

Comparative Study on Robust Ranking Technique and Magnitude Ranking Method for Fuzzy Linear Programming Problem

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ABSTRACT

In this paper, we first consider the fuzzy linear problem, Here coefficients are fuzzy numbers, we convert the problem into crisp form using the robust technique after using the simplex method we obtain the optimal solution for the given problem, secondary we apply the Magnitude method and got the solution comparison study illustrated.

Keywords: Fuzzy Linear programming problem, Robust Ranking Method, Triangular Fuzzy Number, Magnitude Ranking Method, Simplex Method.

1. Introduction:

In fuzzy environment the concept of decision making introduced Bellman and Zadeh. Many researchers used this concept. However, fuzzy linear programming not all parts of the problem to be fuzzy, for example only the right hand side or the objective function coefficients were fuzzy. The fuzzy linear programming problems in which all the parameters as well as the variables are represented by fuzzy numbers is known as FFLP problems. FFLP problems can be divided into two categories:

- (1) FFLP problems with equality constraints
- (2) FFLP problems with inequality constraints.

Some authors have different methods for solving FFLP problems with inequality constraints. In all methods first the FFLP problem is converted into crisp linear programming problem and then the obtained crisp linear programming problem is solved to find the fuzzy optimal solution of the FFLP problems.

Fuzzy set introduced by Zadeh in 1965. A. Nagoor Gani et al created the new operations for the triangular fuzzy numbers for solving fuzzy linear programming. Because there is a some issues for the usual arithmetic operations.

Arun Pratap Singh compared the Robust method and Centroid technique for the Fuzzy assignment problems. K. Kalaiarasi et al used the RRT for the fuzzy optimization in Assignment. Monalisha Pattnaik presented the two phase optimization using the RRT.

Stephen Dinagar et al applied the RRT for Fully Fuzzy Integer Linear Programming Problems. H. R. Maleki et al approached fuzzy linear problem for the parabolic method. D. Selvi et al used the Magnitude ranking for the fuzzy assignment to get the optimum solution. K. Ganesan et al tested the fuzzy linear programming problem for the trapezoidal fuzzy numbers. That fuzzy problem author solved without converting the problem into the crisp model, the solution obtain the best solution in this research. Amit Kumar et al suggested the new method for solving the fuzzy linear problems.

In this paper we have to solve the fuzzy linear problems using the RRT and MRM, finally we have comparison between optimal solution made the table. The further section as follows; section two introduces the concept of fuzzy sets and fuzzy numbers etc., Third section presents the General form of Fuzzy Linear programming Problems. Four investigates a numerical example and five contains the comparison table, six finished the conclusions based on our discussion.

2. Preliminaries:

Definition: 2.1

A fuzzy set \tilde{A} is called fuzzy number if it satisfies the following criteria

1. \tilde{A} must be normal
2. \tilde{A} is convex fuzzy set

3. $\tilde{A}(x)$ is piece wise continuous.

4.

Definition : 2.2

A fuzzy number \tilde{A} on \mathbb{R} is said to be a triangular fuzzy number if its membership function $\tilde{A}: \mathbb{R} \rightarrow [0, 1]$ has the following characteristics:

$$\tilde{A}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 < x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

We denote this triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$.

We use $F(\mathbb{R})$ to denote the set of all triangular fuzzy numbers.

Definition : 2.3

If \tilde{a} is a fuzzy number, the Robust ranking index is defined by

$$R(\tilde{a}) = 0.5 \int_0^1 (a_\alpha^L, a_\alpha^U) d\alpha$$

Where $(a_\alpha^L, a_\alpha^U) = \{(b-a)\alpha + a, d-(d-c)\alpha\}$ is the α -cut of the fuzzy number \tilde{a} .

Definition : 2.4

The magnitude of the triangular fuzzy number \tilde{a} is defined by,

$$\text{Mag } \tilde{a} = \frac{1}{2} \int_0^1 (a_3 + 3a_1 - a_2)f(r)dr$$

Where the function $f(r)$ is a non-negative and increasing function on $(0,1)$. In the real life applications $f(r)$ can be chosen by the decision maker according to the situation.

