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Pythagorean Fuzzy Set and Its Application in Career Placements Using Min-Max-Min Composition

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Abstract

Any decision-making method involves some degree of uncertainity. To deal with the uncertain environment of collective decision making, various tools and modern techniques have been proposed. Among those techniques, Pythagorean Fuzzy sets are one of the most recently using approach for dealing with imprecision. These sets enhance intuitionistic fuzzy sets with a wider range of applications, which is the reason for looking into their applicability in dealing with the problem of career placements. In this paper, we examine at the concept of Pythagorean Fuzzy sets and some definitions about score and accuracy functions of fuzzy sets. Some Pythagorean Fuzzy set properties are discussed. The concept of a relation is established in a Pythagorean Fuzzy set termed as Pythagorean Fuzzy Relation, which is accompanied by numerical examples. Finally, a career placement decision-making process on the proposed Pythagorean fuzzy relation called min-max-min composition can be used to determine the appropriate suitability of careers to the applicants on the basis of skills of academic performance.

Keywords: Fuzzy set, construction of Intuitionistic Fuzzy Set, Pythagorean fuzzy structure. Pythagorean relation on fuzzy, min-max-min composition.

1.Introduction

The fuzzy set was developed due to the uncertainty condition in decision making process. Those fuzzy set contains a membership function that assigns to each element in the unit interval [0,1]. A membership function range is 0 to 1, with 0 indicating that the element does not belong to a class, and 1 indicating that it belong, such that the other values indicating the degree of membership to the class. A membership function (α) and a non membership function (β) and a marginal value (δ) are used to create the Intuitionistic fuzzy set (IFS) where δ is neither a membership nor a non membership function, $\alpha + \beta \leq 1$ and $\alpha + \beta + \delta = 1$. In such odd circumstances like $\alpha + \beta \geq 1$ the constraint in IFS naturally led to the development of Pythagorean Fuzzy sets (PFS).In such a way like, PFS incorporates with the membership and non membership function (α , β) satisfying the constraint conditions , $\alpha + \beta \leq 1$ or $\alpha + \beta \geq 1$ and implies that $\alpha^2 + \beta^2 + \delta^2 = 1$. PFS and IFS have a close link as a generalized set. PFS can be used to characterize uncertain information more thoroughly and precisely than IFS.

Garg proposed an improved scoring function for ranking order of PFS with interval values called Interval-valued Pythagorean fuzzy sets. It is used to describe a PFS strategy by similar to that of TOPSIS. Many research scholars have been drawn to the PFS and the paradigm has now been applied in different disciplines including measure of information in decision making and aggregation operators. Perez-Dominguez talked about MOORA with PFS setting and used it to solve MCDM problems. PFS were proposed by Liang and XU in a tentative manner.

Because of its broader spectrum of applicability in real-life problems involving imprecision, we are encouraged to examine the resourcefulness of PFS in solving career placement challenges using the min-max-min rule in this paper. The purpose of this research is to examine the concept of PFS and its application in job placement based on the academic level skills and performance.

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2. Preliminaries

Fuzzy Set

A fuzzy set F in Z where Z is a non empty set, is defined by the membership function$\left(\alpha_F\,(z)\right)$

$F = \{\left(z, \alpha_F\,(z)\right) | \, z \in Z\}$ or $F = \left\{(\frac{\alpha_F\,(z)}{z}) \mid z \in Z\right\}$

Construction of Intuitionistic Fuzzy sets

Let a non empty set Z be fixed, then the IFS F_I in Z has the form

$F_I = \left\{\left(z, \alpha_{F_I}(z), \beta_{F_I}(z)\right) \middle| \, z \in Z\right\}$or $A = \left\{\frac{\alpha_{F_I}(z),\beta_{F_I}(z)}{z} \middle| \, z \in Z\right\}$where the functions lies in the interval [0,1] ie., $\mathbf{0} \le \boldsymbol{\alpha_{F_I}} + \boldsymbol{\beta_{F_I}} \le \mathbf{1}$

