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NEUTROSOPHIC TRAVELLING SALESMAN PROBLEM IN HEXAGONAL FUZZY NUMBER USING NEAREST NEIGHBOR TECHNIQUE

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Abstract

A hexagonal fuzzy number and its membership function is introduced, which is used to representing the uncertainty with the six points. In this paper the Neutrosophic Hexagonal fuzzy number has been adopted and for finding an optimal solution for travelling salesman problem by using a nearest neighbor method. This method requires the least iterations to obtain the optimality. Results are given in the numerical examples.

Keywords: *Fuzzy number, Hexagonal fuzzy number, Fuzzy optimal solution.*

1. Introduction

Travelling salesman problem is a well-known combinatorial optimization in operations research and theoretical computer science. A map of cities is given, and a task is to find a shortest among all the possible tours visits each city by exactly once. Smarandache introduced the concept of Neutrosophic set. The main concept of Neutrosophic set is characterized by three independent membership degrees namely truth, indeterminacy and falsity membership which are lies between$]\mathbf{0}^{-}, \mathbf{1}^{+}[$. D. Dubais and H. Prade in 1978 have defined the fuzzy numbers as the fuzzy subsets of the real line. A fuzzy number is a quality whose values are imprecise, rather than exact as in the case with single-valued numbers. In this paper, Hexagonal fuzzy numbers have been introduced with its membership function. By using a Nearest Neighboring method, we find the solution of the fuzzy travelling salesman problem. Numerical example shows the easiest way of the proposed method.

2. Preliminaries

2.1 Fuzzy set

A **fuzzy set** is characterized by its membership function taking values from the domain, space or the universe of discourse mapped into the unit interval $[0,1]$.A fuzzy set A in the universal set X is defined as

$$A = \{(x, \mu_A(x))/x \epsilon X\}.$$

Here$\mu_A(x): A \to [0,1]$is the grade of the membership function and $\mu_A(x)$is the grade value of $x \in X$ in the fuzzy set A.

2.2 Normal

A fuzzy set A is called **Normal** if there exists an element $x \in X$ whose membership value is one, i.e., $\mu_A(x) = 1$.

2.3Fuzzy number

A fuzzy set A of real line R with membership function $\mu_A(x): R \to [0,1]$ is called **fuzzy number** if

(i) A is normal and convexity.
(ii) A must be bounded.
(iii) $\mu_A(x)$ is piecewise continuous.

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2.4Trapezoidal fuzzy number

A fuzzy number $A = (a, b, c, d)$ is a **Trapezoidal fuzzy number** where a_1, a_2, a_3, a_4 are real numbers and its membership function is given below,

$$\mu_A(x)=\begin{cases} \frac{x-a}{b-a} \ if \ a \le x \le b \\ 1 \quad if \ b \le x \le c \\ \frac{c-x}{d-c} \ ifc \le x \le d \end{cases}$$

2.5 Pentagonal fuzzy number

A **Pentagonal fuzzy number** of a fuzzy set $P'$is defined as $A_{P'} = (a, b, c, d, e)$and its membership function is given by,

$$\mu_{A_{P'}}(x) = \begin{cases} 0 & for \quad x < a \\ \dfrac{x-a}{b-a} & for \quad a \le x \le b \\ \dfrac{x-b}{c-a} & for \quad b \le x \le c \\ 1 & for \quad x = c \\ \dfrac{d-x}{d-c} & for \quad c \le x \le d \\ \dfrac{e-x}{e-d} & for \quad d \le x \le e \\ 0 & for \quad x > e \end{cases}$$

2.6 Neutrosophic set

Let X be a universe. A **Neutrosophic set** A over X is defined by

$$A^N = \{\langle x: T_{a^N}(x), I_{a^N}(x), F_{a^N}(x)\rangle : x\epsilon X\}$$

where $T_{a^N}, I_{a^N}, F_{a^N}: X \to]0^-, 3^+[$are called the truth, indeterminacy and falsity membership function of the element $x \in X$ to the set $A^N$with $0^- \le T_{a^N}(x) + I_{a^N}(x) + F_{a^N}(x) \le 3^+$.

3.Hexagonal Fuzzy Number

A fuzzy number$A_{H'}$is a **Hexagonal fuzzy number** denoted by $A_{H'} = (a, b, c, d, e, f)$ are real numbers and its membership functionis given by

$$\mu_{A_{H'}}(x) = \begin{cases} 0 & for \quad x < a \\ \frac{1}{2}\left(\frac{x-a}{b-a}\right) & for \quad a \le x \le b \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-b}{c-b}\right) & for \quad b \le x \le c \\ 1 & for \quad c \le x \le d \\ 1 - \frac{1}{2}\left(\frac{x-d}{e-d}\right) & for \quad d \le x \le e \\ \frac{1}{2}\left(\frac{f-x}{f-e}\right) & for \quad e \le x \le f \\ 0 & for \quad x > f \end{cases}$$

4.Fuzzy Travelling Salesman Problem

Suppose a salesman has to visits n cities. He visits one city and return to the hometown within a short period of time. The objective is to select the sequence in which the cities are visited in such a way that his total fuzzy travelling time will be minimized.

