JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2223-2228
https://publishoa.com
ISSN: 1309-3452

A Note on Arithmetic Operations of Octagonal Fuzzy Numbers Using α −Cut Method

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Abstract

In this paper new arithmetic operation on α - Cuts of Octagonal fuzzy numbers are investigated. ACOFNS has also been shown to have several important features. Examples are also provided to demonstrate the outcomes.

Keywords: Fuzzy Set, Fuzzy number, Octagonal fuzzy number, α −Cut of Octagonal fuzzy number.

1.Introduction

In a number of different ways, real-world decision-making challenges are frequently ambiguous or vague. To tackle these issues, Zadeh introduced fuzzy set theory in 1965. The membership of elements in a set described in the interval [0,1] can be gradually assessed using fuzzy set theory. It can be applied to a variety of domains when data is inadequate or imprecise. Zadeh's extension theory was used to propose interval arithmetic. The typical real-number arithmetic operations can be extended to fuzzy-number arithmetic. A fuzzy number is a quantity whose values are ambiguous rather than precise, as with single-valued numbers. The most widely utilized shapes of fuzzy numbers are triangular fuzzy numbers and trapezoidal fuzzy numbers.

This paper is organized as follows: In section 2 the basic definitions of a OFN and some operations on OFNs. In section 3 Alpha Cuts of Octagonal Fuzzy Number is discussed. A new arithmetic operation on Alpha cut is presented in section 4. In the next section, numerical example is solved. Finally in section 6, conclusion is included.

2. Preliminaries

Definition 2.1 Fuzzy Set

A **fuzzy set** is characterized by its membership function taking values from the domain, space or the universe of discourse mapped into the unit interval [0,1].A fuzzy set A in the universal set X is defined as

$$A = \{(x, \mu_A(x))/x\epsilon X\}.$$

Here$\mu_A(x): A \to [0,1]$is the grade of the membership function and $\mu_A(x)$is the grade value of $x \in X$ in the fuzzy set A.

Definition 2.2 Convex fuzzy Set

A fuzzy set $A = \{(x, \mu_A(x))\} \subseteq X$ is called a convex fuzzy set if all A_α are **convex sets**, i.e. for every element $x_1 \in A_\alpha$ and $x_2 \in A_\alpha$ we have $\lambda x_1 + (1-\lambda)x_2 \in A_\alpha$ for all $\lambda \in [0,1]$. Otherwise the fuzzy set is called non-convex fuzzy set.

Definition 2.3 Fuzzy number

A fuzzy set A of real line R with membership function $\mu_A(x): R \to [0,1]$ is called **fuzzy number** if

(i) A is normal and convexity.
(ii) A must be bounded.
(iii) $\mu_A(x)$ is piecewise continuous.

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Definition 2.4 Octagonal Fuzzy Number

A fuzzy number $\tilde{A}^{OFN} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ is said to be **octagonal fuzzy number** if its membership function is given by,

$$\mu_{\tilde{A}^{OFN}}(x) = \begin{cases} 0 & for x < a_1 \\ \frac{1}{3}\frac{(x-a_1)}{(a_2-a_1)} & for a_1 \le x \le a_2 \\ \frac{1}{3} + \frac{1}{3}\frac{(x-a_2)}{(a_3-a_2)} & for a_2 \le x \le a_3 \\ \frac{2}{3} + \frac{1}{3}\frac{(x-a_3)}{(a_4-a_3)} & for a_3 \le x \le a_4 \\ 1 & for a_4 \le x \le a_5 \\ 1 - \frac{1}{3}\frac{(x-a_5)}{(a_6-a_5)} & for a_5 \le x \le a_6 \\ \frac{2}{3} - \frac{1}{3}\frac{(x-a_6)}{(a_7-a_6)} & for a_6 \le x \le a_7 \\ \frac{1}{3}\frac{(a_8-x)}{(a_8-a_7)} & for a_7 \le x \le a_8 \\ 0 & otherwise \end{cases}$$

