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Bounded-Addition Fuzzy Simple Splicing Systems

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ABSTRACT
A splicing system is one of the early theoretical models for DNA computing. Two strings of DNA molecules are cut at specified recognition sites in a splicing system, and the prefix of the first string is attached to the suffix of the second string, generating new strings. The recognition sites for both strings of DNA molecules are the same for a specific type of splicing system, namely simple splicing systems. Splicing systems with finite sets of axioms and splicing rules are known to produce only regular languages. As a result, many forms of restrictions for splicing systems have been considered in order to boost their generative power. Fuzzy splicing systems, in which truth values (i.e., fuzzy membership values) from the closed interval [0, 1] are assigned to the axioms of splicing systems, have been introduced. The truth values of every generated string z from strings x and y are then computed by performing a fuzzy bounded-addition operation over their truth values. The features of bounded-addition fuzzy simple splicing systems are studied in this research. It has been demonstrated that fuzzy simple splicing systems with bounded addition operations can increase the generative power of the splicing languages generated.

Keywords: Formal Language Theory, Fuzzy Splicing System, Restriction, Bounded-Addition, Simple Splicing System

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1. INTRODUCTION

Deoxyribonucleic acid (DNA) contains the genetic material of organisms and it is made up of a chain of nucleotides. The chemical bases of the nucleotides are adenine (A), guanine (G), cytosine (C), and thymine (T). Nucleotides can be joined together in two long strands to form a spiral knownas a double helix where A pairs with T, C pairs with G and vice versa. The double helix's structure is similar to that of a ladder. Restriction enzymes, which are found naturallyin bacteria, cut DNA fragments at certain sequences called restriction sites, while ligase re-join DNA fragments with complementary ends. Head [1] described the recombination behavior of restriction enzymes and ligases as splicing systems and splicing languages in 1987. This splicing system model is created to observe the recombinant behavior of DNA molecules when restriction enzymes and ligases arepresent.

Due to the limitations of unrestricted splicing systems in terms of the generating power, different splicing operation restrictions have been suggested to increase the generative power of splicing systems. This is important from the perspective of DNA computing; as mentioned by Adleman in [2] where splicing systems with restriction can be considered as theoretical models of universal programmableDNA-based computers. A few operations are performed on strings that can be

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combined to generate an expression that denotes a language. Using grammar, the string pattern will be classified into the respective language family according to the Chomsky hierarchy. Pixton [3] defined some of the classes for a family of languages which are recursively enumerable, context-sensitive, context-free, linear, regular and finite languages. Paun [4] mentioned the links between many variants of splicing system such as simple splicing system, semi-simple splicing system and one-sided splicing system.

Based on the concept of fuzzy set formulated by Zadeh in 1965, fuzzy-fuzzy automata are defined and some properties are investigated. It is shown that fuzzy-fuzzy languages characterized by fuzzy-fuzzy automata are closed under the operations of union, intersection, concatenation and Kleene closure [5]. Fuzzy-fuzzy grammars are illustrated and it is shown that fuzzy-fuzzy grammars with context-free rules can generate context-sensitive languages. This fuzzy concept in formal language and automata theory can be used in DNA computing as well. Karimi et al. have proven in [6] that the languages generated by fuzzy splicing systems have a higher generative power as compared to the languages generated by splicing systems without any restriction on the rules.

In this paper, the concept of bounded-addition fuzzy splicing system is introduced by assigning truth values (i.e., fuzzy membership values) to the axioms of splicing systems from the closed interval [0, 1]. Then, using a fuzzy bounded- addition operation over the truth values of strings x and y, the truth value of each created string x is calculated. A bounded-addition fuzzy simple splicing system is investigated in this study.

This paper is organized as follows; First, some necessary definitions and findings from formal language theory, splicing systems, and introduction to fuzzy splicing systems are explained. Next, some definitions of bounded-addition fuzzy simple splicing systems are given. Moreover, some examples and important results concerning the generative power of the languages generated by bounded-addition fuzzy simple splicing system are established. The conclusion of this research is then discussed at the end of the paper.