The mathematical formulation of the fuzzy travelling salesman problem is given by

$$x_{ij} = \begin{cases} 1, & from \ city \ i \ to \ city \ j \\ 0, & otherwise \end{cases}$$

Thus, the above can be expressed as,

Minimize $Z^N = \sum_i \sum_j C_{ij} x_{ij}$

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Subject to the constraints

$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1,2, \dots, n \sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1,2, \dots, m\, x_{ij} = 0 \text{ or } 1$

5. Algorithm

Step1: Find the membership function for the given hexagonal fuzzy numbers.
Step 2: Assigning the membership function for the given hexagonal fuzzy numbers.
Step 3: Finding all the possible tours for the salesman and find their fuzzy costs.
Step 4: Compute the minimum fuzzy costs for the salesman.

6. Numerical Example

Consider the following Neutrosophic Fuzzy travelling salesman problem with 4 cities namely A,B,C and D. The cost matrix $[C_{ij}]$ is given whose elements are hexagonal Neutrosophic fuzzy numbers. The problem is to find a route that starts from city A, passes through each city and return to the hometown exactly once at lowest cost by using membership function.

$$\begin{array}{c|l} & \quad \mathbf{A} \qquad\qquad \mathbf{B} \qquad\qquad \mathbf{C} \qquad\qquad \mathbf{D} \\ \boldsymbol{A} & \infty \quad [(2,4,6,8,10,12); (60,62,66,72,74,7 \\ \boldsymbol{B} & [(2,4,6,8,10,12); (60,62,66,72,74,76); (101,105,110,113,114,118)] \quad \infty \\ \boldsymbol{C} & [(23,26,29,35,37,39); (85,87,88,91,92,93); (122,124,126,133,137,140)] \quad [(65,66,67,69,73,75); (121,124,128,13 \\ \boldsymbol{D} & [(41,44,49,50,53,55); (82,85,89,90,96,99); (100,106,112,114,116,117)] \quad [(86,87,89,90,94,97); (100,101,104,11 \end{array}$$

Solution:
Let us take the Membership function for the given HFN

Fuzzy number	Membership function
0-20	0.6
20-40	0.10
40-60	0.14
60-80	0.18
80-100	0.20
100-120	0.22
120-140	0.24
140-160	0.28
160-180	0.30
180-200	0.36

Assigning the membership function for the given hexagonal Neutrosophic travelling salesman problem.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{bmatrix} \infty & (\mathbf{0.6}, \mathbf{0.18}, \mathbf{0.22}) & (\mathbf{0.10}, \mathbf{0.20}, \mathbf{0.24}) & (\mathbf{0.14}, \mathbf{0.20}, \mathbf{0.22}) \\ (\mathbf{0.6}, \mathbf{0.18}, \mathbf{0.22}) & \infty & (\mathbf{0.18}, \mathbf{0.24}, \mathbf{0.28}) & (\mathbf{0.20}, \mathbf{0.22}, \mathbf{0.28}) \\ (\mathbf{0.10}, \mathbf{0.20}, \mathbf{0.24}) & (\mathbf{0.18}, \mathbf{0.24}, \mathbf{0.28}) & \infty & (\mathbf{0.24}, \mathbf{0.28}, \mathbf{0.36}) \\ (\mathbf{0.14}, \mathbf{0.20}, \mathbf{0.22}) & (\mathbf{0.20}, \mathbf{0.22}, \mathbf{0.28}) & (\mathbf{0.24}, \mathbf{0.28}, \mathbf{0.36}) & \infty \end{bmatrix} \end{array}$$

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	Salesman Routes		**Optimal solution**
1	$A \to B \to C \to D \to A$	(0.6,0.18,0.22)+(0.18,0.24,0.28)+(0.24,0.28,0.36)+(0.14,0.20,0.22)	(1.16,0.9,1.08)
2	$A \to B \to D \to C \to A$	(0.6,0.18,0.22)+(0.20,0.22,0.28)+(0.24,0.28,0.36)+(0.10,0.20,0.24)	(1.14,0.88,1.1)
3	$A \to C \to B \to D \to A$	(0.10,0.20,0.24)+(0.18,0.24,0.28)+(0.20,0.22,0.28)+(0.14,0.20,0.22)	(0.62,0.86,1.02)
4	$A \to C \to D \to B \to A$	(0.10,0.20,0.24)+(0.24,0.28,0.36)+(0.20,0.22,0.28)+(0.6,0.18,0.22)	(1.14,0.88,1.1)
5	$A \to D \to C \to B \to A$	(0.14,0.20,0.22)+(0.24,0.28,0.36)+(0.18,0.24,0.28)+(0.6,0.18,0.22)	(1.16,0.9,1.08)
6	$A \to D \to B \to C \to A$	(0.14,0.20,0.22)+(0.20,0.22,0.28)+(0.18,0.24,0.28)+(0.10,0.20,0.24)	(0.62,0.86,1.02)

All the possible tours for the fuzzy travelling salesman problem and its optimal solution are below

Cheapest tour:

$A \to C \to B \to D \to A$ and $A \to D \to B \to C \to A$

The total distance travelled is (0.62,0.86,1.02)

Better tour:

$A \to B \to D \to C \to A$ and $A \to C \to D \to B \to A$

The total distance travelled is (1.14,0.88,1.1)

Costliest tour:

$A \to B \to C \to D \to A$ and $A \to D \to C \to B \to A$

The total distance travelled is (1.16,0.9,1.08)

5. Conclusion

Using our proposed method, we can find the optimal solution for the fuzzy travelling salesman problem. This method is new and easy to understand the problem. It is very effective to find the minimum cost. And all fuzzy numbers are obtained by using this method.

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