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Comparative Analysis Of Scheduling Problem Under Linguistic Environment

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Abstract

This paper deals with the methodlogy to solve the scheduling problems in modified algorithm under the intuitionistic trapezoid fuzzy linguistic and intuitionitic linguistic environment.In many real life decision making analysis the application of Intuitionistic Linguistic Variables is used to get the appropriate answers quickly.Anumerical example is given to illustrate the solution of scheduling problems under Intuitionistic Trapezoid Fuzzy Linguistic and Intuitionistic Linguistic environment

Keywords **:** Processing time, Intuitionistic Trapezoid Fuzzy Linguistic Variable, Modified Algorithm, Rental Cost**.**

1.INTRODUCTION

To deal with vague problems and problems with uncertainity zadeh [8] developed the idea of fuzzy set theory which is majorly characterised by membership degree .Linguistic variable is an another important tool to express the most preference information to the decision makers under uncertain environments. Linguistic Variables properly describes the qualitative linguistic information from 'extremely low' to 'extremely high'.

Later Atanassov [1]developed the idea of intuitionistic fuzzy set which characterise both the membership and non-membership function.The decision makers can clearly express the information by combining the idea of Linguistic variables and Intuitionistic fuzzy set .The concept of intuitionistic linguistic set was developed by Wang and Li.

To overcome uncertain and inaccuracy information more effectively, the combination of trapezoid fuzzy linguistic variables and intuitionistic fuzzy set is necessary. For example ,the mere application of

$$S = \begin{Bmatrix} s_0(extermely\ low); s_1(very\ low); s_2(low): s_3(medium); \\ s_4(high); s_5(very\ high); s_6(extermely\ high) \end{Bmatrix}$$

in the linguistic range of trapezoid fuzzy linguistic $\left|s_\alpha, s_\beta, s_\gamma, s_\delta\right|$ $(0 \leq \alpha \leq \beta \leq \gamma \leq \delta)$ set is not accurate .The introduction of membership and non membership degree such as $u\ and\ v$ is needed to combine with $\left|s_\alpha, s_\beta, s_\gamma, s_\delta\right|$ to describe the idea of intuitionistic trapezoid fuzzy linguistic set as $\langle\left|s_\alpha, s_\beta, s_\gamma, s_\delta\right|, (u, v)\rangle$.

To minimise the production time and to increase the profit many production sectors are widely using the idea of scheduling which gives the perfect sequencial operation in a particular manner. Sameer Sharma and Deepak Gupta [3] analysed rental cost with break down interval and job block criteria. To get the solution of scheduling problems various algorithms has been developed .Nagoor Gani and Mohamed [2]solved assignment problem with the modified algorithm in a efficient manner .Application of modified algorithm in flow shop scheduling problems provides the best way to calculate the total elapsed time and rental.

This paper specifies the application of ITrFL and ILN information in a production sector to calculate total elapsed time and rental cost under modified algorithm for scheduling problems.

2. BASIC DEFINITIONS:-

2.1 Let X be a nonempty set, a **Fuzzy set** $\tilde{A}$ is defined by $\tilde{A}$ = {(x, $\mu_{\tilde{A}}$(x)): x∈A}. In the pair (x, $\mu_{\tilde{A}}$(x)), the first element belongs to the classical set A, the second element $\mu_{\tilde{A}}$(x), belong to the interval [0, 1] is called the membership function.

2.2 Fuzzy number $\tilde{A}$ is a fuzzy set on the real line$\mathfrak{R}$, must satisfy the following conditions.

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(i) $\mu_{\tilde{A}}(x_0)$ is piecewise continuous

(ii) There exist at least one $x_0 \in \Re$ with $\mu_{\tilde{A}}(x_0)$ =1

(iii) $\widetilde{A}$ must be normal & convex

2.3 Intuitionistic Fuzzy number

An Intuitionistic fuzzy subset $A^I = \{(x_i, \mu_{A^I}(x), \gamma_{A^I}(x) \,/ x_i \in X)$ of the real line R is named as an intuitionistic fuzzy number if the following holds.