2. PRELIMINARIES

In this section, some prerequisites were covered by outlining the basic concepts and notations of the formal language and the fuzzy splicing system theories that will be used later. More details can be referred to [1, 6–9].

The following general notations are used throughout the paper. The term $\in$ denotes an element in a member of a set, whereas $\notin$ denotes not an element in a member of a set, while $\subseteq$ stands for (proper) inclusion, $\subset$ specifies the strictness of the inclusion. The term |X| denotes the cardinality of a set X, while the symbol $\varnothing$ represents an empty set.

The families of finite, regular, linear, context-free, context-sensitive and recursively enumerable languages are denoted by **FIN**, **REG**, **LIN**, **CF**, **CS** and **RE** respectively. For these family languages, the next strict inclusions, namely Chomsky hierarchy (see [7]), holds:

$$\mathbf{FIN} \subset \mathbf{REG} \subset \mathbf{LIN} \subset \mathbf{CF} \subset \mathbf{CS} \subset \mathbf{RE}.$$

Further, a basic definition of splicing system and a theorem on the family of languages generated by a splicing language are recalled.

Definition 1 [1]: A splicing system (EH) is a 4-tuple $\gamma = (V, T, A, R)$ where V is an alphabet, $T \subseteq V$ is a terminal alphabet. A is a finite subset of V^+ and $R \subseteq V^*\#V^*\$V^*\#V^*$ is the set of splicing rules.

Theorem 1 [4]: The relations in the following table hold, where at the intersection of the row marked with F_1 with the column marked with F_2, there appear either the family $\mathrm{EH}(F_1, F_2)$ or two families F_3, F_4 such that $F_3 \subset \mathrm{EH}(F_1, F_2) \subseteq F_4$.

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Table 1: The family of languages generated by splicing systems

$F_1 \backslash F_2$	**FIN**	**REG**	**LIN**	**CF**	**CS**	**RE**
FIN	**REG**	**RE**	**RE**	**RE**	**RE**	**RE**
REG	**REG**	**RE**	**RE**	**RE**	**RE**	**RE**
LIN	**LIN, CF**	**RE**	**RE**	**RE**	**RE**	**RE**
CF	**CF**	**RE**	**RE**	**RE**	**RE**	**RE**
CS	**RE**	**RE**	**RE**	**RE**	**RE**	**RE**
RE	**RE**	**RE**	**RE**	**RE**	**RE**	**RE**

Next, the definition of a fuzzy splicing system is presented.

Definition 2 [6]: A fuzzy splicing system (a fuzzy H system) is a 6-tuple $\gamma = (V, T, A, R, \mu, \odot)$ where V, T, R are defined as for a usual extended H system, $\mu : V^* \to [0,1]$ is a fuzzy membership function, A is a subset of $V^* \times [0, 1]$ and $\odot$ is a fuzzy operation over [0, 1].

Strings x, y and z are written as $(x, \mu(x))$, $(y, \mu(y))$, $(z, \mu(z))$ and a fuzzy splicing operation is defined as follows.

Definition 3 [6]: For $(x, \mu(x)), (y, \mu(y)), (z, \mu(z)) \in V^* \times [0, 1]$ and $r \in R$,

$$[(x, \mu(x)), (y, \mu(y))] \mapsto_r z(z, \mu(z))$$

if and only if $(x, y) \mapsto_r z$ and $\mu(Z) = \mu(X) \odot \mu(Y)$

Thus, the fuzzy of the string $z \in V^*$ obtained by splicing operation on two strings $x, y \in V^*$ is computed by multiplying their fuzzy membership values. The language generated by the fuzzy splicing system is defined next.

Definition 4 [6]: The fuzzy language generated by fuzzy splicing system $\gamma = (V, T, A, R, \mu, \odot)$ is defined as

$$L_f(\gamma) = \{(z, \mu(z)) \in \sigma_f^*(A^{\oplus}) : z \in T^*\}.$$

Further, the definition of simple splicing system is recalled.

Definition 5 [8]: A simple splicing system (SEH) is a triple $\gamma = (V, M, A)$ where V is an alphabet, $M \subseteq V$, A is a finite language over V. The elements of M are called markers in the form $(a, 1; a, 1)$ and those of A are called *axioms*.

