$ and $F(\xi)$ is Mellin transform of f (u).

Now we shall use operator $\Re$ defined in (1.2) and the following definition of Beta function [5]

$$(2.4) \qquad \int_x^\infty \left(v^{\frac{1}{f_1}} - x^{\frac{1}{f_1}}\right)^{b_1-e_1-1} v^{\frac{1}{f_1}-\frac{b_1}{f_1}-\xi-1} = \frac{f_1\Gamma(b_1-e_1)\Gamma(e_1+f_1\xi)}{\Gamma(b_1+f_1\xi)} x^{-\frac{\mu_1}{f_1}-\xi}.$$

For convergence of (2.4), we require $b_1 > e_1$ at the lower limit and $f_1\,Re(\xi) + e_1 > 0$ at the upper limit. But when the fractional integral operator $\Re$ is introduced, some of these conditions may no longer be necessary.

Now we replace x by v in (2.3), multiply both the sides by

$$\int_x^\infty \left(v^{\frac{1}{f_1}} - x^{\frac{1}{f_1}}\right)^{b_1-e_1-1} v^{\frac{1}{f_1}-\frac{b_1}{f_1}-1}$$

and integrate with respect to v from x to ∞ and using (2.4).Consequently we have

$$(2.5) \qquad \frac{1}{2\pi i}\int \frac{\Gamma(\alpha+\xi)\Gamma(e_1+f_1\xi)}{\Gamma(1+\mu-\lambda\alpha-\lambda\xi)\Gamma(b_1+f_1\xi)} x^{-\xi}F(1-\xi)d\xi$$

$$= \frac{1}{f_1\Gamma(b_1-e_1)} x^{\frac{\mu_1}{f_1}} \int_x^\infty \left(v^{\frac{1}{f_1}} - x^{\frac{1}{f_1}}\right)^{b_1-e_1-1} v^{\frac{1}{f_1}-\frac{b_1}{f_1}-1} g(v)dv \qquad ,$$

Now, we introduce fractional integral operator $\Re$ defined in (1.2), we write

$$(2.6) \qquad \Re\left[b_1 - e_1, \frac{\mu_1}{f_1} : \frac{1}{f_1}\right] g(x) = \Re_1\,[g(x)] \;.$$

Again, if we apply the operator

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$$\Re\left[b_2 - e_2, \frac{e_2}{f_2}: \frac{1}{f_2}\right]$$

Then the equation (2.5) becomes

$$(2.7)\ \frac{1}{2\pi i}\int_L \frac{\Gamma(\alpha+\xi)\Gamma(e_1+f_1\xi)\Gamma(e_2+f_2\xi)}{\Gamma(1+\mu-\lambda\alpha-\lambda\xi)\Gamma(b_1+f_1\xi)\Gamma(b_2+f_2\xi)} x^{-\xi} F(1-\xi)d\xi = \Re_2\Re_1[g(x)],$$

Continuing in same manner up to m-times, we obtain

$$(2.8)\ \frac{1}{2\pi i}\int_L \frac{\Gamma(\alpha+\xi)\prod_{k=1}^{m}\Gamma\{e_k+f_k\xi\}}{\Gamma(1+\mu-\lambda\alpha-\lambda\xi)\prod_{k=1}^{m}\Gamma\{b_k+f_k\xi\}} x^{-\xi} F(1-\xi)d\xi = \prod_{k=1}^{m}\{\Re_k\,[g(x)]\,\}, x>1\,,$$

where

$$(2.9)\qquad g_1(x) = \prod_{k=1}^{m}\{\Re_k\,[g(x)]\,\}, x>1.$$

Now applying Parseval's theorem of Mellin transforms (1.7), we have obtained the transformation of the given equation (2.1) as

$$(2.10)\quad \int_0^\infty H_{m,m+2}^{m+1,0}\left[ux\left|\begin{matrix}(\cdots\cdots),(b_k,f_k)_{1,m},(\cdots\cdots)\\(\alpha,1),(e_k,f_k)_{1,m},(1+\mu-\lambda\alpha,\lambda)\end{matrix}\right.\right] f(u)du = g_1(x), x>1.$$