(i) There exist θ ∈R, $\mu_{A^I}(\theta)$ = 1 and $\gamma_{A^I}(\theta)$ = 0 . Where θ is the mean value of A^I.

(ii) μ_{A^I} is continuous mapping from R to [0,1] for all x∈R, the relation

0≤ $\mu_{A^I}(x) + \gamma_{A^I}(x)$ ≤1 holds. The membership and non-membership function of A^I is of the following form,

$$\mu_{A^I}(x) = \begin{cases} 0, & if -\alpha < x < \theta - \alpha \\ f_1(x), & if\ x \in [\theta - \alpha, \theta] \\ 1, & if\ \ x = \theta \\ g_1(x), & if\ x \in [\theta, \theta + \beta] \\ 0, & if \theta + \beta \leq x < \alpha \end{cases}$$

$$\gamma_{A^I}(x) = \begin{cases} 1, & if -\alpha < x < \theta - \alpha' \\ f_2(x), & if\ x \in [\theta - \alpha', \theta]; 0 \leq f_1(x) + f_2(x) \leq 1 \\ 0, & if\ x = \theta \\ g_2(x), & if\ x \in [\theta, \theta + \beta']; 0 \leq g_1(x) + g_2(x) \leq 1 \\ 1, & if \theta + \beta' \leq x \leq \alpha \end{cases}$$

Where $f_i(x)$ and , , $g_i(x)$; i =1,2 which are strictly increasing and decreasing functions in $[\theta - \alpha, \theta]$, $[\theta, \theta + \beta]$, $[\theta - \alpha', \theta]$ and $[\theta, \theta + \beta']$ respectively. $\alpha, \beta, \alpha' and\ \beta'$ are left and right spreads of $\mu_{A^I}(x)$ and $\gamma_{A^I}(x)$.

2.4.Definition:Intuitionistic Linguistic Variable:

Intuitionistic linguistic set T in X can be defined as $T = \{\langle x[s_\theta(x), (u(x), v(x))]\rangle / x \epsilon X\}$

Where $s_\theta(x) \epsilon [0,1], u(x) \epsilon [0,1], and v(x) \epsilon [0,1]$ .Let $\pi(x) = 1 - u(x) - v(x)$ where $\pi(x) = [0,1]$ is called hesistancy degree of x to linguistic term $s_\theta(x)$.

2.5 Trapezoid Fuzzy Linguistic Variable

A finite ,completely ordered discerte linguistic set is termed as $S = \{s_0, s_1, \dots \dots s_{l-1}\}$ where l is the odd value. For instance when l = 7 the linguistic term set S can be defined as follows $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$.

Definition: Let $\bar{S} = \{s_\theta / s_0 \leq s_\theta \leq s_{l-1}\}, \theta \in [0, l-1]$ which is the continuous form of linguistic set S. $s_\alpha, s_\beta, s_\gamma, s_\delta$ are four linguistic terms in $\bar{S}$ and $s_0 \leq s_\alpha \leq s_\beta \leq s_\gamma \leq s_\delta \leq s_{l-1}$ then the trapezoid fuzzy linguistic is defined as $\bar{S} = |s_\alpha, s_\beta, s_\gamma, s_\delta|$ and $\bar{S}$ denote a set of trapezoid fuzzy linguistic variables. If any two of $\alpha, \beta, \gamma, \delta$ are equal ,then $\bar{S}$ is reduced to a triangular fuzzy linguistic variable ,if any three are equal,it is uncertain linguistic variable.

2.6 Definition :Intuitionistic Trapezoid Fuzzy Linguistic Number

An intuitionistic trapezoid fuzzy linguistic set T in X can be defined as

as $T = \{\langle x[|s_{\alpha(x)}, s_{\beta(x)}, s_{\gamma(x)}, s_{\delta(x)}|(u(x), v(x))]\rangle / x \epsilon X\}$ where $s_{\alpha(x)}, s_{\beta(x)}, s_{\gamma(x)}, s_{\delta(x)} \epsilon \bar{S}$

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and $u(x) + v(x) \leq 1 \ \forall x \in X$ $\left|s_{\alpha(x)}, s_{\beta(x)}, s_{\gamma(x)}, s_{\delta(x)}\right|$ is a trapezoid linguistic fuzzy linguistic term ,where $u(x), v(x)$ are membership and non membership function and

$\pi(x) = 1 - u(x) - v(x)$ where $\pi(x) = [0,1]$ is called hesistancy degree of x to linguistic term $s_\theta(x)$.