3. MAIN RESULTS

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In this section, the concept of bounded-addition fuzzy simple splicing system is introduced by first assigning the truth values (i. e., fuzzy membership values) to the axioms of splicing systems from the closed interval [0, 1]. Then, using a fuzzy bounded-operation over the truth value of strings *x* and *y*, the truth value of each created string *z* is calculated.

Definition 6: A bounded-addition fuzzy simple splicing system is a 5-tuple $\gamma_s^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ where V is defined as the usual extended splicing systems, R is the rule in the form (a, 1; a, 1), $\mu : V^* \to [0,1]$ is a fuzzy membership function, $A^{\oplus}$ is a subset of $V^* \times [0,1]$ such that

$$\sum_{i=1}^{n} \mu(x_i) \leq 1$$

and $\oplus$ is a bounded-addition fuzzy operation on [0, 1] defined by

$$\mu_{A+B} = \mu_A + \mu_B - \mu_A \mu_B \text{ where } \mu_A \mu_B \in \mu(x_i).$$

A bounded-addition fuzzy simple splicing operation is defined next.

Definition 7: For strings (x, μ(x)), (y, μ(y)), (z, μ(z)) $\in V^* \times [0, 1]$ and $r \in R$, the bounded-addition fuzzy simple splicing operation is defined as

$$[(x, \mu(x)), (y, \mu(y))] \mapsto_r z(z, \mu(z))$$

if and only if $(x, y) \mapsto_r z$ and $\mu(z) = \mu(x) \oplus \mu(y)$ is defined by

$$\mu_{x+y} = \mu_x + \mu_y - \mu_x \mu_y \text{ where } \mu_x \mu_y \in \mu(x_i)$$

and $r = (a, 1; a, 1) \in R$.

The value of string z, μ(z), is computed using a simple splicing operation on two strings $x, y \in V^*$ using the $\oplus$ operation on these two strings.

Definition 8: The language generated by bounded-addition fuzzy simple splicing system $\gamma_s^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ is defined as

$$L_f(\gamma_s^{\oplus}) = \{z \in T \mid (z, \mu(z)) \in \sigma^*(A^{\oplus})\}.$$

From the definition of bounded-addition fuzzy simple splicing system, an example is given to illustrate the application of bounded-addition fuzzy simple splicing systems. Here, the symbol $\mapsto$ denotes the splicing operations on the strings.

Example 1: Consider the bounded-addition fuzzy simple splicing system

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$$\gamma_s^{\oplus} = (\{a,b,c\},\{(aab,\frac{1}{4}),(abb,\frac{1}{5}),(bcc,\frac{1}{6})\}, R, \mu, \oplus)$$

Where $R = \{r_1, r_2, r_3\}$ and $r_1 = a\#1\$b\#1, r_2 = b\#1\$a\#1, r_3 = c\#1\$a\#1, r_4 = a\#1\$c\#1$ and $r_5 = b\#1\$d\#1$.

When the first rule r_1 is applied in string *aab*, the string obtained is

$$[(aab,\frac{1}{4}),(aab,\frac{1}{4})] \mapsto_{r_1} (aaab,\frac{7}{16}).$$

By iterative splicing operation between the same string using the rule r_1, the string

$$(a^{k+1}b, 1-\frac{3^k}{4^k}), k \geq 1$$

is obtained.

By applying the rule r_2 to the strings *abb*, the string obtained is

$$[(abb,\frac{1}{5}),(abb,\frac{1}{5})] \mapsto_{r_2} (abbb,\frac{9}{25}).$$

By iterative splicing operation between the same string using the rule r_2, the string

$$(ab^{m+1}, 1-\frac{4^m}{5^m}), m \geq 1$$

is obtained.

By applying the rule r_3 to the strings *bcc* and, the string obtained is

$$[(bcc,\frac{1}{6}),(bcc,\frac{1}{6})] \mapsto_{r_3} (bccc,\frac{11}{36}).$$

By iterative splicing operation between the same string using the rule r_3, the string

$$(bc^{n+1}, 1-\frac{5^n}{6^n}), n \geq 1$$

is obtained.