In this way our problem has been reduced to finding the solution of integral equation (2.10).Now from (1.5) and (2.8), we get

$$(2.11)\ G(\xi) = \frac{\Gamma(\alpha+\xi)\prod_{k=1}^{m}\Gamma\{e_k+f_k\xi\}}{\Gamma(1+\mu-\lambda\alpha-\lambda\xi)\prod_{k=1}^{m}\Gamma\{b_k+f_k\xi\}} F(1-\xi)\ ,$$

where $G(\xi)$ is the Mellin transform of $g_1(x)$ and the parameters e_k, b_k and f_k are chosen in such a way that the kernel fulfills the conditions for inversion.

From (2.11), we have

$$(2.12)\quad F(s) = \frac{\Gamma(1+\mu-\lambda\alpha-\lambda+\lambda s)\prod_{k=1}^{m}\Gamma\{b_k+f_k-f_k s\}}{\Gamma(\alpha+1-s)\prod_{k=1}^{m}\Gamma\{e_k+f_k-f_k s\}}\ G(1-s)\quad ,$$

Taking Mellin inverse on both sides, we have

$$(2.13)\ f(x) = \frac{1}{2\pi i}\int_L \frac{\Gamma(1+\mu-\lambda\alpha-\lambda+\lambda s)\prod_{k=1}^{m}\Gamma\{b_k+f_k-f_k s\}}{\Gamma(\alpha+1-s)\prod_{k=1}^{m}\Gamma\{e_k+f_k-f_k s\}}\ x^{-s}\,G(1-s)ds\ \ ,$$

Again, applying Parseval's theorem of defined in (1.7), we find that (2.13) takes the form

$$(2.14)\ f(x) = \int_0^\infty H_{m,m+2}^{1,m}\left[ux\left|\begin{matrix}(\cdots\cdots),(b_k+f_k,f_k)_{1,m},(\cdots\cdots)\\(1+\mu-\lambda\alpha-\lambda,\lambda),(e_k+f_k,f_k)_{1,m}\,,(\alpha+1,1)\end{matrix}\right.\right] g_1(u)du\,.$$

Now we will extend the above result. If we take an integral equation involving summation of N-Bessel Maitland functions instead of single function and apply the same process of above result, then we will find the transformation of this equation in terms of summation of H-functions and solution in term of I-function.

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Theorem I. If f(x) is the solution of the integral equation

$$(2.15)\qquad \int_0^\infty \sum_{j=1}^N u^{\alpha_j} J_{\mu_j}^{\lambda_j}(ux) f(u)du = g(x), \quad x > 1,$$

where α, λ and μ are arbitrary real numbers.

Then

$$(2.16)\quad f(x) = \int_0^\infty I_{m+N-1,m+N+1;r}^{N,m} \left[ux \left| \begin{matrix} (A_j, \gamma_j)_{1,m}, (A_{ji}, \gamma_{ji})_{1,N-1} \\ (B_j, \delta_j)_{1,N}, (B_{ji}, \delta_{ji})_{1,m+1} \end{matrix} \right. \right] g_1(u)du ,$$

where

$g_1(x) = \prod_{k=1}^m \{\mathfrak{R}_k [g(x)]\}$, $\mathfrak{R}\left[b_k - e_k, \frac{\mu_k}{f_k} : \frac{1}{f_k}\right] g(x) = \mathfrak{R}_k [g(x)], A_j = 1 - b_k - f_k, \gamma_j = f_k, (j, k = 1,2, \dots m); B_j = 1 - \lambda_j + \mu_j - \lambda_j \alpha_j, \delta_j = \lambda_j, (j = 1,2, \dots, N);$ $A_{ji} = 1 - \lambda_{ji} + \mu_{ji} - \lambda_{ji}\alpha_{ji}, \gamma_{ji} = \lambda_{ji}, (j = 1,2, \dots, N-1; i = 1,2, \dots, r), B_{ji} = 1 - e_{ki} - f_{ki}, \delta_{ji} = f_{ki}, (j, k = 1,2, \dots m; i = 1,2, \dots, r);$ $B_{m+1,i} = -\alpha_i, \delta_{m+1,i} = 1, (i = 1,2, \dots, r),$

