JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2216-2222
https://publishoa.com
ISSN: 1309-3452

A New Technique for Solving Fuzzy Transportation Problem Using Trapezoidal Fuzzy Numbers

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Abstract

Here we propose a new kind of approach to solve the fuzzy transportation problem with an imprecise environment, where the transportation costs are in the form of trapezoidal fuzzy numbers. In real life situation, because of many reasons, supply, demand, and unit transportation costs may become inconsistent. These inaccurate data can be represented as fuzzy numbers. The fuzzy numbers and values were majorly used in various fields such as experimental sciences, artificial intelligence, etc. Here, we converted the trapezoidal fuzzy numbers into crisp values by using the magnitude ranking function and by applying Max-min method the initial basic feasible solution to the fuzzy transportation problem were obtained. The numerical illustration demonstrates the new projected way for managing transportation problems on fuzzy algorithms.
Keywords Fuzzy set, Fuzzy number, Triangular fuzzy number, Trapezoidal fuzzy number, Magnitude ranking function.

1. Introduction

Transportation models have a wide range of applications in supply chain management and logistics for reducing the cost-efficient algorithms developed to solve the transportation problems when the cost coefficients, supply, demand are known precisely. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. In that case, the cost coefficients, supply, and demand are uncertain because of uncontrollable factors. To deal with these ambiguous situations, Bellmann and Zadeh [2] introduced the notion of fuzziness. In 1941, Hitchcock [6] initiated the fundamental transportation problem. In 1951, G.B Dantzig [3] had first provided the linear programming formulation and systematic approach for the transportation problem.

In 2004, Liu and Kao [8] Proposed a new fuzzy approach to the multi-objective transportation problem. In 2006, Gani and Razak [5] had presented the fuzzy transportation problem in which two-stage cost-minimizing whose supplies and demands are taken as trapezoidal fuzzy numbers. Their main objective is to minimize transportation costs in two stages. In 2010, Kumar, Bansal, and Neetu [7] studied a new method to solve the fully fuzzy linear systems with trapezoidal fuzzy numbers. In 2010, De and Yadav [4] initiated to defuzzify the trapezoidal fuzzy number in the transportation problem. In 2012, Amarpeet Kaur and Amit Kumar [1] introduced a new approach to solving fuzzy transportation problems by generalized trapezoidal fuzzy numbers. In 2016, Neetum Sharma [9] proposed a new method for solving transportation problems, an alternative method for the North West Corner method. Here a new approach, namely the max-min method were being used to solve the fuzzy transportation problem by using the magnitude ranking function. The supply, demand, and unit transportation costs are trapezoidal fuzzy numbers. The max-min method gives the minimum value for comparing all other existing methods such as the Northwest corner, LCM, and VAM. An illustrative example is given for the best understanding of the given algorithm.

2. Preliminaries

2.1 Definition: Fuzzy set

A fuzzy set A in R (real line) is defined as a set of ordered pair A = $\{x_0, \mu_A(x_0)/x_0 \in A, \mu_A(x_0) \to [0,1]\}$,Where $\mu_A(x_0)$ is said to be the membership function.

2.2 Definition: Fuzzy number

A is a fuzzy set on the real line$R$, must satisfy the following conditions.

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(i) $\mu_A(x_0)$ is piecewise continuous

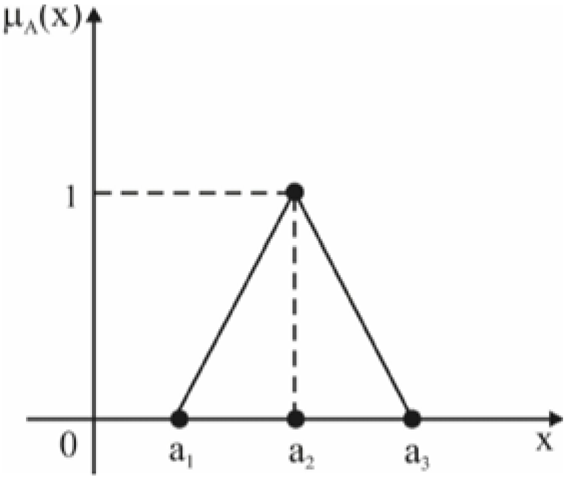
(ii) There exist at least one $x_0 \in \Re$ with $\mu_A(x_0) = 1$