where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ are real numbers

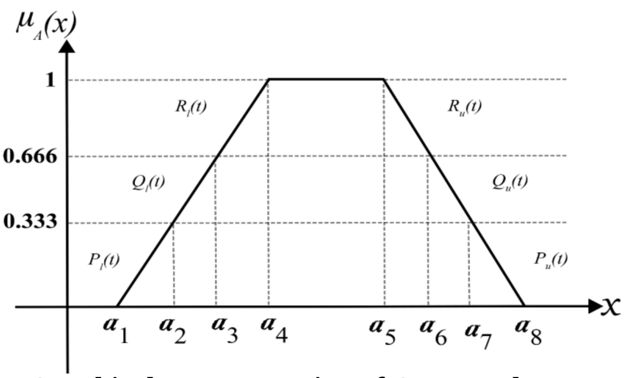

Figure : Graphical Representation of Octagonal Fuzzy Numbers

2.5 Arithmetic Operations on Hexagonal Fuzzy Numbers

If $\tilde{A}_o = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\tilde{B}_o = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ then basic arithmetic operations between them are defined as follows:

i. **Addition:**

$$\tilde{A}_o + \tilde{B}_o = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8)$$

ii. **Subtraction:**

$$\tilde{A}_o - \tilde{B}_o = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6, a_7 - b_7, a_8 - b_8)$$

iii. Multiplication:

$$\tilde{A}_o(X)\tilde{B}_o = \left(\frac{a_1}{8}\sigma_b, \frac{a_2}{8}\sigma_b, \frac{a_3}{8}\sigma_b, \frac{a_4}{8}\sigma_b, \frac{a_5}{8}\sigma_b, \frac{a_6}{8}\sigma_b, \frac{a_7}{8}\sigma_b, \frac{a_8}{8}\sigma_b\right)$$

Where $\sigma_b = b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8$ (or)

$$\tilde{A}_o(X)\tilde{B}_o = \left(a_1\tilde{R}(b), a_2\tilde{R}(b), a_3\tilde{R}(b), a_4\tilde{R}(b), a_5\tilde{R}(b), a_6\tilde{R}(b), a_7\tilde{R}(b), a_8\tilde{R}(b)\right)$$

Where

$$\tilde{R}(\tilde{B}_o) = \frac{(b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8)}{8}$$

(or)

$$R(b) = \frac{(b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8)}{8}$$

iv. Division:

$$\tilde{A}_o(/)\tilde{B}_o = \left(\frac{8a_1}{\sigma_b}, \frac{8a_2}{\sigma_b}, \frac{8a_3}{\sigma_b}, \frac{8a_4}{\sigma_b}, \frac{8a_5}{\sigma_b}, \frac{8a_6}{\sigma_b}, \frac{8a_7}{\sigma_b}, \frac{8a_8}{\sigma_b}\right)$$

Where $\sigma_b = b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8$

(or)

$$\tilde{A}_o(/)\tilde{B}_o = \left(\frac{a_1}{\tilde{R}(b)}, \frac{a_2}{\tilde{R}(b)}, \frac{a_3}{\tilde{R}(b)}, \frac{a_4}{\tilde{R}(b)}, \frac{a_5}{\tilde{R}(b)}, \frac{a_6}{\tilde{R}(b)}, \frac{a_7}{\tilde{R}(b)}, \frac{a_8}{\tilde{R}(b)}\right)$$

Where

$$\tilde{R}(\tilde{B}_o) = \frac{(b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8)}{8}$$