The hesitation margin $\delta_{F_I}(z)$ *is* the degree of non-determinacy of z ϵ Z , where it also lies in the interval [0,1] thus $\delta_{F_I}(z)$can be calculated by $\boldsymbol{\alpha_{F_I}(z)} + \boldsymbol{\beta_{F_I}(z)} + \boldsymbol{\delta_{F_I}(z)}$ **= 1**and hence $\boldsymbol{\delta_{F_I}(z)}$**= 1 -** $\boldsymbol{\alpha_{F_I}} - \boldsymbol{\beta_{F_I}}$

Example 1: Let Z = {z_1, z_2, z_3} , F_I ={ $\frac{\{0.2,0.7\}}{z_1}, \frac{\{0.6,0.2\}}{z_2}, \frac{\{0.5,0.2\}}{z_3}$ } then the hesitation margin of the intuitionistic fuzzy set is given by $\delta_{F_I}(z_1) = 0.1 \delta_{F_I}(z_2) = 0.2 \delta_{F_I}(z_3) = 0.3$

Pythagorean fuzzy sets Structure

Let Z be a universal set, then the PFS $F_p$is defined as

$F_p = \left\{\left(z, \alpha_{F_p}(z), \beta_{F_p}(z)\right) \middle| \, z \in Z\right\}$or $F_p = \left\{\frac{\alpha_{F_p}(z),\beta_{F_p}(z)}{z} \middle| \, z \in Z\right\}$ where $\alpha_{F_p}(z)$ and $\beta_{F_p}(z)$ defines the membership and non-membership functions of the element of $z \in Z$, then for every z $\in Z$,

$$\mathbf{0} \le (\boldsymbol{\alpha_{F_p}(z)})^2 + (\boldsymbol{\beta_{F_p}(z)})^2 \le \mathbf{1}$$

then as discussed above there is a degree of indeterminacy in PFs for $z \in Z$and hence it can be defined by $\boldsymbol{\delta_{F_p}(z)}$ $= \sqrt{1 - [(\boldsymbol{\alpha_{F_p}}\,(\boldsymbol{z}))^2 + (\boldsymbol{\beta_{F_p}(z)})^2]}$which follows that $(\boldsymbol{\alpha_{F_p}(z)})^2 + (\boldsymbol{\beta_{F_p}(z)})^2 + (\boldsymbol{\delta_{F_p}(z)})^2 = \mathbf{1}$

Example 3: Suppose $\alpha_{F_p}(z)$ = 0.6 and $\beta_{F_p}(z)$ = 0.5 where 0.6 + 0.5 $\nleq$ 1 thus $(0.6)^2 + (0.5)^2 < 1$ thus $\delta_{F_p}(z)$is calculated by$\delta_{F_p}(z) = \sqrt{1 - [(\alpha_{F_p}\,(z))^2 + (\beta_{F_p}(z))^2]}$, then $\delta_{F_p}(z) = \sqrt{1 - ((0.6)^2 + (0.5)^2)} = \sqrt{1 - 0.61} = \sqrt{0.39}$ = 0.62449.

Difference between Pythagorean fuzzy sets and intuitionistic fuzzy sets

Intuitionistic fuzzy sets	**Pythagorean fuzzy sets**
$\alpha + \beta \le 1$	$\alpha + \beta \le 1$ or $\alpha + \beta \ge 1$
$0 \le \alpha + \beta \le 1$	$0 \le \alpha^2 + \beta^2 \le 1$
$\delta_{F_I}(z) = 1 - \alpha_{F_I} - \beta_{F_I}$	$\delta_{F_I}(z) = \sqrt{1 - [(\alpha_{F_p}\,(z))^2 + (\beta_{F_p}(z))^2]}$
$\alpha_{F_I}(z) + \beta_{F_I}(z) + \delta_{F_I}(z)$	$(\alpha_{F_p}(z))^2 + (\beta_{F_p}(z))^2 + (\delta_{F_p}(z))^2 = 1$

Complementary function:

Let $F \in PFS(Z)$ then the complementary is denoted by F^c and it may be defined as follows

$$F^c = \{z, \beta_F(z), \alpha_F(z) | \, z \in Z\}$$

Note : $(F^c)^c$ = F

Union and Intersection:

Let $E, F \in PFS(Z)$ then the union and intersection of the sets E, F is:

$$E \cup F = \{[z, \max(\alpha_E(z), \alpha_F(z)), \min(\beta_E(z), \beta_F(z))]\} \mid z \in Z$$
$$E \cap F = \{[z, \min(\alpha_E(z), \alpha_F(z)), \max(\beta_E(z), \beta_F(z))]\} \mid z \in X$$

