

Fuzzy Time series and its Methods

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ABSTRACT

In this paper, introduced the fuzzy time series in moving average and ratio to moving average method. Fuzzy time series forecasting methods have been developed and applied in a wide variety of applications. This paper also outlines the current company profits, productions, achievements, limitations, and suggestions for future research associated with the fuzzy time series forecasting.

Keywords: Time series, Fuzzy Time series, Ranking Function, Moving Average, Odd period Moving average, Even period Moving average, Seasonal Index.

[1] Introduction

In 1965 by Zadeh, fuzzy set theory has enjoyed fruitful achievement both in theory and applications. The motivation of fuzzy set theory is mainly to provide a format, powerful and quantitative framework.

A fuzzy set theory is being applied into wider and wider areas, such as decision making, planning, logic, system theory, economics, and control theory and so on. The use of fuzzy sets for modeling and predicting time series arises almost intuitively, first based on the ability of fuzzy models to approximate functions, but also on the readability of rules using linguistic variables that make them more accessible to experts and non-experts analysis. Time series forecasting models based on a linear relationship model show great performance. However, these models cannot handle the data that are incomplete, imprecise, and ambiguous as the interval-based fuzzy time series models since the process of fuzzification is abandoned. This article proposes a novel fuzzy time series forecasting model based on multiple linear regression and time series clustering for forecasting market prices. Q. Cai, D. Zhang, B. Wu, and S. Leung[2], proposed a fuzzy time series forecasting model based on intuitionistic fuzzy logical relations, respectively. Fuzzy time-series approach is an effective tool to overcome the above mentioned drawbacks of classical time series models. Some other advantages of fuzzy time series models can be interpreted as follows: fuzzy models are effectively applied in complex and optimization problems [3]; Traditional time series data are based on numerical values which are unreliable whereas fuzzy time series data are based on linguistic values. The use of fuzzy time series was expanded due to its potential to cope with incomplete and ambiguous data [4] Time Series Analysis is used for many applications such as: Economic Forecasting, Sales Forecasting, Budgetary Analysis, Stock Market Analysis, Census Analysis and etc...

Fuzzy time series is developed rapidly since their introduction by Song and Chissom Current fuzzy time series methods have benefited from both theoretical developments as well as relevant applications in research which has led to more diverse uses. This trend indicates that the development of fuzzy time series has markedly improved.

In this paper, some preliminaries presented in section 2. Section 3, illustrates the new approach with numerical examples. Conclusions are discussed in section 4.

[2] Preliminaries

Definition 2.1. An arrangement of statistical data in accordance with time of occurrence is called a time series. It consists data arranged chronologically.

Definition 2.2. A time series is a series of data points indexed in time order. Most commonly, a time series is a sequence taken at successive equally spaced points in time. Thus it is a sequence of discrete-time data.

Definition2.3. A Time-Series represents a series of time-based orders. It would be Years, Months, Weeks, Days, Hours, Minutes, and Seconds. A time series is an observation from the sequence of discrete-time of successive intervals. A Time Series Analysis (TSA) is used in different fields for time-based predictions – like Weather Forecasting, Financial, Signal processing, Engineering domain – Control Systems, Communications Systems.

Definition2.4. Components of Time Series Analysis:(i) Secular Trend (T)(ii) Seasonal Variations (S)(iii) Cyclical Variations (C)(iv) Irregular Variations (I)

Definition2.5.Secular Trend: The basic tendency of a seires to grow or decline over a period of time and In which there is no fixed interval and any divergence within the given dataset is a continuous timeline. The trend would be Negative or Positive or Null Trend. In Secular Trend can be measured using the following methods (i) Freehand or graphic method (ii) Method of Semi –averages (iii) Method of moving average (iv) Method of Least squares

Definition2.6. Seasonal Variations (S):Seasonal Variations are variations which occur with some degree of regularity within a specific period of one year or shorter. Seasons could be weekly, monthly, quarterly or half-yearly depending on the nature. Variations that occur regularly and periodically with period less than one year are Seasonal Variations in a time series. Seasonal movements affect economic and business phenomena and so knowledge about them is necessary in decision making. The following are the important methods of studying seasonal variations (i) Method of Simple average (ii) Ratio to Trend Method (iii) Ratio to Moving Average Method (iv) Link Relative Method

Definition2.7.Cyclical Variations (C): Cyclical Variations in a time series are the recurrent variations whose duration is more than one year.