The non-terminals *a* and *b* from these strings are eliminated by rules r_1 and r_2. Hence, the following results are obtained:

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$$(a^{k+1}b, 1-\frac{3^k}{4^k}), (ab^{m+1}, 1-\frac{4^m}{5^m}) \mapsto_{r_1} (a^{k+1}b^{m+1}, 1-\frac{3^k 4^m}{4^k 5^m})$$

and

$$(a^{k+1}b^{m+1}, 1-\frac{3^k 4^m}{4^k 5^m}), (bc^{n+1}, 1-\frac{5^n}{6^n}) \mapsto_{r_2} (a^{k+1}b^{m+1}c^{n+1}, 1-\frac{3^k 4^m 5^n}{4^k 5^m 6^n}), k, m, n \geq 1.$$

Then, the language generated by the bounded-addition fuzzy simple splicing system $\gamma_s^{\oplus}$,

$$L_f(\gamma_s^{\oplus}) = \{(a^{k+1}b^{m+1}c^{n+1}, 1-\frac{3^k 4^m 5^n}{4^k 5^m 6^n}) \mid k, m, n \geq 1\}.$$

When bounded-addition fuzzy simple splicing systems with different thresholds and modes are considered, the threshold languages generated are

1. $L_f(\gamma_s^{\oplus} = 0) = \varnothing \in$ **FIN,**
2. $L_f(\gamma_s^{\oplus} = 1-\frac{3^k 4^m 5^n}{4^k 5^m 6^n}) = \{(a^{k+1}b^{m+1}c^{n+1} \mid k, m, n \geq 1\} \in$ **REG,**
3. $L_f(\gamma_s^{\oplus} = \frac{1}{2}) = \{a^2 b^2 c^2\} \in$ **FIN,**
4. $L_f(\gamma_s^{\oplus} = 1-(\frac{1}{2})^n) = \{a^{n+1}b^{n+1}c^{n+1} \mid n \geq 1\} \in$ **CS - CF.**

The example above demonstrated that with this restriction, the generative power of bounded-addition fuzzy simple splicing systems can be increased up to the context-sensitive languages. The next lemma follows immediately.

Lemma 1: For all families $F \in \{\mathbf{FIN}, \mathbf{REG}, \mathbf{CF}, \mathbf{LIN}, \mathbf{CS}, \mathbf{RE}\}$,

$$\mathrm{SEH}(FIN, F) \subseteq f^{\oplus}\mathrm{SEH}(F).$$

Proof: Let $\gamma_s = (V, A, R)$ be a simple splicing system generating the language $L(\gamma_s) \in \mathrm{SSEH}(\mathbf{FIN}, \mathrm{F})$ where $F \in \{\mathbf{FIN}, \mathbf{REG}, \mathbf{CF}, \mathbf{CS}, \mathbf{RE}\}$. Let $A = \{x_1, x_2, \ldots, x_n\}$, $n \geq 1$. A bounded-addition fuzzy simple splicing system is defined by $\gamma_s^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ where the set of axioms is defined by

$$A^{\oplus} = \{(x_i, \mu(x_i)) : x_i \in A^{\oplus}, 1 \leq n\}$$

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where $\mu(x_i) = 1/n$ for all $1 \le n$, then

$$\sum_{i=1}^{n} \mu(x_i) \le 1$$

and $\oplus$ is a bounded-addition fuzzy operation on [0, 1] defined by

$$\mu_{A+B} = \mu_A + \mu_B - \mu_A \mu_B \text{ where } \mu_A \mu_B \in \mu(x_i).$$

The threshold language generated by $\gamma_s^{\oplus}$ is defined as $L_f(\gamma_s^{\oplus} > 0)$, then it is clear that $L(\gamma_s) = L_f(\gamma_s^{\oplus} > 0)$. Hence, $\mathrm{SEH}(FIN, F) \subseteq f^{\oplus}\mathrm{SEH}(F)$ for all families $F \in \{\mathbf{FIN}, \mathbf{REG}, \mathbf{CF}, \mathbf{LIN}, \mathbf{CS}, \mathbf{RE}\}$.