Definition : 2.5

Define a ranking function $R: F(\mathbb{R}) \rightarrow \mathbb{R}$ which maps each fuzzy number into the real line, where a natural order exists and defining order on $F(\mathbb{R})$ by

1. $\tilde{a} \geq \tilde{b}$ iff $R(\tilde{a}) \geq R(\tilde{b})$
2. $\tilde{a} \leq \tilde{b}$ iff $R(\tilde{a}) \leq R(\tilde{b})$
3. $\tilde{a} = \tilde{b}$ iff $R(\tilde{a}) = R(\tilde{b})$

3. Fuzzy Linear Programming Problems:

The general form fuzzy linear programming problem defined as

$$\text{Maximize } \tilde{W}(X) = \sum_{i=1}^n \tilde{c}_i \tilde{a}_i$$

$$\text{Subject to } \tilde{g}_j(\tilde{c}_j) \leq \text{or } \approx \text{ or } \geq \tilde{b}_i, \quad i = 1, 2, 3, \dots, m$$

$$\& \tilde{c}_j \geq \tilde{0}, \text{ for } j = 1, 2, 3, \dots, n$$

4. Numerical Examples:

Consider a fuzzy linear programming problem

$$\text{Max } \tilde{W} = (5, 7, 9) \tilde{c}_1 + (6, 8, 10) \tilde{c}_2$$

Subject to

$$(1, 2, 3) \tilde{c}_1 + (2, 3, 4) \tilde{c}_2 \leq (4, 6, 8)$$

$$(4, 5, 6) \tilde{c}_1 + (3, 4, 5) \tilde{c}_2 \leq (8, 10, 12) \quad \& \tilde{c}_1, \tilde{c}_2 \geq 0$$

Now convert the given fuzzy problem into crisp form using the Robust Ranking method,

$$\text{Max } \tilde{W} = R(5, 7, 9) \tilde{c}_1 + R(6, 8, 10) \tilde{c}_2$$

Subject to

$$R(1, 2, 3) \tilde{c}_1 + R(2, 3, 4) \tilde{c}_2 \leq R(4, 6, 8)$$

$$R(4, 5, 6) \tilde{c}_1 + R(3, 4, 5) \tilde{c}_2 \leq R(8, 10, 12) \quad \& \tilde{c}_1, \tilde{c}_2 \geq 0$$

The membership function of the triangular fuzzy number (5,7,9) is

$$\tilde{A}(x) = \begin{cases} \frac{x-5}{2}, & 5 \leq x \leq 7 \\ \frac{9-x}{2}, & 7 < x \leq 9 \\ 0, & \text{otherwise} \end{cases}$$

$$R(\tilde{a}) = 0.5 \int_0^1 (a_\alpha^L, a_\alpha^U) d\alpha$$

Where $(a_\alpha^L, a_\alpha^U) = \{(b-a)\alpha + a, d-(d-c)\alpha\}$

The α - cut of the fuzzy number (5,7,9) is $(a_\alpha^L, a_\alpha^U) = (2\alpha+5, 9-2\alpha)$.

$$\begin{aligned} R(5,7,9) &= 0.5 \int_0^1 \{(5-7)\alpha + 5 + 9 - (9-7)\alpha\} d\alpha \\ &= 0.5 \int_0^1 2\alpha + 14 - 2\alpha d\alpha = 0.5 \int_0^1 (14) d\alpha = 7. \end{aligned}$$

The membership function of the triangular fuzzy number (6,8,10) is

$$\tilde{A}(x) = \begin{cases} \frac{x-6}{2}, & 6 \leq x \leq 8 \\ \frac{10-x}{2}, & 8 < x \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

The α - cut of the fuzzy number (6,8,10) is $(a_\alpha^L, a_\alpha^U) = (2\alpha+6, 10-2\alpha)$.

$$\begin{aligned} R(6,8,10) &= 0.5 \int_0^1 \{(8-6)\alpha + 6 + 10 - (10-8)\alpha\} d\alpha \\ &= 0.5 \int_0^1 2\alpha + 16 - 2\alpha d\alpha = 0.5 \int_0^1 (16) d\alpha = 8. \end{aligned}$$

Similarly we get,

$R(1,2,3)=2$, $R(2,3,4)=3$, $R(4,6,8)=6$, $R(4,5,6)=5$, $R(3,4,5)=4$, $R(8,10,12)=10$.

we obtain crisp linear programming problem is

$$\begin{aligned} \text{Max } \tilde{W} &= 7 \tilde{c}_1 + 8 \tilde{c}_2 \\ \text{Subject to} \\ 2 \tilde{c}_1 + 3 \tilde{c}_2 &\leq 6 \\ 5 \tilde{c}_1 + 4 \tilde{c}_2 &\leq 10 \text{ \& } \tilde{c}_1, \tilde{c}_2 \geq 0 \end{aligned}$$

