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Bi-Domination in Brick product graphs

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ABSTRACT

A non-empty set of vertices is a bi-domination set if D_{bi} is dominating set of $G=(V,E)$ and every $v \in D_{bi}$ dominates exactly two vertices in $V-D_{bi}$ such that $|N(v)\cap(V-D_{bi})|=2$.The bi-domination number $\gamma_{bi}(G)$ is the minimum cardinality over all bi-dominating set in G. In this paper we determine bi-domination number $\gamma_{bi}(G)$ for the brick product graph of even cycle graphs.

Key Words: Dominating set, bi-dominating set, minimal bi-domination number.

AMS Subject Classification: 05C69.

1. Introduction

All graphs considered in this paper are simple connected graphs without loops and multiple edges. The concept of a dominating set is well known in graph theoretic literature. In this paper we study the bi- domination number of a graph G and determine the bi- domination in brick product of even cycle graphs where $C(2k,p,q), q=3,5,7,11$.

The concept of brick product of even cycles was introduced by Alspach et.al. [2] in which the Hamiltonian laceability properties of brick products was explored. Using the concept of brick-products, Alspach and Zhang show in [3] that all cubic Cayley graphsover dihedral groups are Hamiltonian. It was also conjectured that all brick product graphs C (2n, m, r) are Hamiltonian laceable. Chen et.al. in [4] have shown that the conjecture is true for m is even. In [6] the authors Leena Shenoy and Murali and in [5] the authors Girisha and Murali studied the Hamiltonian laceability properties in cyclic product graphs associated with even cycles.

Definition 1.1. A set D_{bi} of vertices in a graph G is a **bi- dominating set** [1] if every $v \in D_{bi}$ dominates exactly two vertices in $V-D_{bi}$ such that $|N(v)\cap(V-D_{bi})|=2$. The **bi-domination number** $\gamma_{bi}(G)$ is the minimum size of a bi-dominating set. Throughout this paper we will denote **dominating set** by Dst **.**

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Definition 1.2. Let $k, p\, and\, q$ be positive integers. $Let\, C_{2k} = v_0, v_1, v_2 ... v_{2k-1}, v_0$ denote a cycle of order $2k$. The (p,q) -brick product of C_{2k} denoted by $C(2k, p, q)$ is defined as follows:

For $p=1$, we require that q be odd and greater than 1. Then, $C(2k, p, q)$ is obtained from C_{2k} by adding chords $(v_{2r}, v_{2r+q}), r = 1, 2....k$, where the computation is performed under modulo $2k$.

For $p>1$, we require that $p+q$ be even. Then $C(2k, p, q)$ is obtained by first taking disjoint union of k copies of C_{2k} namely , $C_{2k}(1), C_{2k}(2), C_{2k}(3)......C_{2k}(p)$ where for each $i = 1, 2, ...p$, $C_{2k}(i) = v_i(1), v_i(2), v_i(3)........v_i(2k)$. Next , for each odd $i = 1, 2, ...p-1$ and each even $r = 0, 1, 2.....2k-2$ an edge (called a brick edge) drawn to join $v_{ir}\; to\; v_{(i+1)r}$, whereas , for each even $i = 1, 2, ...p-1$ and each odd $r = 0, 1, 2.....2k-1$, an edge (also called a brick edge) is drawn to join $v_{ir}\; to\; v_{(i+1)r}$.

Finally , for each odd $r = 0, 1, 2.....2k-1$, an edge (called a hooking edge) is known to join $v_{1r}\; to\; v_{p(r+q)}$. An edge in $C(2k, p, q)$ which is neither a brick edge nor a hooking edge is called a flat edge.

2. Main Results

Theorem 2.1.

Let $G = C(2k, p, q)$ then for $p = 1, k \geq 3$ and $q = 3$

$$\gamma_{bi}(G) = \begin{cases} 2, & k = 3 \\ 4, & k = 4, 5, 6 \\ \dfrac{2k}{3}, & k \equiv 0 (\bmod 3) \\ \dfrac{2(k-1)}{3} + 2, & k \equiv 1 (\bmod 3) \\ \dfrac{2(k-2)}{3} + 2, & k \equiv 2 (\bmod 3) \end{cases}$$

Proof.

We Consider the vertex set G as $V(G) = \{v_0, v_1, v_2, ..., v_{2k-1}, v_{2k} = v_0\}$ and

the edge set of G as $E(G) = \{e_j : 1 \leq j \leq 2k\} \cup \{e'_j : 1 \leq j \leq k\}$ where e_j is the edge $(v_{i-1}.v_i)$ and e'_j is the edge (v_{2r}, v_{2r+q})

$r = 0, 1, 2, 3...k.\; Here\; 2r+q$ is computed modulo $2k$.