Score Function:

Let $B \in PFS(Z)$ then the score function s of B is defined as $s\,(B) = (\alpha_B(z))^2 - (\beta_B(z)^2)$

Accuracy Function:

Let $B \in PFS(Z)$ then the accuracy function a of B is defined as $a\,(B) = (\alpha_B(x))^2 + (\beta_B(x)^2)$

Example 3 : If $\alpha_B(x) = 0.3775, \beta_B(x) = 0.7062$ then the score and the accuracy function of the Set B is given and concluded as s(B) = - 0.35 where as a(B) = 0.64

3. Pythagorean Fuzzy Relation

The Pythagorean fuzzy relation is a combination of fuzzy and intuitionistic fuzzy sets and relations.

Extension Principle:

Let ρ be a function from the two sets T and U, C and Dare Pythagorean fuzzy sets of T and U then:

i. The image of C under the function ρ is denoted by ρ (C), the Pythagorean fuzzy set of U could be defined as

$$\alpha_{\rho(C)}(u) = \begin{cases} \bigvee_{x \in \rho^{-1}(u)} \alpha_C(t) & ; \quad \rho^{-1}(u) \neq 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

And

$$\beta_{\rho(C)}(u) = \begin{cases} \bigwedge_{x \in \rho^{-1}(u)} \beta_C(t) & ; \quad \rho^{-1}(u) \neq 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

ii. The inverse image of D under ρ and it is denoted by ρ^{-1} (D) , the Pythagorean fuzzy set of T could be defined as

$$\alpha_{\rho^{-1}(D)}(t) = \alpha_D(\rho(t)) \quad \text{and} \quad \beta_{\rho^{-1}(D)}(t) = \beta_D(\rho(t)) \quad \text{where } t \in T$$

Pythagorean Fuzzy Relation:

Let the two non empty sets be T and U , the relation of pythogorean fuzzy P from T to U is a PFS of $T \times U$ is defined by the membership function α_P, and the non membership function β_P. A Pythagorean fuzzy relation from the sets T to U is denoted by P (T→ U)

4. Min-Max-Min Composition defined for the Member and the non membership function

Min-Max-Min composition of a relation to the element in a PFS:

Let $F \in PFS\,(Z)$ then the min-max-min composition of R (S→ A) with F is a PFS B of A denoted by B = $R \circ F$ and its member and non-membership functions are given by

$$\alpha_B(a) = \bigwedge(\max[\,\alpha_F(z), \alpha_R(z,a)])$$

and

$$\beta_B(a) = \bigvee(\min[\,\beta_F(z), \beta_R(z,a)])$$

For every $z \in Z$ *and* $a \in A$, where $\wedge$ = minimum ; $\vee$ = maximum

Min-Max-Min composition of two Pythagorean fuzzy relation:
Let R (S → A) and P (A → G) be two PFRs then the min-max-min composition $P \circ R$ is a relation S to G then its membership and non membership functions are defined by:

$$\alpha_{P\circ R}(s,g) = \Lambda(\max[\,\alpha_R(s,a)\,,\alpha_P(a,g)])$$

And

$$\beta_{P\circ R}(s,g) = \Lambda(\min[\,\beta_R(s,a)\,,\beta_P(a,g)])$$

For all $(s,g) \in S \times G \ and \ a \in A$

From the definitions above defined the min-max-min composition B or $P \circ R$can be calculated by:

$$B = \alpha_B\,(a) - \beta_B\,(a)\delta_B\,(a) \quad for\ all\ a \in A$$

Or

$$P \circ R = \alpha_{P\circ R}(s,g) - \beta_{P\circ R}(s,g)\delta_{P\circ R}(s,g) \qquad for\ all\ (s,g) \in S \times G$$

Example 4:
Let A, S be the two non empty sets where $A, S \in PFS\,(Z)$ for Z = { z_1, z_2, z_3} such that

$$A = \left\{\frac{(0.8,0.2)}{z_1}, \frac{(0.6,0.3)}{z_2}, \frac{(0.4,0.5)}{z_3}\right\}$$

$$S = \left\{\frac{(0.6,0.2)}{z_1}, \frac{(0.4,0.6)}{z_2}, \frac{(0.7,0.3)}{z_3}\right\}$$