(or)

$$R(b) = \frac{(b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8)}{8}$$

v. Join Operator:

$$\tilde{A}_o(\vee)\tilde{B}_o = (a_1\vee b_1, a_2\vee b_2, a_3\vee b_3, a_4\vee b_4, a_5\vee b_5, a_6\vee b_6, a_7\vee b_7, a_8\vee b_8)$$

$$= \left(max(a_1, b_1), max(a_2, b_2), max(a_3, b_3), max(a_4, b_4), max(a_5, b_5), max(a_6, b_6), max(a_7, b_7), max(a_8, b_8)\right)$$

vi. Meet Operator:

$$\tilde{A}_o(\wedge)\tilde{B}_o = (a_1\wedge b_1,\ a_2\wedge b_2,\ a_3\wedge b_3,\ a_4\wedge b_4,\ a_5\wedge b_5,\ a_6\wedge b_6,\ a_7\wedge b_7,\ a_8\wedge b_8)$$

$$= \left(min(a_1, b_1), min(a_2, b_2), min(a_3, b_3), min(a_4, b_4), min(a_5, b_5), min(a_6, b_6), min(a_7, b_7), min(a_8, b_8)\right)$$

2.6 Ranking Function

We define a ranking function $\tilde{R}: F(R) \to R$ which maps each fuzzy number to real line $F(R)$ represent the set of all hexagonal fuzzy numbers. If R be any linear ranking functions, then

$$\tilde{R}(\tilde{A}_o) = \left(\frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8}{8}\right)$$

Also we define orders on $F(R)$ by

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ISSN: 1309-3452

$\tilde{R}(\tilde{A}_o) \geq \tilde{R}(\tilde{B}_o)$ if and only if $\tilde{A}_o \geq \tilde{B}_o$,

$\tilde{R}(\tilde{A}_o) \leq \tilde{R}(\tilde{B}_o)$ if and only if $\tilde{A}_o \leq \tilde{B}_o$ and

$\tilde{R}(\tilde{A}_o) = \tilde{R}(\tilde{B}_o)$ if and only if $\tilde{A}_o = \tilde{B}_o$.

3. α −Cut of Octagonal Fuzzy Number

The crisp set A_o called alpha cut is defined as

$$A_\alpha = \{x \in X / \mu_{\tilde{A}_o}(x) \geq \alpha\}$$

$$A_\alpha = \begin{Bmatrix} [g_l(\alpha), g_u(\alpha)] & for \quad \alpha \in [0,0.33] \\ [h_l(\alpha), h_u(\alpha)] & for \quad \alpha \in [0.33,0.66] \\ [I_l(\alpha), I_u(\alpha)] & for \quad \alpha \in [0.66,1] \end{Bmatrix}$$

3.1 α −Cut Operations

The intervalA_α, for all $\alpha \in [0,1]$ is obtained as follows:

Consider,

$$[g_l(\alpha), g_u(\alpha)] = \left[\frac{1}{3}\frac{(x-a_1)}{(a_2-a_1)} \;\; and \;\; \frac{1}{3}\frac{(a_8-x)}{(a_8-a_7)}\right]$$

Simplifying, we get

$$[g_l(\alpha), g_u(\alpha)] = [a_1 + 3\alpha(a_2 - a_1), a_8 - 3\alpha(a_8 - a_7)]$$

Consider $[h_l(\alpha), h_u(\alpha)] = \left[\frac{1}{3} + \frac{1}{3}\frac{(x-a_2)}{(a_3-a_2)} \;\; and \;\; \frac{2}{3} - \frac{1}{3}\frac{(x-a_6)}{(a_7-a_6)}\right]$

Simplifying, we get

$$[h_l(\alpha), h_u(\alpha)] = [a_2 + (3\alpha - 1)(a_3 - a_2), a_7 - (2 - 3\alpha)(a_7 - a_6)]$$

Consider,

$$[I_l(\alpha), I_u(\alpha)] = \left[\frac{2}{3} + \frac{1}{3}\frac{(x - a_3)}{(a_4 - a_3)}, 1 - \frac{1}{3}\frac{(x - a_5)}{(a_6 - a_5)}\right]$$

Simplifying, we get

$$[I_l(\alpha), I_u(\alpha)] = [a_3 + (3\alpha - 2)(a_4 - a_3), a_6 - (3 - 3\alpha)(a_6 - a_5)]$$