From Theorem 1, Lemma 1 and Example 1, the following two theorems are obtained.

Theorem 2: Let $\gamma_s^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ be a bounded-addition fuzzy simple splicing system, where $0 < \mu(x) < 1$ for all $x \in A^{\oplus}$ and $\alpha \in [0,1]$. Then,

1. $L_f(\gamma_s^{\oplus} > \alpha)$ is a finite language,
2. $L_f(\gamma_s^{\oplus} \le \alpha)$ is a regular language,
3. $L_f(\gamma_s^{\oplus} \in I)$ is a regular language where I is a subsegment of [0, 1].

Proof:

Case 1: Let $\gamma_s^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ be a bounded-addition fuzzy semi-simple splicing system where

$$A^{\oplus} = \{(x_1, \mu(x_1)), (x_2, \mu(x_2)), ..., (x_n, \mu(x_n))\}$$

and $\mu(x_i) = \mu_i$, $1 \le i \le n$. Since $0 < \mu(x_i) < 1$,

$$\sum_{j=1}^{k+1} \mu(x_{ij}) > \sum_{j=1}^{k} \mu(x_{ij}), \mu_{ij} \in \{\mu_1, ..., \mu_n\}.$$

Then, there exists a positive integer $m = k + 1 \in \mathrm{N}$ such that

$$\sum_{j=1}^{m} \mu(x_{ij}) < \alpha, \mu_{ij} \in \{\mu_1, ..., \mu_n\}$$

where $1 \le j \le m$.

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For any string $x \in \sigma_f^i(A^{\oplus})$, $i \geq m$ that was obtained from some strings of $\sigma_f^{i-1}(A^{\oplus})$ using more than or equal to *m* splicing operations, $\mu(x) < \alpha$ is produced. Thus, $L_f(\gamma_s^{\oplus} > \alpha)$ contains a finite number of strings.

Case 2: Let $L_f(\gamma_s^{\oplus}) = L_f(\gamma_s^{\oplus} > \alpha) \cup L_f(\gamma_s^{\oplus} \leq \alpha)$. Since $L_f(\gamma_s^{\oplus})$ is regular and $L_f(\gamma_s^{\oplus} > \alpha)$ is finite, then $L_f(\gamma_s^{\oplus} \leq \alpha)$ is regular.

Case 3: If $I = (\alpha_1, \alpha_2)$, then $L_f(\gamma_s^{\oplus} \in I) = L_f(\gamma_s^{\oplus} > \alpha) \cap L_f(\gamma_s^{\oplus} < \alpha)$. Hence, according to **Case 1** and **Case 2**, $L_f(\gamma_s^{\oplus} \in I)$ is regular.

Theorem 3: Let $\gamma_s^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ be a bounded-addition fuzzy simple splicing system and $L_f(\gamma_s^{\oplus} * \alpha)$ be a threshold language where $\oplus \in \{min, max\}$, $* \in \{>, <, =\}$ and $\alpha \in [0,1]$. Then,

1. $L_f(\gamma_s^{\oplus} * \alpha)$ is a regular language,
2. If α is *max*, then $L_f(\gamma_s^{\oplus} > \alpha) = \varnothing$ and $L_f(\gamma_s^{\oplus} \leq \alpha) = L_f(\gamma_s^{\oplus})$,
3. If α is *min*, then $L_f(\gamma_s^{\oplus} \leq \alpha) = \varnothing$ and $L_f(\gamma_s^{\oplus} > \alpha) = L_f(\gamma_s^{\oplus})$,
4. If *I* is a subsegment of [0, 1], then $L_f(\gamma_s^{\oplus} \in I)$ is a regular language.