3.Arithmetic Operators:

Let $a_i = \langle\left|s_{\alpha(a_i)}, s_{\beta(a_i)}, s_{\gamma(a_i)}, s_{\delta(a_i)}\right|, u(a_i), v(a_i))\rangle$ and

$a_j = \langle\left|s_{\alpha(a_j)}, s_{\beta(a_j)}, s_{\gamma(a_j)}, s_{\delta(a_j)}\right|, u(a_j), v(a_j))\rangle$ be two ITrFLN's then

1. $a_i + a_j = \langle\left|s_{\alpha(a_i)+\alpha(a_j)}, s_{\beta(a_i)+\beta(a_j)}, s_{\gamma(a_i)+\gamma(a_j)}, s_{\delta(a_i)+\delta(a_j)}\right|, u(a_i) + u(a_j) - u(a_i)u(a_j), v(a_i)v(a_j))\rangle$
2. $a_i * a_j = \langle\left|s_{\alpha(a_i)*\alpha(a_j)}, s_{\beta(a_i)*\beta(a_j)}, s_{\gamma(a_i)*\gamma(a_j)}, s_{\delta(a_i)*\delta(a_j)}\right|, u(a_i)u(a_j), v(a_i) + v(a_j) - v(a_i)v(a_j))\rangle$

4. Ranking Forrmula:

The normalised Hamming Distance between $a_i \ and \ a_j$ is given by

$$d(a_i, a_j) = \frac{1}{2(l-1)}\left|(1 + u(a_i) - v(a_i)) * \frac{\alpha(a_i) + \beta(a_i) + \gamma(a_i) + \delta(a_i)}{4} - \left(1 + u(a_j) - v(a_j)\right) * \frac{\alpha(a_j) + \beta(a_j) + \gamma(a_j) + \delta(a_j)}{4}\right|$$

5. Notations

f_{ij} - Processing time of i^{th} job on $a \ j^{th}$ machine

R(S)- Total rental cost for the sequence (S)

$U_k(S_K)$ -Utilisation time of each machine

Cm -Cost for each rent (m =1…..4)

6. Problem Formulation

Assume that some jobs $i(i = 1,2, \ldots\ldots n)$ are to be processed on machines $j(j = 1,2, \ldots\ldots m)$ under the specified rental policy.

Let f_{ij} be the processing time of i^{th} job on j^{th} machine described by the ITrFLN and ILN. Our aim is to find the minimal rental cost

$$R(S) = \sum_{i=1}^{n} f_{ij} * C1 + U_2(S_K) * C2 + U_3(S_K) * C3 + U_4(S_K) * C4$$

7. Algorithm

Step 1: Defuzzify the ITrFL number in to a crisp number by using normalised hamming distance formula.

Step 2: Form three columns ,the first column represent the jobs ,second column represent the minimum processing time in each row ,and the third column allocation of jobs.

Step 3:If there is a tie in the minimum processing time ,choose the minimum and the next minimum of the corresponding rows ,find the difference .The highest difference will get the allocation.

Step 4:Form the sequence from step.2 and step.3 ,until all the jobs are arranged .

Step 5:Calculate the minimum total elapsed time and the rental cost.