$Case(i):$ For $k = 3a+4,\; a = 1, 2, 3, 4......$

We consider the set $D_{bi} = \{\{v_{3j-2}\} \cup \{v_2\}\}$ where $1 \leq j \leq 2\left\lceil \dfrac{k}{3} \right\rceil - 1$

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$Case(ii)$: For $k = 3b + 5$, $b = 1, 2, 3, 4.......$

We consider the set $D_{bi} = \{v_{3j-2}\}$ where $1 \le j \le 2\left\lceil \frac{k}{3} \right\rceil$

$Case(iii)$: For $k = 3c + 6$, $c = 1, 2, 3, 4......$

We consider the set $D_{bi} = \{v_{3j-2}\}$ where $1 \le j \le \frac{2k}{3}$

The above cases of D_{bi} are the minimal bi-Dst . Hence, for every $u, w \in V - D_{bi}$ is adjacent to $v \in D_{bi}$ such that $|N(v) \cap (V - D_{bi})| = 2$ and every $u - v$ path contain a vertex of D_{bi} .

Therefore , D_{bi} is minimal bi-Dst and Since $$|D_{bi}| = \begin{cases} 2, & k = 3 \\ 4, & k = 4, 5, 6 \\ \frac{2k}{3}, & k \equiv 0(\bmod 3) \\ \frac{2(k-1)}{3} + 2, & k \equiv 1(\bmod 3) \\ \frac{2(k-2)}{3} + 2, & k \equiv 2(\bmod 3) \end{cases}$$

We immediately obtain $$\gamma_{bi}(G) = \begin{cases} 2, & k = 3 \\ 4, & k = 4, 5, 6 \\ \frac{2k}{3}, & k \equiv 0(\bmod 3) \\ \frac{2(k-1)}{3} + 2, & k \equiv 1(\bmod 3) \\ \frac{2(k-2)}{3} + 2, & k \equiv 2(\bmod 3) \end{cases}$$

Hence the proof.

Theorem 2.2.

Let $G = C(2k, p, q)$ then for $p = 1, k \ge 5$ and $q = 5$

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$$\gamma_{bi}(G)) = \begin{cases} 4, & k=5 \\ 5, & k=6 \\ 6, & k=7 \\ k, & k \equiv 0 (\bmod 2) \\ k-1, & k \equiv 1 (\bmod 2) \end{cases}$$

Proof.

We Consider the vertex set G as $V(G) = \{v_0, v_1, v_2, ..., v_{2k-1}, v_{2k} = v_0\}$ and

the edge set of G as $E(G) = \{e_j : 1 \le j \le 2k\} \cup \{e'_j : 1 \le j \le k\}$ where e_j is the edge (v_{i-1}, v_i) and e'_j is the edge (v_{2r}, v_{2r+q})

$r = 0,1,2,3...k.$ *Here* $2r+q$ is computed modulo $2k$.

$Case(i)$: For $k = 2a+6$, $a = 1,2,3,4......$

We consider the set $D_{bi} = \{\{v_1\} \cup \{v_{4j-1}\} \cup \{v_{4j'}\} \cup \{v_{2k-6}\} \cup \{v_{2k}\}\}$

Where $1 \le j \le \frac{k}{2} - 1$, $1 \le j' \le \frac{k}{2} - 2$

$Case(ii)$: For $k = 2b+7$, $b = 1,2,3,4......$

We consider the set $D_{bi} = \{\{v_1\} \cup \{v_{4j-1}\} \cup \{v_{4j'}\} \cup \{v_{2k-4}\}\}$

Where $1 \le j \le \left\lfloor \frac{k}{2} \right\rfloor - 1, 1 \le j' \le \left\lfloor \frac{k}{2} \right\rfloor - 1$

The above cases of D_{bi} are the minimal bi-Dst . Hence, for every $u, w \in V - D_{bi}$ is adjacent to $v \in D_{bi}$ such that $|N(v) \cap (V - D_{bi})| = 2$ and every $u-v$ path contain a vertex of D_{bi} .

Therefore , D_{bi} is minimal bi-Dst and Since $|D_{bi}| = \begin{cases} 4, & k=5 \\ 5, & k=6 \\ 6, & k=7 \\ k, & k \equiv 0 (\bmod 2) \\ k-1, & k \equiv 1 (\bmod 2) \end{cases}$

We immediately obtain $\gamma_{bi}(G) = \begin{cases} 4, & k=5 \\ 5, & k=6 \\ 6, & k=7 \\ k, & k \equiv 0 (\bmod 2) \\ k-1, & k \equiv 1 (\bmod 2) \end{cases}$

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Hence the proof.

Theorem: 2.3.