Hence,

$$A_\alpha = \begin{Bmatrix} [a_1 + 3\alpha(a_2 - a_1), a_8 - 3\alpha(a_8 - a_7)] & for \quad \alpha \in [0, 0.33] \\ [a_2 + (3\alpha - 1)(a_3 - a_2), a_7 - (2 - 3\alpha)(a_7 - a_6)] & for \quad \alpha \in [0.33, 0.66] \\ [a_3 + (3\alpha - 2)(a_4 - a_3), a_6 - (3 - 3\alpha)(a_6 - a_5)] & for \quad \alpha \in [0.66, 1] \end{Bmatrix}$$

4 New Arithmetic Operations on Octagonal Fuzzy Numbers using α −Cut

The arithmetic operations among α −Cuts of Octagonal fuzzy numbers are given below:

Let $\tilde{A}_o = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and

$\tilde{B}_o = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ be two Octagonal fuzzy numbers for all $\alpha \in [0,1]$. Here, we use interval arithmetic.

$$A_\alpha = \begin{Bmatrix} [a_1 + 3\alpha(a_2 - a_1), a_8 - 3\alpha(a_8 - a_7)] & for \quad \alpha \in [0, 0.33] \\ [a_2 + (3\alpha - 1)(a_3 - a_2), a_7 - (2 - 3\alpha)(a_7 - a_6)] & for \quad \alpha \in [0.33, 0.66] \\ [a_3 + (3\alpha - 2)(a_4 - a_3), a_6 - (3 - 3\alpha)(a_6 - a_5)] & for \quad \alpha \in [0.66, 1] \end{Bmatrix}$$

$$B_\alpha = \begin{Bmatrix} [b_1 + 3\alpha(b_2 - b_1), b_8 - 3\alpha(b_8 - b_7)] & for \quad \alpha \in [0, 0.33) \\ [b_2 + (3\alpha - 1)(b_3 - b_2)\ , b_7 - (2 - 3\alpha)(b_7 - b_6)] & for \ \ \alpha \in [0.33, 0.66) \\ [b_3 + (3\alpha - 2)(b_4 - b_3), b_6 - (3 - 3\alpha)(b_6 - b_5)] & for \ \ \alpha \in [0.66, 1] \end{Bmatrix}$$

5. Numerical Example

Let $A_\alpha = (2,3,4,5,6,7,8,9)$ and $B_\alpha = (3,5,7,9,11,13,15,17)$ be two fuzzy numbers.

By the Arithmetic operations on OFNs we have,

$A_\alpha\ (+)\ B_\alpha = (2,3,4,5,6,7,8,9)\ (+)\ (3,5,7,9,11,13,15,17)$

$A_\alpha\ (+)\ B_\alpha = (5,8,11,14,17,20,23,26)$

By the new arithmetic operations on ACOFNs. We have the same illustration numbers as,

$$A_\alpha\ (+)\ B_\alpha = \begin{cases} [9\alpha + 5, 26 - 9\alpha] & for \ \ \alpha \in \ [0, 0.33) \\ [9\alpha + 5, 9\alpha + 15\ ] & for \ \ \alpha \in \ [0.33, 0.66) \\ [9\alpha + 5, 9\alpha + 11] & for \ \ \alpha \in \ [0.66,1] \end{cases}$$

When

$\alpha = 0, \quad A_0\ (+)\ B_0 = (5, 26)$

$\alpha = 0.33, \qquad A_{0.33}\ (+)\ B_{0.33} = (8, 17)$

$\alpha = 0.66, \qquad A_{0.66}\ (+)\ B_{0.66} = (11, 23)$

$\alpha = 1, \qquad A_1\ (+)\ B_1 = (14, 20)$

Hence

$A_\alpha\ (+)\ B_\alpha = (5,8,11,14,17,20,23,26)$

Hence all the points coincide with the sum of the two octagonal fuzzy numbers. A similar procedure can be attempted for difference of both OFNs and ACOFNs.

6. Conclusion

In this paper, a new octagonal fuzzy number is utilized to study the arithmetic operations on fuzzy numbers. Moreover, the $\alpha -$ cut of the octagonal fuzzy number is also studied and the relevant operations are presented. This work could be extended to the domain of fuzzy number matrices.

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