8. **Numerical Example:**

Consider 4 jobs and 4 machines problem to minimise the rental cost ,here the processing time are being given in ITrFL numbers whose ranges are from [0,1].Obtain the optimal sequence of the jobs and the minimum rental cost of the machines if the rental charges are given as Rs.100,Rs.50,Rs.150 and Rs.200 respectively. The nature of the processing time are in the following manner

$$S = \left\{\begin{matrix} s_0(extermely\ low); s_1(very\ low); s_2(low): s_3(medium); \\ s_4(high); s_5(very\ high); s_6(extermely\ high) \end{matrix}\right\}$$

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Machines :

I $< s_1, s_3, s_5, s_6, (0.6,0.2) >$ II $< s_2, s_3, s_4, s_5, (0.7,0.3) >$

III $< s_1, s_4, s_5, s_6, (0.1,0.6) >$ **IV** $< s_3, s_4, s_5, s_6, (0.5,0.3) >$

Jobs :

1 $< s_3, s_4, s_5, s_6, (0.4,0.5) >$ **2** $< s_1, s_2, s_3, s_4, (0.2,0.5) >$ Type equation here.

3 $< s_2, s_3, s_4, s_5, (0.3,0.5) >$ **4** $< s_1, s_3, s_4, s_5, (0.7,0.2) >$

Solution:

The crisp value of the ITrFL numbers are

1. $< s_1, s_3, s_5, s_6, (0.6,0.2) >$ = 0.438 ,2. $< s_2, s_3, s_4, s_5, (0.7,0.3) >$ = 0.408
3. $< s_1, s_4, s_5, s_6, (0.1,0.6) >$ =0.167 , 4. $< s_3, s_4, s_5, s_6, (0.5,0.3) >$ = 0.450
5. $< s_3, s_4, s_5, s_6, (0.4,0.5) >$ = 0.338, 6. $< s_1, s_2, s_3, s_4, (0.2,0.5) >$ = 0.146
7. $< s_2, s_3, s_4, s_5, (0.3,0.5) >$ = 0.233, 8. $< s_1, s_3, s_4, s_5, (0.7,0.2) >$ = 0.406

The processing time of the jobs are given below

		0.438	0.408	0.167	0.450
	Machines Jobs	I	II	III	IV
0.338	1	0.148	0.138	0.056	0.152
0.146	2	0.064	0.060	0.024	0.066
0.233	3	0.102	0.095	0.039	0.105
0.406	4	0.178	0.166	0.068	0.183

Allocate the jobs with minimum processing time.Since there is a tie we need to form the difference table

Jobs	Minimum time	Allocation	Min-Next Min (if tie) - (Max diff)
1	0.056	III	0.082
2	0.024	III	0.036
3	0.039	III	0.056
4	0.068	III	**0.098**

Here 4^{th} job is allocated to III.So the sequence is $\boldsymbol{s_3}$.

Delete fourth row and third column

The remaining cells are

Machines Jobs	I	II	IV
1	0.148	0.138	0.152
2	0.064	0.060	0.066
3	0.102	0.095	0.105

Allocation of jobs

Jobs	Minimum time	Allocation	Min-Next Min (if tie) - (Max diff)
1	0.138	II	**0.010**
2	0.060	II	0.004
3	0.095	II	0.007

Here job 1is allocated to II.the sequence is s_3- s_2. Eliminate first row and second column
The remaining allocations are

Machines Jobs	I	IV
2	0.064	0.066
3	0.102	0.105

Jobs	Minimum time	Allocation	Min-Next Min (if tie) - (Max diff)
2	0.064	I	0.02
3	0.102	I	**0.03**

Here third job is allocated to I.Therefore s_3- s_2-s_1Type equation here.
The sequence is s_3- s_2-s_1- s_4.
The in-out flow table for the above formed sequence is given below

Machines	I		II		III		IV	
JOBS	In	Out	In	Out	In	Out	In	Out
3	--	0.102	0.102	0.197	0.197	0.236	0.236	0.341
2	0.102	0.166	0.197	0.257	0.257	0.281	0.341	0.407
1	0.166	0.314	0.314	0.452	0.452	0.508	0.508	0.660
4	0.314	0.492	0.492	0.658	0.658	0.726	0.726	**0.909**