Definition2.8.Irregular Variations (I): Irregular Variations refer to those variations in business or other activities, which do not repeat in a definite pattern.

Definition2.9. The characteristic function μ_A of a crisp set $A \subset X$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned value indicate the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for $x \in X$ is called fuzzy set.

Definition2.10. An effective approach for ordering the elements of $F(R)$ is also to define a ranking function $\mathfrak{R} : F(R) \rightarrow R$ which maps each fuzzy number into the real line, where a natural order exists. We define orders on $F(R)$ by:

$$\tilde{a} \geq \tilde{b} \quad \text{if and only if } R(\tilde{a}) \geq R(\tilde{b})$$

$$\tilde{a} > \tilde{b} \quad \text{if and only if } R(\tilde{a}) > R(\tilde{b})$$

$$\tilde{a} = \tilde{b} \quad \text{if and only if } R(\tilde{a}) = R(\tilde{b})$$

Definition2.11. A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by

$\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $(a_1, a_2, a_3, a_4, a_5, a_6)$ are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below.

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0 & x < a_1 \\ \frac{1}{2} \left(\frac{x-a_1}{a_2-a_1} \right) & a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x-a_2}{a_3-a_2} \right) & a_2 \leq x \leq a_3 \\ 1 & a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2} \left(\frac{x-a_4}{a_5-a_4} \right) & a_4 \leq x \leq a_5 \\ \frac{1}{2} \left(\frac{a_6-x}{a_6-a_5} \right) & a_5 \leq x \leq a_6 \\ 0 & x > a_6 \end{cases}$$

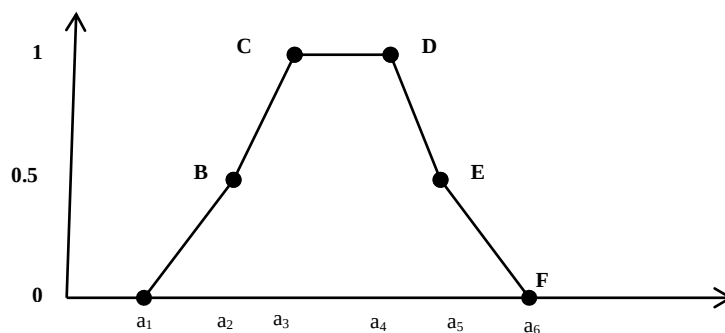


Figure1: Graphical representation of a hexagonal fuzzy number for $x \in [0, 1]$

[3] Numerical Examples

Definition3.1. Method of moving average

This is a simple method of obtaining trend values with a fair degree of accuracy by eliminating fluctuation. The average value of a number of years is taken as the trend value and placed at the middle point of the period of moving average.

Definition3.2. Odd period Moving average

The periods are odd 5, then take the total of the first 5 items and place it against the third year. Then we take the total of the second set of 5 items and place it against the fourth year. Continue the process till the last 5 years have been taken into account.

Definition3.3. Even period Moving average

The periods are even 7, then take the total of the first 4 items will be placed between the second and third years. The total of the second set of 4 items will be placed between the third and fourth year, and so on. This gives the 4- year moving totals. Now the first and second totals are added and placed against the third year. This value is divided by 8 to get 4-year moving average. This process is replaced for other 4-year totals.

Definition 3.4. Seasonal Index

Seasonal variation is measured in terms of an index, called a seasonal index. It is an average that can be used to compare an actual observation relative to what it would be if there were no seasonal variations. An index value is attached to each period of the time series within a year.

Example 1: The following table gives the number of profits with hexagonal fuzzy numbers in an industry during Sunday to Saturday in shift wise .Calculate a three days moving averageusing hexagonal fuzzy number for the data.