Proof:

Case 1: Let $\gamma_s^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ be a bounded-addition fuzzy simple splicing system with

$$A^{\oplus} = \{(x_1, \mu(x_1)), (x_2, \mu(x_2)), \ldots, (x_n, \mu(x_n))\}.$$

Consider *max* as a bounded-addition fuzzy operation and > as a threshold mode. Then, the set of $\sigma_f^*(A^{\oplus})$ can be represented as $\sigma_f^*(A^{\oplus}) = \sigma_{f1}^*(A^{\oplus}) \cup \sigma_{f2}^*(A^{\oplus})$ where

$$\sigma_{f1}^*(A^{\oplus}) = \{(x, \mu(x)) \in \sigma_{f1}^*(A^{\oplus}) : \mu(x) > \alpha\}, \text{ and}$$

$$\sigma_{f2}^*(A^{\oplus}) = \{(x, \mu(x)) \in \sigma_{f1}^*(A^{\oplus}) : \mu(x) \leq \alpha\}.$$

Let $\sigma_{f1}^*(A^{\oplus}) = A_i^{\oplus}, i = 1, 2$. Then, $A^{\oplus} = A_1^{\oplus} \cup A_2^{\oplus}$ where $A_1^{\oplus} = \{x \in A^{\oplus} : \mu(x) > \alpha\}$, and $A_2^{\oplus} = \{x \in A^{\oplus} : \mu(x) \leq \alpha\}$.

The simple splicing system $\gamma_s = (V, A_2, R)$ is constructed where $L(\gamma_s) = \alpha^*(A_2) \cap T^*$ is regular. Moreover, it is shown that $\sigma_{f2}^*(A^{\oplus}) = \sigma_f^*(A_2^{\oplus})$. First, $\sigma_f^*(A_2^{\oplus}) \subseteq \sigma_{f2}^*(A^{\oplus})$ since $A_2^{\oplus} \subseteq A^{\oplus}$. On the other hand, $\sigma_{f2}^*(A^{\oplus}) \subseteq \sigma_f^*(A_2^{\oplus})$. Let $x \notin \sigma_f^*(A_2^{\oplus})$. Then, there is an axiom $(x, \mu(x)) \in A_1^{\oplus}$ such that

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$$((x_1, \mu(x_1)), (x_2, \mu(x_2))) \mapsto (z_1, \mu(z_1)),\ ((z_1, \mu(z_1)), (z_2, \mu(z_2))) \mapsto (z_3, \mu(z_3)),$$
$$\vdots$$
$$((z_k, \mu(z_k)), (z_{k+1}, \mu(z_{k+1}))) \mapsto (x, \mu(x)),$$

where $(x_2, \mu(x_2)) \in A^{\oplus}$ and $(z_1, \mu(z_1)) \in \sigma_f^*(A^{\oplus})$. Then,

$$max\{\mu(x_1), \mu(x_2)\} = \mu(z_1) > \alpha,$$
$$\vdots$$
$$max\{\mu(z_{k+1}), \mu(z_{k+1})\} = \mu(z_1) > \alpha.$$

Consequently, $(x, \mu(x)) \notin \sigma_{f2}^*(A^{\oplus})$. Thus, $\sigma_{f2}^*(A^{\oplus}) = \sigma_f^*(A_2^{\oplus})$. It follows that the language $L_f(\gamma_s^{\oplus} \leq \alpha) = \sigma_{f2}^*(A^{\oplus}) \cap T^*$ is regular. Hence, $\sigma_{f1}^*(A^{\oplus}) = \sigma_f^*(A^{\oplus}) \setminus \sigma_{f2}^*(A^{\oplus})$ and the language $L_f(\gamma_s^{\oplus} > \alpha) = L_f(\gamma_s^{\oplus}) \setminus L_f(\gamma_s^{\oplus} \leq \alpha)$ is also regular. Similarly, if the bounded-addition fuzzy operation is *min*, it can be proven that $L_f(\gamma_s^{\oplus} > \alpha)$ and $L_f(\gamma_s^{\oplus} \leq \alpha)$ are regular.