(iii) A must be normal & convex

2.3 Definition: Triangular fuzzy number

A fuzzy number A is a triangular fuzzy number which is called as $(a_{1,} a_{2,} a_3)$ if its membership function $\mu_A(x)$ has the following characteristic

$$\mu_A(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, \ if\ a_1 \le x < a_2 \\ 1,\ if\ x = a_2 \\ \frac{a_3 - x}{a_3 - a_2}, \ if\ a_2 < x \le a_3 \\ 0,\ if\ Otherwise \end{cases}$$



2.4 Definition: Trapezoidal fuzzy number

A fuzzy number A is a trapezoidal fuzzy number which is named as ($a_1, a_2, a_{3,} a_4$) where $a_1, a_2, a_{3,} a_4$ are real numbers whose membership function $\mu_A(x)$ is given by

$$\mu_A(x) = \begin{cases} 0, \quad if\ x < a_1 \\ \frac{x - a_1}{a_2 - a_1}, \quad if\ a_1 \le x \le a_2 \\ 1, \quad if a_2 \le x \le a_3 \\ \frac{a_4 - x}{a_4 - a_3}, \quad if\ a_3 \le x \le a_4 \\ 0, \quad if\ x > a_4 \end{cases}$$

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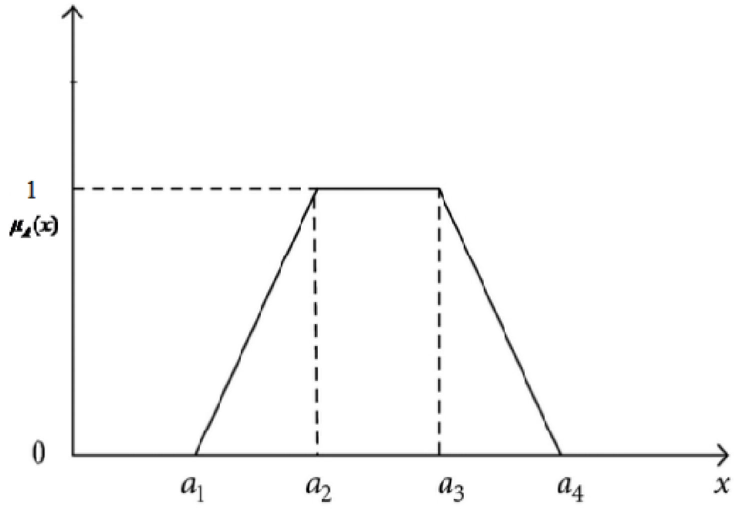

2.5 Arithmetic operations of triangular fuzzy numbers:

Addition:

Let $\bar{A} = (a_1, a_2, a_3,)$ and $\bar{B}$ = $(b_1, b_2, b_3,)$ be triangular fuzzy numbers, then

$\bar{A}$ + $\bar{B}$ = $(a_1 + b_1, a_2 + b_2, a_3 + b_3)$

Subtraction:

$\bar{A}$ - $\bar{B}$ = $(a_1 - b_3, a_2 - b_2, a_3 - b_1)$

Multiplication:

$\bar{A} * \bar{B}$ = {min$(a_1b_1, a_1b_3, a_3b_1, a_3b_3)$, a_2b_2, max$(a_1b_1, a_1b_3, a_3b_1, a_3b_3)$}

2.6 Arithmetic operations of trapezoidal fuzzy numbers:

Addition**:**

Let$\bar{A} = (a_1, a_2, a_3, a_4)$and $\bar{B}$ = (b_1, b_2, b_3, b_4)be trapezoidal fuzzy numbers, then

$\bar{A}$ + $\bar{B}$ = $(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

Subtraction**:**

$\bar{A}$ - $\bar{B}$ = $(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

Multiplication**:**

$\bar{A} * \bar{B}$ ={min$(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$, min$(a_2b_2, a_3b_2, a_2b_3, a_3b_3)$,
max($a_2b_2, a_3b_2, a_2b_3, a_3b_3$) , max$(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$}

3. Definition: Ranking Formula

Let $\bar{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number, then

$\bar{V}$ = $(x_0 - \alpha, x_0, y_0, y_0 + \alpha)$ with parametric form $\bar{V} = \left(\underline{V}(h), \overline{V}(h)\right)$

Where $\underline{V}(h)$ = $(x_0 - \alpha + \alpha h$) and $\overline{V}(h)$ = $(y_0 + \beta - \beta h)$ are defined as