Days	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Profits('000) in lakhs	(0.5,0.7,0.9,0.9,0.7,0.5)	(0.2,0.3,0.6,0.6,0.5,0.4)	(0.4,0.5,0.6,0.6,0.5,0.4)	(0.5,0.6,0.7,0.7,0.6,0.5)	(0.2,0.2,0.4,0.4,0.2,0.1)	(0.1,0.2,0.4,0.4,0.2,0.1)	(0.1,0.2,0.3,0.3,0.2,0.1)

Solution:

Days	Profits('000) in lakhs	$\Re$	Three Days Total	Moving Average
Sunday	(0.5,0.7,0.9,0.9,0.7,0.5)	0.7	9.4	3.133
Monday	(0.2,0.3,0.6,0.6,0.5,0.4)	0.366		

Tuesday	(0.4,0.5,0.6,0.6,0.5,0.4)	0.5	8.8	2.933
Wednesday	(0.5,0.6,0.7,0.7,0.6,0.5)	1.2	8.2	2.733
Thursday	(0.2,0.2,0.4,0.4,0.2,0.1)	0.266	6.6	2.2
Friday	(0.1,0.2,0.4,0.4,0.2,0.1)	0.233	4.2	1.4
Saturday	(0.1,0.2,0.3,0.3,0.2,0.1)	0.2		

Example 2: The following series of observations is known to have a business cycle with a period of 4 days. Find the trend values using hexagonal fuzzy number by the moving average method.

Days	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Productions ('000) in tons	(0.5,0.6,0.7,0.7,0.6,0.5)	(0.4,0.6,0.7,0.7,0.6,0.4)	(0.4,0.5,0.6,0.6,0.5,0.4)	(0.3,0.2,0.5,0.5,0.4,0.3)	(0.2,0.3,0.4,0.4,0.3,0.2)	(0.7,0.8,0.9,0.9,0.8,0.7)	(0.1,0.3,0.8,0.8,0.3,0.1)

Solution:

Days	Productions ('000) in tons	$\mathfrak{R}$	Four Days Total	Four days Moving total centred	Four days Moving Average
Sunday	(0.5,0.6,0.7,0.7,0.6,0.5)	0.6	2.066	3.832	0.479
Monday	(0.4,0.6,0.7,0.7,0.6,0.4)	0.566			
Tuesday	(0.4,0.5,0.6,0.6,0.5,0.4)	0.5			
Wednesday	(0.3,0.2,0.5,0.5,0.4,0.3)	0.4			
Thursday	(0.2,0.3,0.4,0.4,0.3,0.2)	0.3	1.766	3.766	0.4705
Friday	(0.7,0.8,0.9,0.9,0.8,0.7)	0.8	2	3.9	0.4875
Saturday	(0.1,0.3,0.8,0.8,0.3,0.1)	0.4	1.9		

Example 3: Calculate five days moving average using hexagonal fuzzy number and determine short time oscillations of the following data

Days	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Productions ('000) in tons	(0.5,0.7,0.9,0.9,0.7,0.5)	(0.2,0.4,0.8,0.8,0.4,0.2)	(0.3,0.5,0.9,0.9,0.5,0.3)	(0.4,0.5,0.7,0.7,0.5,0.4)	(0.3,0.4,0.6,0.6,0.4,0.3)	(0.2,0.3,0.6,0.6,0.3,0.2)	(0.1,0.2,0.4,0.4,0.2,0.1)

Solution:

Days	Productions ('000) in tons	$\mathfrak{R}$	Five Days Total	Five Days Moving Average	Short time oscillations
Sunday	(0.5,0.7,0.9,0.9,0.7,0.5)	0.7	2.6998 2.3664 2.0331	0.5399 0.4732 0.4066	0.0267 0.0601 0.0267
Monday	(0.2,0.4,0.8,0.8,0.4,0.2)	0.566			
Tuesday	(0.3,0.5,0.9,0.9,0.5,0.3)	0.466			
Wednesday	(0.4,0.5,0.7,0.7,0.5,0.4)	0.533			
Thursday	(0.3,0.4,0.6,0.6,0.4,0.3)	0.433			
Friday	(0.2,0.3,0.6,0.6,0.3,0.2)	0.366			
Saturday	(0.1,0.2,0.4,0.4,0.2,0.1)	0.233			

Example 4: Calculate the seasonal index from the following table using hexagonal fuzzy number by ratio to moving average method.