Have to find the composition B using min-max-min principle

Solution:
$\max[\mu_R(a_i,x_j), \mu_S(x_j,s_k)]$ = 0.8, 0.6, 0.7
$\boldsymbol{\alpha_B(a_i,x_j)}$ = ∧ ($\mathbf{0.8, 0.6, 0.7}$) = **0.6**
Then,
$\min[\upsilon_R(a_i,x_j), \upsilon_S(x_j,s_k)] =$ 0.2, 0.3, 0.3
$\boldsymbol{\beta_B(a_i,x_j)}$ = ∨ **(0.2, 0.3, 0.3) = 0.3**
We have the formula to find the composition of the relation B where $\boldsymbol{B = \alpha_B - \beta_B\delta_B}$ suchthat the indeterminancyδ can be calculated by $\boldsymbol{\delta} = \sqrt{\mathbf{1} - [\,\boldsymbol{\alpha}^2 + \boldsymbol{\beta}^2]}$
Here the value of δ is **0.7416**
Then
B = 0.6 – (0.3 × 0.7416) = 0.3775

5. Algorithm

Step:1
Determine the membership and non membership function with respect to the relation **R(S→A)**
Step :2
Determine the another member and non membership function with respect to **P (A → G)**
Step:3
Calculate the composition function $\boldsymbol{\alpha}_{P\circ R}\,(\boldsymbol{s_i}\,,\boldsymbol{g_k})$ **and**$\boldsymbol{\beta}_{P\circ R}\,(\boldsymbol{s_i},\ \boldsymbol{g_k})ie\boldsymbol{Q}\,(\boldsymbol{S} \rightarrow \boldsymbol{G})$ for the two given PFR using min-max-min principle.
Step:4
The career placement is given by ξ, where $\boldsymbol{\xi} = \boldsymbol{\alpha}_\xi - \boldsymbol{\beta}_\xi\boldsymbol{\delta}_\xi$

6. Application of Min-Max-Min Composition for Pythagorean Fuzzy Sets to Career Placements

General Illustration:
Let A = { a_1, a_2, ….. a_j} , G = { g_1, g_2,….g_k} , S = { s_1, s_2,….s_i} be a finite set of subject related to the course, and the finite set of courses, and finite set of applicants respectively.Suppose we are having two PFRs, R(S → A) and P (A→ G), where
R = { $[(\boldsymbol{s},\boldsymbol{a}), \boldsymbol{\alpha_R}(\mathbf{s},\mathbf{a}),\ \boldsymbol{\beta}_{\mathbf{R}}(\mathbf{s},\mathbf{a})] \mid (\mathbf{s},\mathbf{a}) \in \mathbf{S} \times \mathbf{A}$ **}**
P = $\{\,[(\boldsymbol{a},\boldsymbol{g}), \boldsymbol{\alpha_P}(\mathbf{a},\mathbf{g}),\ \boldsymbol{\beta}_{\mathbf{P}}(\mathbf{a},\mathbf{g})] \mid (\mathbf{a},\mathbf{g}) \in \mathbf{A} \times \mathbf{G}\}$
Where

- $\alpha_R\,(s,a)$denotes the degree to which the applicant - s, passes the related subject requirement- a
- $\beta_R\,(s,a)$represents the degree to which the applicant - s, does not passes the related subject requirement- a

Similarly,

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- $\alpha_P\ (a, g)$represents the degree to which the related subject requirement – a, determines the course –g
- $\beta_P\ (a, g)$denotes the degree to which the related subject reuirement – a, does not determine the course – g

Therefore there exists a composition of ξ, of R and P , it can be given as $\xi = R \circ P$. This describes the state in which the applicants - s_i with respect to the related subject requirement – a_j, fit the courses - g_k. Thus,

$$\boldsymbol{\alpha_\xi(s_i\,, g_k) = \wedge\ \{max\,[\alpha_R\,(s_i, a_j),\ \alpha_P\,(a_j\,, c_k)]\,\}}$$

$$\boldsymbol{\beta_\xi\,(s_i\,, g_k) = \vee\ \{\min\ [\ \beta_R\,(s_i, a_j)\,,\ \beta_P\,(a_j\,, c_k)]\,\}}$$