Mag$(\bar{V})$ = $\frac{1}{2}\left[\int_0^1\left(\underline{V}(h) + \overline{V}(h) + x_0 + y_0\right)\right]$hdh. Where h$\in$ [0,1]

The magnitude of the trapezoidal fuzzy number

$\bar{V}$= (a_1,a_2,a_3,a_4) is given by Mag $\bar{V}$ = $\frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$

4. Mathematical Formulation

. Consider a fuzzy transportation problem with m sources and n destinations with trapezoidal fuzzy numbers. Let a_i, ($a_i \geq 0$) be the fuzzy availability at source i and b_j, ($b_j \geq 0$) be the fuzzy requirement at destination j. Let c_{ij} be the fuzzy unit transportation cost from source i to destination j. Let x_{ij} denote the number of fuzzy units transported from source i to destination j. Then the problem is to find a feasible way of transporting the available amount at each supply to satisfy the demand at each destination so that the total transportation cost is minimized.

The mathematical formulation of the fuzzy transportation whose parameters are trapezoidal fuzzy numbers under the case that the total supply is equivalent to the total demand is given by

Min z = $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i$, i =1,2,……m.
$\sum_{i=1}^{m} x_{ij} = b_j$, j =1,2,…….n.
$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$; i =1,2,……m; j =1,2,……n and $x_{ij} \geq 0$.

The fuzzy transportation table explicitly represents the fuzzy transportation problem:

	1	⋯	**N**	**Supply**
1	c_{11}	⋯	c_{1n}	a_1
⋮	⋮	⋯	⋮	⋮
M	c_{m1}	⋯	c_{mn}	a_m
Demand	b_1	⋯	b_n	

5.Proposed - Max- min method-Algorithm
Step(1)
Construct the transportation table, we examine whether total demand equals total supply then go to step 2.
Step(2)
Applying magnitude ranking function, we convert the fuzzy cost values into crisp values to the given transportation problem.
Step(3)
For the row-wise difference between the maximum and minimum of each row and it is divided by a number of columns in the cost matrix.
Step(4)
For the column-wise difference between the maximum and minimum of each column and is divided by a number of rows in the cost matrix.
Step(5)
We find the maximum of the resultant values, find the corresponding minimum cost value and allocate that particular cell of the given matrix. Suppose we have more than one maximum resulting value. We can select anyone.
Step(6)
Repeated procedures 1 to 5 until all the allocations are completed.

5. Numerical example

An oil Product is manufactured by five factories Factory1, Factory2, Factory3, Factory4, Factory5. The production capacity of the five factories are **(38,39,40,44), (22,24,25,30), (30,34,35,36), (16,20,21,22, (27,29,31,33)** respectively. The product is supplied to five retail stores Store1, Store2, Store3, Store4, Store5. The requirements of Demands are **(21,23,25,27), (43,44,45,49), (23,24,25,29), (31,32,34,35), (18,22,23,24)** respectively. The unit costs of fuzzy transportation are represented as trapezoidal fuzzy numbers are given below. Find the fuzzy transportation plan such that the total production and transportation cost is minimum.

	Store1	**Store2**	**Store3**	**Store4**	**Store5**	**Capacity**
Factory1	(1,2,3,6)	(3,6,7,8)	(2,4,6,8)	(0,1,3,6)	**(5,7,9,12)**	**(38,39,40,44)**
Factory2	(2,4,6,8)	(1,3,4,5)	(4,6,8,9)	(2,6,7,9)	**(3,4,7,8)**	**(22,24,25,30),**
Factory3	(3,4,5,6)	(5,6,8,9)	(5,8,9,10)	(1,3,4,6)	**(2,3,4,5)**	**(30,34,35,36)**
Factory4	(2,5,6,7)	(2,3,4,7)	(3,6,8,9)	(4,7,8,11)	**(6,7,8,10)**	**(16,20,21,22)**
Factory5	(4,6,7,9)	(0,2,4,6)	(1,4,5,6)	(3,5,7,10)	**(1,3,5,9)**	**(27,29,31,33)**
Demand	**(21,23,25,27)**	**(43,44,45,49)**	**(23,24,25,29)**	**(31,32,34,35)**	**(18,22,23,24)**	

Solution:

We have to convert trapezoidal fuzzy numbers into crisp values using the magnitude ranking function.