provided

(i) $\lambda' > 0,\ |arg\, x| < \frac{1}{2}\lambda'\pi$

(ii) $\lambda' \geq 0,\ |arg\, x| \leq \frac{1}{2}\lambda'\pi, Re\,(\mu' + 1) < 0$

where

$$(2.17)\quad \lambda' = \sum_{j=1}^m \gamma_j + \sum_{j=1}^N \delta_j - \max_{1\leq i\leq r}\left[\sum_{j=1}^{N-1} \gamma_{ji} + \sum_{j=1}^{m+1} \delta_{ji}\right]$$

and

$$(2.18)\quad \mu' = \sum_{j=1}^N B_j - \sum_{j=1}^m A_j - \min_{1\leq i\leq r}\left[\sum_{j=1}^{N-1} A_{ji} - \sum_{j=1}^{m+1} B_{ji} - 1\right]$$

Proof. Applying Parseval's theorem of the Mellin transforms in (2.15) from (1.7), we obtain

$$(2.19)\quad \lim_{t\to\infty} \frac{1}{2\pi i} \int_{\sigma_0 - it}^{\sigma_0 + it} \sum_{j=1}^N \frac{\Gamma\left(\alpha_j + \xi\right)}{\Gamma\left(1 + \mu_j - \lambda_j\alpha_j - \lambda_j\xi\right)} x^{-\xi} F(1-\xi)d\xi = g(x), x > 1$$

where $\xi = \sigma_0 + it$ and $F(\xi)$ is Mellin transform of *f (u).*

Proceeding on the same lines of above and apply the second fractional integral operator $\mathfrak{R}$ defined in (1.2).we obtain the transformation of the equation (2.15) as

$$(2.20)\quad \int_0^\infty \sum_{j=1}^N H_j{}_{m+2,m}^{0,m+1} \left[ux \left| \begin{matrix} (\cdots\cdots), (b_k, f_k)_{1,m}, (\cdots\cdots) \\ (\alpha_j, 1), (e_k, f_k)_{1,m}, \left(1 + \mu_j - \lambda_j\alpha, \lambda_j\right) \end{matrix} \right. \right] f(u)du = g_1(x),$$

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where $g_1(x)$ is defined in (2.9).Proceeding on the similar lines as above, we obtain

$$(2.21)\quad f(x) = \int_L \frac{\prod_{j=1}^{N} \Gamma\left(1+\mu_j-\lambda_j\alpha_j-\lambda_j+\lambda_j s\right) \prod_{k=1}^{m} \Gamma\{b_k+f_k-f_k s\}\ x^{-s}\ G(1-s)ds}{\sum_{i=1}^{r}\ [\Gamma(\alpha_i+1-s) \prod_{j=1}^{N-1} \Gamma\left(1+\mu_{ji}-\lambda_{ji}\alpha_{ji}-\lambda_{ji}+\lambda_{ji}s\right) \prod_{k=1}^{m} \Gamma\{e_{ki}+f_{ki}-f_{ki}s\}]}\ .$$

Again, applying Parseval's theorem defined in (1.7), in this way we finally obtain solution of Theorem I.