Let $G = C(2k, p, q)$ then for $p = 1, k \geq 7 \; and \; q = 7$

$$\gamma_{bi}(G)) = \begin{cases} 6, & k = 7 \\ k, & k \equiv 0(\mathrm{mod}\,2) \\ k-1, & k \equiv 1(\mathrm{mod}\,2) \end{cases}$$

Proof.

We Consider the vertex set G as $V(G) = \{v_0, v_1, v_2, ..., v_{2k-1}, v_{2k} = v_0\}$ and

the edge set of G as $E(G) = \{e_j : 1 \leq j \leq 2k\} \cup \{e'_j : 1 \leq j \leq k\}$ where e_j is the edge (v_{i-1}, v_i) and e' is the edge (v_{2r}, v_{2r+q})

$r = 0,1,2,3...k. \; Here \; 2r + q$ is computed modulo $2k$.

$Case(i)$: For $k = 2a + 6$, $a = 1,2,3,4......$

We consider the set $D_{bi} = \{\{v_{4j-3}\} \cup \{v_{4j'}\}\}$ where $1 \leq j \leq \frac{k}{2}, 1 \leq j' \leq \frac{k}{2}$

$Case(ii)$: $k = 2b + 7$, $b = 1,2,3,4......$
we consider the set $D_{bi} = \{\{v_{4j-3}\} \cup \{v_{4j'}\} \cup \{v_{2k}\} \cup \{v_{2k-4}\}\}$

where $1 \leq j \leq \left\lfloor \frac{k}{2} \right\rfloor, 1 \leq j' \leq \left\lfloor \frac{k}{2} \right\rfloor - 2$

The above cases of D_{bi} are the minimal bi- Dst . Hence, for every $u, w \in V - D_{bi}$ is adjacent to $v \in D_{bi}$ such that $|N(v) \cap (V - D_{bi})| = 2$ and every $u - v$ path contain a vertex of D_{bi} .

Therefore , D_{bi} is minimal bi- Dst and Since $|D_{bi}| = \begin{cases} 6, & k = 7 \\ k, & k \equiv 0(\mathrm{mod}\,2) \\ k-1, & k \equiv 1(\mathrm{mod}\,2) \end{cases}$

We immediately obtain $\gamma_{bi}(G)) = \begin{cases} 6, & k = 7 \\ k, & k \equiv 0(\mathrm{mod}\,2) \\ k-1, & k \equiv 1(\mathrm{mod}\,2) \end{cases}$

Hence the proof.

Theorem: 2.4.

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Let $G = C(2k, p, q)$ then for $p = 1, k \geq 11$ and $q = 11$

$$\gamma_{bi}(G) = \begin{cases} k, & k \equiv 0 (\bmod 2) \\ k-1, & k \equiv 1 (\bmod 2) \end{cases}$$

Proof.

We Consider the vertex set G as $V(G) = \{v_0, v_1, v_2, ..., v_{2k-1}, v_{2k} = v_0\}$ and

the edge set of G as $E(G) = \{e_j : 1 \leq j \leq 2k\} \cup \{e'_j : 1 \leq j \leq k\}$

where e_j is the edge (v_{i-1}, v_i) and e'_j is the edge (v_{2r}, v_{2r+q}) $r = 0,1,2,3...k.$ *Here* $2r + q$ is computed modulo $2k$.

$Case(i) : k = 2a + 6,\ a = 1,2,3,4......$

We consider the set $D_{bi} = \{\{v_{4j-3}\} \cup \{v_{4j'}\}\}$ where $1 \leq j \leq \frac{k}{2}, 1 \leq j' \leq \frac{k}{2}$

$Case(ii): k = 2b + 7,\ b = 1,2,3,4......$

We consider the set $D_{bi} = \{\{v_{4j-3}\} \cup \{v_{4j'}\} \cup \{v_{2k}\} \cup \{v_{2k-4}\}\}$

Where $1 \leq j \leq \left\lfloor \frac{k}{2} \right\rfloor, 1 \leq j' \leq \left\lfloor \frac{k}{2} \right\rfloor - 2$

The above cases of D_{bi} are the minimal bi-Dst. Hence, for every $u, w \in V - D_{bi}$ is adjacent to $v \in D_{bi}$ such that $|N(v) \cap (V - D_{bi})| = 2$ and every $u - v$ path contain a vertex of D_{bi}.

Therefore , D_{bi} is minimal bi-Dst and Since $|D_{bi}| = \begin{cases} k, & k \equiv 0 (\bmod 2) \\ k-1, & k \equiv 1 (\bmod 2) \end{cases}$

We immediately obtain $\gamma_{bi}(G) = \begin{cases} k, & k \equiv 0 (\bmod 2) \\ k-1, & k \equiv 1 (\bmod 2) \end{cases}$

Hence the proof.

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