	Store1	Store2	Store3	Store4	Store5	capacity
Factory1	2.83	6.83	5	1.67	8.17	**40**
Factory2	5	3.33	6.83	6.17	5.50	**25**
Factory3	4.50	7	8.17	3.50	3.50	**34**
Factory4	5.17	3.83	6.67	7.50	7.67	**20**
Factory5	6.50	3	4.17	6.17	4.33	**30**
Demand	**24**	**45**	**25**	**33**	**22**	

Find the maximum of the resultant values, find the corresponding minimum cost value, and allocate the particular cost cell of the given matrix. If we have more than one maximum resulting value, we can select anyone.

	Store1	Store2	Store3	Store4	Store5	Capacity	$\frac{\boldsymbol{Max-Min}}{\boldsymbol{5}}$
Factory1	2.83	6.83	5	1.67 **33**	8.17	**40**	$\frac{\mathbf{6.50}}{\mathbf{5}}$ **=1.30←**
Factory2	5	3.33	6.83	6.17	5.50	**25**	$\frac{3.50}{5} = 0.70$
Factory3	4.50	7	8.17	3.50	3.50	**34**	$\frac{4.67}{5} = 0.93$
Factory4	5.17	3.83	6.67	7.50	7.67	**20**	$\frac{3.67}{5} = 0.74$
Factory5	6.50	3	4.17	6.17	4.33	**30**	$\frac{3.17}{5} = 0.64$
Demand	**24**	**45**	**25**	**33**	**22**		
$\frac{\boldsymbol{Max-Min}}{\boldsymbol{5}}$	$\frac{3.67}{5} = 0.74$	$\frac{4}{5} = 0.80$	$\frac{4}{5} = 0.80$	$\frac{5.83}{5} = 1.17$	$\frac{4.67}{5} = 0.93$		

We find the next maximum of the resultant values, find the corresponding minimum cost value, and allocate the particular cost cell of the given matrix. If we have more than one maximum resulting value, we can select anyone.

	Store1	Store2	Store3	Store5	Capacity	$\frac{\boldsymbol{Max-Min}}{\boldsymbol{4}}$
Factory1	2.83 **7**	6.83	5	8.17	**7**	$\frac{\mathbf{5.34}}{\mathbf{4}}$ **=1.34←**
Factory2	5	3.33	6.83	5.50	**25**	$\frac{3.50}{4} = 0.88$
Factory3	4.50	7	8.17	3.50	**34**	$\frac{4.67}{4} = 1.17$
Factory4	5.17	3.83	6.67	7.67	**20**	$\frac{3.84}{4} = 0.96$
Factory5	6.50	3	4.17	4.33	**30**	$\frac{3.50}{4} = 0.88$
Demand	**24**	**45**	**25**	**22**		
$\frac{\boldsymbol{Max-Min}}{\boldsymbol{5}}$	$\frac{3.67}{5} = 0.74$	$\frac{4}{5} = 0.80$	$\frac{4}{5} = 0.80$	$\frac{4.67}{5} = 0.93$		

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https://publishoa.com
ISSN: 1309-3452

We will follow the same procedure until we reach the final allocation.

	Store1	**Store2**	**Store3**	**Store4**	**Store5**	**Supply**
Factory1	2.83 7	6.83	5	1.67 33	8.17	**40**
Factory2	5	3.33 25	6.83	6.17	5.50	**25**
Factory3	4.50 12	7	8.17	3.50	3.50 22	**34**
Factory4	5.17 5	3.83 15	6.67	7.50	7.67	**20**
Factory5	6.50	3 5	4.17 25	6.17	4.33	**30**
Demand	**24**	**45**	**25**	**33**	**22**	

The fuzzy transportation cost

$$Z = (1.67\times 33) + (2.83\times 7) + (3.50 \times 22) + (4.50 \times 12) + (3.33 \times 25) +$$

$$(4.17 \times 25) + (3 \times 5) + (5.17\times 5) + (3.83 \times 15)$$

$$Z = \text{Rs.}491.72$$

6. Results and Discussion

The comparison of the max-min method with the existing methods is tabulated below, in which it is clearly shown that the proposed method provides the optimal results.

Methods	**Optimal solution**
Northwest corner method	804.82
VAM method	496.77
LCM method	538.42
Proposed method	**838**

7. Conclusion

This paper provides the application of new algorithm to solve the transportation problem .In this analysis the result obtained by the proposed algorithm yields better results when compared to the exisiting method.

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