Days	1 st Quarter	2 nd Quarter	3 rd Quarter	4 th Quarter
Monday	(0.2,0.3,0.5,0.5,0.3,0.2)	(0.2,0.3,0.6,0.6,0.3,0.2)	(0.3,0.4,0.6,0.6,0.4,0.3)	(0.3,0.6,0.7,0.7,0.6,0.3)
Tuesday	(0.3,0.4,0.6,0.6,0.4,0.3)	(0.2,0.4,0.6,0.6,0.4,0.2)	(0.3,0.5,0.9,0.9,0.5,0.3)	(0.3,0.6,0.7,0.7,0.6,0.3)
Wednesday	(0.4,0.5,0.7,0.7,0.5,0.4)	(0.3,0.4,0.6,0.6,0.4,0.3)	(0.2,0.4,0.6,0.6,0.4,0.2)	(0.3,0.5,0.9,0.9,0.5,0.3)
Thursday	(0.2,0.4,0.6,0.6,0.4,0.2)	(0.3,0.4,0.6,0.6,0.4,0.3)	(0.4,0.5,0.7,0.7,0.5,0.4)	(0.4,0.6,0.7,0.7,0.6,0.4)

Solution:

Days	Quarter	$\Re$	4-Quarter moving total	2-Quarter moving total	Moving average centred	Percentage of actual to moving average
Monday	(0.2,0.3,0.5,0.5,0.3,0.2)	0.3333	1.6665	3.4333	0.4291	100.97
	(0.2,0.3,0.6,0.6,0.3,0.2)	0.3666				
	(0.3,0.4,0.6,0.6,0.4,0.3)	0.4333				
	(0.3,0.6,0.7,0.7,0.6,0.3)	0.5333				
Tuesday	(0.3,0.4,0.6,0.6,0.4,0.3)	0.4333	1.6665	3.4333	0.4291	100.97
	(0.2,0.4,0.6,0.6,0.4,0.2)	0.4				
	(0.3,0.5,0.9,0.9,0.5,0.3)	0.5666				
	(0.3,0.6,0.7,0.7,0.6,0.3)	0.5333				
Wednesday	(0.4,0.5,0.7,0.7,0.5,0.4)	0.5333	1.7665	3.5664	0.4458	119.62
	(0.3,0.4,0.6,0.6,0.4,0.3)	0.4333				
	(0.2,0.4,0.6,0.6,0.4,0.2)	0.4				
	(0.3,0.5,0.9,0.9,0.5,0.3)	0.5666				
Thursday	(0.4,0.5,0.7,0.7,0.5,0.4)	0.5333	1.7999	3.7331	0.4666	92.86
	(0.2,0.4,0.6,0.6,0.4,0.2)	0.4				
	(0.3,0.5,0.9,0.9,0.5,0.3)	0.5666				
	(0.2,0.4,0.6,0.6,0.4,0.2)	0.4				
	(0.3,0.4,0.6,0.6,0.4,0.3)	0.4333	1.7999	3.8664	0.4833	82.76
	(0.4,0.5,0.7,0.7,0.5,0.4)	0.5333				
	(0.2,0.4,0.6,0.6,0.4,0.2)	0.4				
	(0.3,0.4,0.6,0.6,0.4,0.3)	0.4333				
	(0.4,0.5,0.7,0.7,0.5,0.4)	0.5333	1.9332	3.9664	0.4958	114.27
	(0.2,0.4,0.6,0.6,0.4,0.2)	0.4				
	(0.3,0.4,0.6,0.6,0.4,0.3)	0.4333				
	(0.4,0.5,0.7,0.7,0.5,0.4)	0.5333				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666	1.9332	4.0997	0.5124	104.07
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666	1.9332	3.9664	0.4958	107.56
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666	1.8999	3.8321	0.4790	90.45
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666	1.9332	3.7321	0.4665	85.74
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666	1.9332	3.5998	0.4499	125.93
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666	1.7999	3.7331	0.4666	85.72
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666	1.7999	3.8664	0.4833	89.65
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				
	(0.4,0.6,0.7,0.7,0.6,0.4)	0.5666				

Calculation of Seasonal Indices

Days	Percentage of actual to moving average			
	1 st Quarter	2 nd Quarter	3 rd Quarter	4 th Quarter
Monday	-	-	100.97	119.62
Tuesday	92.86	82.76	114.27	104.07
Wednesday	107.56	90.45	85.74	125.93
Thursday	85.72	89.65	-	-
Total	286.14	262.86	300.98	349.62
Seasonal Average	95.38	87.62	100.32	116.54
Seasonal Index	95.42	87.66	100.36	116.58

Conclusion

This paper proposes a fuzzy time series forecasting model based on moving average and ratio to moving average methods and it makes three main contributions with numerical examples. This paper also outlines the current company profits, productions, achievements, limitations, and suggestions for future research associated with the fuzzy time series forecasting.

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