Now we solve the problem using the simplex method,

The problem is converted to canonical form by adding slack variables. As the constraints are ' \leq ' type we should add slack variables \tilde{s}_1 and \tilde{s}_2 .

$$\begin{aligned} \text{Max } \tilde{W} &= 7 \tilde{c}_1 + 8 \tilde{c}_2 + 0 \tilde{s}_1 + 0 \tilde{s}_2 \\ \text{Subject to} \\ 2 \tilde{c}_1 + 3 \tilde{c}_2 + \tilde{s}_1 &= 6 \\ 5 \tilde{c}_1 + 4 \tilde{c}_2 + \tilde{s}_2 &= 10 \text{ \& } \tilde{c}_1, \tilde{c}_2, \tilde{s}_1, \tilde{s}_2 \geq 0 \end{aligned}$$

Therefore we get the Iteration 1 table as follows:

Table 4.1

Iteration-1		C_j	7	8	0	0	
B	C_B	X_B	\tilde{c}_1	\tilde{c}_2	\tilde{s}_1	\tilde{s}_2	MinRatio $\frac{X_B}{\tilde{c}_2}$
\tilde{s}_1	0	6	2	(3)	1	0	$\frac{6}{3}=2 \rightarrow$
\tilde{s}_2	0	10	5	4	0	1	$\frac{10}{4}=2.5$

Z=0		Z_j	0	0	0	0	
		$Z_j - C_j$	-7	-8↑	0	0	

Negative minimum $Z_j - C_j$ is -8 and its column index is 2. So, the entering variable is \tilde{c}_2 .

Minimum ratio is 2 and its row index is 1. So, the leaving basis variable is \tilde{s}_1 .

∴ The pivot element is 3.

Table 4.2

Iteration-2		C_j	7	8	0	0	
B	C_B	X_B	\tilde{c}_1	\tilde{c}_2	\tilde{s}_1	\tilde{s}_2	MinRatio $\frac{XB}{\tilde{c}_1}$
\tilde{c}_2	8	2	0.6667	1	0.3333	0	$\frac{2}{0.6667}=3$
\tilde{s}_2	0	2	(2.3333)	0	-1.3333	1	$\frac{2}{2.3333}=0.8571 \rightarrow$
Z=16		Z_j	5.3333	8	2.6667	0	
		$Z_j - C_j$	-1.6667↑	0	2.6667	0	

Negative minimum $Z_j - C_j$ is -1.6667. So, the entering variable is \tilde{c}_1 .

Minimum ratio is 0.8571. So, the leaving basis variable is \tilde{s}_2 .

∴ The pivot element is 2.3333.

Table 4.3

Iteration-3		C_j	7	8	0	0	
B	C_B	X_B	\tilde{c}_1	\tilde{c}_2	\tilde{s}_1	\tilde{s}_2	MinRatio
\tilde{c}_2	8	1.4286	0	1	0.7143	-0.2857	
\tilde{c}_1	7	0.8571	1	0	-0.5714	0.4286	
Z=17.4286		Z_j	7	8	1.7143	0.7143	
		$Z_j - C_j$	0	0	1.7143	0.7143	

Since all $Z_j - C_j \geq 0$

Hence, optimal solution is

$\tilde{c}_1 = 0.8571, \tilde{c}_2 = 1.4286$

Max $\tilde{W} = 17.4286$

Magnitude Ranking Method:

Max $\tilde{W} = (5,7,9) \tilde{c}_1 + (6,8,10) \tilde{c}_2$
Subject to

$$(1,2,3) \tilde{c}_1 + (2,3,4) \tilde{c}_2 \leq (4,6,8)$$

$$(4,5,6) \tilde{c}_1 + (3,4,5) \tilde{c}_2 \leq (8,10,12)$$

$$\text{Applying Mag } \tilde{a} = \frac{1}{2} \int_0^1 (a_3 + 3a_1 - a_2)f(r)dr$$

$$\rightarrow \text{Mag}(5,7,9) = \frac{1}{2} \int_0^1 (9 + 3(5) - 7)r dr = \frac{1}{2} \int_0^1 (17)r dr = 4.25$$

$$\text{Mag}(6,8,10) = \frac{1}{2} \int_0^1 (10 + 3(6) - 8)r dr = \frac{1}{2} \int_0^1 (20)r dr = 5$$