Case 2: Let $\alpha > max\{\mu_1, \mu_2, ..., \mu_n\}$. Based on **Case 1**, *max* as bounded-addition fuzzy operation was considered. Then, it produces an axiom such that

$$max\{\mu(x_1), \mu(x_2)\} = \mu(z_1) < \alpha,$$
$$\vdots$$
$$max\{\mu(z_{k+1}), \mu(z_{k+1})\} = \mu(z_1) < \alpha.$$

Thus, the language $L_f(\gamma_s^{\oplus} \leq \alpha)$ is regular and $L_f(\gamma_s^{\oplus} > \alpha)$ is an empty set produced. The language $L_f(\gamma_s^{\oplus}) = L_f(\gamma_s^{\oplus} > \alpha) \cup L_f(\gamma_s^{\oplus} \leq \alpha)$. Since $L_f(\gamma_s^{\oplus} > \alpha)$ is empty and $L_f(\gamma_s^{\oplus} \leq \alpha)$ is regular, then $L_f(\gamma_s^{\oplus})$ is regular. Hence, $L_f(\gamma_s^{\oplus}) = L_f(\gamma_s^{\oplus} \leq \alpha)$.

Case 3: Let $\alpha > min\{\mu_1, \mu_2, ..., \mu_n\}$. Based on **Case 1**, *min* as bounded-addition fuzzy operation was considered. Then, it produces an axiom such that

$$min\{\mu(x_1), \mu(x_2)\} = \mu(z_1) > \alpha,$$
$$\vdots$$
$$min\{\mu(z_{k+1}), \mu(z_{k+1})\} = \mu(z_1) > \alpha.$$

Thus, the language $L_f(\gamma_s^{\oplus} > \alpha)$ is regular and $L_f(\gamma_s^{\oplus} \leq \alpha)$ is an empty set produced. The language $L_f(\gamma_s^{\oplus}) = L_f(\gamma_s^{\oplus} > \alpha) \cup L_f(\gamma_s^{\oplus} \leq \alpha)$. Since $L_f(\gamma_s^{\oplus} \leq \alpha)$ is empty and $L_f(\gamma_s^{\oplus} > \alpha)$ is regular, then $L_f(\gamma_s^{\oplus})$ is regular.

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Hence, $L_f(\gamma_s^{\oplus}) = L_f(\gamma_s^{\oplus} > \alpha)$.

Case 4: Let $L_f(\gamma_s^{\oplus} \in I) = L_f(\gamma_s^{\oplus} > \alpha_1) \cap L_f(\gamma_s^{\oplus} < \alpha_2)$ where $I = (\alpha_1, \alpha_2)$. From **Case 1**, $L_f(\gamma_s^{\oplus} > \alpha_1)$ and $L_f(\gamma_s^{\oplus} < \alpha_2)$ are regular. Therefore, their intersections are also regular.

As direct consequences of the theorems above, interesting facts of bounded-addition fuzzy simple splicing systems are obtained, as stated in Corollary 1, Corollary 2 and Corollary 3.

Corollary 1: If the fuzzy membership of each axiom $\mu(x) \in A^{\oplus}$ in a bounded-addition fuzzy simple splicing system $\gamma_s^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ is nonzero, then the threshold language $L_f(\gamma_s^{\oplus} = 0)$ is an empty set, i.e., $L_f(\gamma_s^{\oplus} = 0) = \varnothing$.

Corollary 2: If the fuzzy membership of each axiom $\mu(x) \in A^{\oplus}$ in a bounded-addition fuzzy simple splicing system $\gamma_s^{\oplus} = (V, A^{\oplus}, R, \mu, \oplus)$ is not greater than 1, then every threshold language $L_f(\gamma_s^{\oplus} = \alpha)$ with $\alpha \in [0,1]$ is finite.

Corollary 3: Every fuzzy simple splicing system with the bounded-addition operation *max* or *min*, and the cut-points of any number in [0, 1] or any subinterval of [0, 1], generates a regular language.

4. CONCLUSION

The concept of bounded-addition fuzzy simple splicing systems has been established and its preliminary properties are determined in this study. The truth values from closed interval [0, 1] are associated with each axiom, and the truth value of a string *z* generated from strings *x* and y is calculated by applying fuzzy bounded-addition operation over the truth values. It has been shown that by introducing bounded addition fuzzy simple splicing systems, the generative power can be increased up to the context-sensitive languages. Besides, some threshold languages with the selection of appropriate cut-points can also generate non-regular languages.

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