Theorem II. $If\ \alpha, \lambda\ and\ \mu\ are\ arbitrary\ real\ numbers\ and\ if$

$$(2.22)\qquad \int_0^\infty \sum_{j=1}^{N}\ u^{\alpha_j} J_{\mu_j}^{\lambda_j}(ux) f(u)du\ = g(x),\ 0<x<1\ , Then$$

$$(2.23)\quad f(x) = \int_0^\infty I_{m+N-1,m+N+1;r}^{m+N,0}\left[ux\left|\begin{matrix}\left(a_k-g_k, g_k\right)_{1,m}, \left(1-\lambda_{ji}+\mu_{ji}-\lambda_{ji}\alpha_{ji}, \lambda_{ji}\right)_{1,N-1}\\ \left(1-\lambda_j+\mu_j-\lambda_j\alpha_j, \lambda_j\right)_{1,N}, (\tau_k-g_k, g_k)_{1,m}, \left(-\alpha_j, 1\right)\end{matrix}\right.\right] h_1(u)du\ ,$$

where

$$h_1(x) = \prod_{k=1}^{m} \{\Im_k\ [g(x)]\ \},\ \Im_k[g(x)] = \Im[a_k-\tau_k, \tau_k g_k^{-1}-1\ : g_k^{-1}]\ g(x),\ \ 0<x<1.$$

provided $a_k > \tau_k$ *and* $\frac{\tau_k}{g_k} > c, (\xi = c+it), m>0, N>0, Re(\tau_k - min\ a_k) > 0, (k=1,2,\dots,m)$ *and other conditions of I-function are same as given in (1.9).*

Proof. Now, to establish the next inversion, we shall use the operator $\Im$, defined in (1.1) and the definition of Beta function (1.3).

Further we replace *x* by *v* in (2.19) and multiply both the sides by

$$\left(x^{\frac{1}{g_1}} - v^{\frac{1}{g_1}}\right)^{a_1-\tau_1-1} v^{\frac{\tau_1}{g_1}-1}$$

and integrate under the integral sign with respect to 0 to *x* and apply the same process as above.

We obtain the transformations of the integral equation (2.22) is given as

$$(2.24)\quad \int_0^\infty \sum_{j=1}^{N} H_{j\ \ m,m+2}^{\ 1,m}\left[ux\left|\begin{matrix}(\dots\dots), (\tau_k, g_k)_{1,m}, (\dots\dots)\\ (\alpha_j, 1), (a_k, g_k)_{1,m}, \left(1+\mu_j-\lambda_j\alpha, \lambda_j\right)\end{matrix}\right.\right] f(u)du = \ h_1(x)\ ,$$

where

$$(2.25)\qquad h_1(x) = \prod_{k=1}^{m} \{\Im_k\ [g(x)]\ \},\qquad 0<x<1\ .$$

by proceeding on similar lines as above, we will obtain solution of Theorem II.

***Particular case*:** If we put $N=1$ in (2.22), then solution (2.23) of the theorem II reduces to the form (2.26)

$$f(x) = \int_0^\infty H_{m,m+2}^{m+1,0}\left[ux \left| \begin{matrix} (\cdots\cdots), (a_k - g_k, g_k)_{1,m} \\ (1+\mu-\lambda\alpha-\lambda, \lambda), (\tau_k - g_k, g_k)_{1,m}, (-\alpha, 1) \end{matrix} \right.\right] h_1(u)du,$$

where $h_1(x)$ is defined in (2.25). Provided $m > 0, Re(\tau_k - min\, a_k) > 0, (k = 1,2, \dots, m)$ and other conditions of *I*-function are same as given in (1.9) with $r = 1$.

Conclusion

The integral transform theory in mathematical analysis was given time to time which involves various transcendental functions as kernel. The theory of fractional integral operators is very useful in the solution of various integral equations. In this work, we define the integral transform involving Bessel Maitland function as unsymmetrical Fourier kernel. We have used Erdélyi-Kober operators in the form of fractional integral to make head way to arrive at an integral equation which involves *H*-function and solutions in terms of *I*-function has been obtained. These results may be useful in finding solutions of problems in mathematical physics and engineering which are expressed as integral equations.

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