Therefore we get in the similar manner,

$$\text{Mag}(1,2,3)=1,$$

$$\text{Mag}(2,3,4)=1.5,$$

$$\text{Mag}(4,6,8)=3.5,$$

$$\text{Mag}(4,5,6)=2.6,$$

$$\text{Mag}(3,4,5)=3.25,$$

$$\text{Mag}(8,10,12)=6.5$$

$$\text{Max } \tilde{W} = 4.25 \tilde{c}_1 + 5 \tilde{c}_2 + 0 \tilde{s}_1 + 0 \tilde{s}_2$$

Subject to

$$1 \tilde{c}_1 + 1.5 \tilde{c}_2 + \tilde{s}_1 = 3.5$$

$$2.6 \tilde{c}_1 + 3.25 \tilde{c}_2 + \tilde{s}_2 = 6.5 \text{ \& } \tilde{c}_1, \tilde{c}_2, \tilde{s}_1, \tilde{s}_2 \geq 0$$

The iteration table given below as follows:

Table 4.4

Iteration-1		C_j	4.25	5	0	0	
B	C_B	X_B	\tilde{c}_1	\tilde{c}_2	\tilde{s}_1	\tilde{s}_2	MinRatio $\frac{X_B}{\tilde{c}_2}$
\tilde{s}_1	0	3.5	1	(1.75)	1	0	$\frac{3.5}{1.75}=2 \rightarrow$
\tilde{s}_2	0	6.5	2.6	3.25	0	1	$\frac{6.5}{3.25}=2$
$Z=0$		Z_j	0	0	0	0	
		$Z_j - C_j$	-4.25	-5↑	0	0	

Table 4.5

Iteration-2		C_j	4.25	5	0	0	
B	C_B	X_B	\tilde{c}_1	\tilde{c}_2	\tilde{s}_1	\tilde{s}_2	MinRatio $\frac{X_B}{\tilde{c}_1}$
\tilde{c}_2	5	2	0.5714	1	0.5714	0	$\frac{2}{0.5714}=3.5$
\tilde{s}_2	0	0	(0.7429)	0	-1.8571	1	$\frac{0}{0.7429}=0 \rightarrow$
$Z=10$		Z_j	2.8571	5	2.8571	0	
		$Z_j - C_j$	-1.3929↑	0	2.8571	0	

Table 4.6

Iteration-3		C_j	4.25	5	0	0	
B	C_B	X_B	\tilde{c}_1	\tilde{c}_2	\tilde{s}_1	\tilde{s}_2	MinRatio $\frac{X_B}{\tilde{s}_1}$
\tilde{c}_2	5	2	0	1	(2)	-0.7692	$\frac{2}{2}=1 \rightarrow$
\tilde{c}_1	4.25	0	1	0	-2.5	1.3462	---
Z=10		Z_j	4.25	5	-0.625	1.875	
		$Z_j - C_j$	0	0	-0.625↑	1.875	

Table 4.7

Iteration-4		C_j	4.25	5	0	0	
B	C_B	X_B	\tilde{c}_1	\tilde{c}_2	\tilde{s}_1	\tilde{s}_2	MinRatio
\tilde{s}_1	0	1	0	0.5	1	-0.3846	
\tilde{c}_1	4.25	2.5	1	1.25	0	0.3846	
Z=10.625		Z_j	4.25	5.3125	0	1.6346	
		$Z_j - C_j$	0	0.3125	0	1.6346	

Hence, optimal solution is

$$\tilde{c}_1 = 4.25, \tilde{c}_2 = 0$$

$$\text{Max } \tilde{W} = 10.625$$

Example 2:

$$\text{Min } z = (1, 1, 1) x_1 + (2, 1, 2) x_2 \text{ Subject to } (4, 1, 0)x_1 + (-3, 2, 1)x_2 \geq (2, 1, 2)$$

$$(-3, 1, 2) x_1 + (2, 1, 1) x_2 \geq (1, 0, 1) \text{ \& } x_1, x_2 \geq 0$$

Converting to crisp problem using RRT, we have

$$\text{Min } z = x_1 + 1.5x_2$$

$$0.75x_1 + 0.5x_2 \geq 1.5$$

$$0.5x_1 + 1.25x_2 \geq 0.5$$

Therefore we could solve the problem now

$$\text{Min } z = x_1 + 1.5x_2 + 0s_1 + 0s_2 + MA_1 + MA_2$$

Sub to

$$0.75x_1 + 0.5x_2 - s_1 + A_1 = 1.5$$

$$0.5x_1 + 1.25x_2 - s_2 + A_2 = 0.5$$

$$\text{and } x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$$

Table 4.8

Iteration-1		C_j	1	1.5	0	0	M	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	MinRatio $\frac{XB}{x_2}$
A_1	M	1.5	0.75	0.5	-1	0	1	0	1.5/0.5=3
A_2	M	0.5	0.5	(1.25)	0	-1	0	1	0.5/1.25=0.4 →
$Z=2M$		Z_j	$1.25M$	$1.75M$	$-M$	$-M$	M	M	
		Z_j-C_j	$1.25M-1$	$1.75M-1.5 \uparrow$	$-M$	$-M$	0	0	