Minimum total elapsed time = 0.909 hrs
Idle time of I = 0.417 hrs, Idle time of II = 0.450 hrs, Idle time of III = 0.722 hrs, Idle time of IV = 0.403 hrs
Total rental cost
R = 0.492*100+0.208*50+0.004*150+0.506*200 **= Rs. 161.40**
Comparison with the Intuitionistic Linguistic Number
Numerical Example :
Consider the 4 jobs and 4 machine problem to minimise the rental cost.The processing time are Intuitionistic Linguistic Number .Calculate the total elapsed time and the rental cost.the cost per unit hour of each machine is Rs.100, Rs.50, Rs.150, Rs.200,

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Machines Jobs	I	II	III	IV
A	$< s_5, (0.6,0.2) >$	$< s_3, (0.3,0.4) >$	$< s_4, (0.6,0.3) >$	$< s_1, (0.5,0.2) >$
B	$< s_1, (0.6,0.2) >$	$< s_4, (0.5,0.4) >$	$< s_5, (0.3,0.2) >$	$< s_2, (0.2,0.5) >$
C	$< s_3, (0.1,0.7) >$	$< s_2, (0.2,0.5) >$	$< s_4, (0.1,0.6) >$	$< s_5, (0.3,0.5) >$
D	$< s_2, (0.6,0.2) >$	$< s_3, (0.3,0.4) >$	$< s_5, (0.5,0.4) >$	$< s_6, (0.6,0.1) >$

Solution :
The above linguistic values can be defuzzifyed using
The normalised Hamming Distance between a_i *and* a_j

$$d(a_i, a_j) \quad = \frac{1}{2(l-1)}\left|\left(1+u(a_i)-v(a_i)\right)*\alpha(a_j)-\left(1+u(a_j)-v(a_j)\right)*\alpha(a_j)\right|$$

Machines/ Jobs	I	II	III	IV
A	0.458	0.225	0.433	0.058
B	0.050	0.367	0.375	0.117
C	0.100	0.117	0.167	0.333
D	0.100	0.225	0.458	0.250

Allocate the job with minimum processing time

Jobs	Minimum time	Allocation
A	0.058	IV
B	0.050	I
C	0.100	I
D	0.100	I

Now job A is allocated to IV .We can form the sequence as $\boldsymbol{s_4}$ and delete the first row and fourth column.

Machines Jobs	I	II	III
B	0.050	0.367	0.375
C	0.100	0.117	0.167
D	0.100	0.225	0.458

Allocations of jobs

Jobs	Minimum time	Allocation	Min-Next Min (if tie) - (Max diff)
B	0.050	I	**0.317**
C	0.100	I	0.017
D	0.100	I	0.125

Here Job B is allocated to I. The sequence is $\boldsymbol{s_4 - s_1}$ .Deleting second row and first column.

Machines Jobs	II	III
C	0.117	0.167
D	0.225	0.458

Allocations of jobs

Jobs	Minimum time	Allocation	Min-Next Min (if tie) - (Max diff)
C	0.117	II	0.050
D	0.225	II	**0.233**

Now D is allocated to II
Therefore the complete sequence is $\mathbf{s_4 - s_1 - s_2 - s_3}$.
The in-out table is given below for the sequence

Machines	I		II		III		IV	
JOBS	In	Out	In	Out	In	Out	In	Out
D	--	0.100	0.100	0.325	0.325	0.783	0.783	1.033
A	0.100	0.558	0.558	0.783	0.783	1.216	1.216	1.274
B	0.558	0.608	0.783	1.150	1.216	1.591	1.591	1.708
C	0.608	0.708	1.150	1.267	1.591	1.758	1.758	**2.091**

Minimum Total Elapsed Time = 2.091 hrs
Idle time of I = 1.383 hrs, Idle time of II = 1.157 hrs, Idle time of III = 0.658hrs, Idle time of IV = 1.333 hrs
Total Rental Cost
R = 0.708*100+0.110*50+1.100*150+0.758*200 = **Rs.392.90**

9. Conclusion

Solving flow shop scheduling problem in ITrFL numbers is very efficient when compared with the problem solved in ILN .Future work deals with the proposal of new algorithm in interval valued intuitionistic fuzzy numbers

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