Table 4.9

Iteration-2		C_j	1	1.5	0	0	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	MinRatio $\frac{XB}{x_1}$
A_1	M	1.3	0.55	0	-1	0.4	1	1.3/0.55=2.3636
x_2	1.5	0.4	(0.4)	1	0	-0.8	0	0.4/0.4=1 →
$Z=1.3M+0.6$		Z_j	$0.55M+0.6$	1.5	$-M$	$0.4M-1.2$	M	
		Z_j-C_j	$0.55M-0.4 \uparrow$	0	$-M$	$0.4M-1.2$	0	

Table 4.10

Iteration-3		C_j	1	1.5	0	0	M	
B	C_B	X_B	x_1	x_2	x_2	S_2	A_1	MinRatio $\frac{XB}{s_2}$
A_1	M	0.75	0	-1.375	-1	(1.5)	1	0.75/1.5=0.5 →
x_1	1	1	1	2.5	0	-2	0	---
$Z=0.75M+1$		Z_j	1	$-1.375M+2.5$	$-M$	$1.5M-2$	M	
		Z_j-C_j	0	$-1.375M+1$	$-M$	$1.5M-2 \uparrow$	0	

Table 4.11

Iteration-4		C_j	1	1.5	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	MinRatio
S_2	0	0.5	0	-0.9167	-0.6667	1	

x_1	1	2	1	0.6667	-1.3333	0	
Z=2		Z_j	1	0.6667	-1.3333	0	
		$Z_j - C_j$	0	-0.8333	-1.3333	0	

Hence the solution is
 $x_1=2, x_2=0 \rightarrow \text{Min } Z=2$

Method :2

Applying Magnitude method, $\text{Min } z=0.75x_1+1.75x_2$
 $2.75x_1-2.5x_2 \geq 1.75$
 $-2.5x_1 + 1.5x_2 \geq 1$

$\text{Min } z=0.75x_1+1.75x_2+0s_1 + 0s_2 + MA_1+MA_2$
 $2.75x_1-2.5x_2 - s_1+A_1 \geq 1.75$
 $-2.5x_1 + 1.5x_2 - s_2 + A_2 \geq 1$

Table 4.12

Iteration 1		C_j	0.75	1.75	0	0	M	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	MinRatio $\frac{X_B}{x_2}$
A_1	M	1.75	(2.75)	-2.5	-1	0	1	0	$1.75/2.75=0.6364 \rightarrow$
A_2	M	1	-2.5	1.5	0	-1	0	1	---
Z=2.75M		Z_j	0.25M	-M	-M	-M	M	M	
		$Z_j - C_j$	$0.25M - 0.75 \uparrow$	$-M - 1.75$	$-M$	$-M$	0	0	

Table 4.13

Iteration-2		C_j	0.75	1.75	0	0	M	
B	C_B	X_B	x_1	x_2	S_1	S_2	A_2	MinRatio
x_1	0.75	0.6364	1	-0.9091	-0.3636	0	0	
A_2	M	2.5909	0	-0.7727	-0.9091	-1	1	
Z=2.5909M+0.4773		Z_j	0.75	-0.7727M-0.6818	-0.9091M-0.2727	-M	M	
		$Z_j - C_j$	0	$-0.7727M - 2.4318$	$-0.9091M - 0.2727$	$-M$	0	

Hence, optimal solution is arrived as :

$x_1=0.6364, x_2=0$

Min Z=0.4773

5. Comparison:

Table 5.1 comparison table

	Robust ranking technique	Magnitude ranking method
\tilde{c}_1	0.8571	4.25
\tilde{c}_2	1.4286	0
Max \widetilde{W}	17.4286	10.625

	Robust ranking technique	Magnitude ranking method
x_1	2	0.6364
x_2	0	0
Min Z	2	0.4773

Conclusion

In the article, we compare the two ranking methods for the fuzzy linear programming maximization problem we obtain the best solution for the Robust ranking technique and minimization problem we obtain the best solution for the Magnitude method. We confirmed the accuracy of the problem using numerical examples.

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