For every $a_j \in A$, $g_k \in G$, where i , j takes the values from 1,2….n
The values$\alpha_{R\circ P}\ (s_i\,, g_k)$ and$\beta_{R\circ P}\ (s_i,\ g_k)$ of the composition function $\xi = R \circ P$ are as follows:
The career placement could be achieved if the value of ξ is given by the following

$$\xi = \boldsymbol{\alpha_\xi(s_i, g_k) - \beta_\xi(s_i, g_k)\delta_\xi\,(s_i, g_k}$$

Numerical Example

Let

- the set S be the set of all applicants for the course of placements
  S = { Aparna, Anju, Abi, Shree, Sathya }
- the set G be the set of courses the applicants are applying for
  G = { Medicine, Pharmacy, Nursing , Optometry, Cardiology}
- the set A be the set of related subjects requirement to the set of courses
  A = { English, Maths, Biology, Physics, Chemistry, Health science}

R	**English**	**Maths**	**Biology**	**Physics**	**Chemistry**	**Health**
Aparna	(0.8,0.1)	(0.7,0.2)	(0.8,0.2)	(0.7,0.1)	(0.6,0.1)	(0.8,0.1)
Anju	(0.7,0.3)	(0.7,0.2)	(0.7,0.3)	(0.5,0.4)	(0.4,0.5)	(0.6,0.3)
Abi	(0.6,0.3)	(0.5,0.4)	(0.6,0.3)	(0.5,0.3)	(0.5.0.5)	(0.6,0.2)
Shree	(0.5,0.3)	(0.6,0.3)	(0.5,0.3)	(0.4,0.5)	(0.7,0.2)	(0.7,0.1)
Sathya	(0.6,0.4)	(0.8,0.2)	(0.6,0.3)	(0.6,0.3)	(0.5,0.3)	(0.7,0.2)

P	**Medicine**	**Pharmacy**	**Nursing**	**Optometry**	**Cardiology**
English	(0.8,0.2)	(0.5,0.4)	(0.8,0.1)	(0.9,0.1)	(0.7,0.2)
Maths	(0.5,0.3)	(0.5,0.3)	(0.7,0.2)	(0.8,0.1)	(0.5,0.4)
Biology	(0.9,0.1)	(0.9,0.1)	(0.9,0.1)	(0.8,0.2)	(0.8.0.2)
Physics	(0.6,0.2)	(0.5,0.4)	(0.6,0.3)	(0.5,0.2)	(0.6,0.3)
Chemistry	(0.7,0.2)	(0.7,0.3)	(0.8,0.2)	(0.7,0.2)	(0.8,0.2)
Health	(0.8,0.2)	(0.7,0.3)	(0.8,0.1)	(0.8,0.2)	(0.9,0.1)

P	**Medicine**	**Pharmacy**	**Nursing**	**Optometry**	**Cardiology**
Aparna	(0.7,0.2)	(0.7,0.2)	(0.7,0.2)	(0.7,0.2)	(0.7,0.2)
Anju	(0.5,0.2)	(0.5,0.3)	(0.6,0.3)	(0.5,0.2)	(0.6,0.3)

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Abi	(0.5,0.3)	(0.5,0.3)	(0.6,0.3)	(0.5,0.2)	(0.5.0.4)
Shree	(0.6,0.3)	(0.5,0.4)	(0.6,0.3)	(0.5,0.2)	(0.6,0.3)
Sathya	(0.6,0.2)	(0.6,0.4)	(0.6,0.3)	(0.6,0.2)	(0.6,0.3)

To find the indeterminate degree δ, for the applicants against each courses ; it can be given by the formula

$$\boldsymbol{\delta} = \sqrt{\mathbf{1} - [\,\boldsymbol{\alpha}^2 + \boldsymbol{\beta}^2]}$$

T	Medicine	Pharmacy	Nursing	Optometry	Cardiology
Aparna	0.5629	0.5629	0.5629	0.5629	0.5629
Anju	0.3315	0.2563	0.3775	0.3315	0.3775
Abi	0.2563	0.2563	0.3775	0.3315	0.1928
Shree	0.3775	0.1928	0.3775	0.3315	0.3775
Sathya	0.4451	0.3228	0.3775	0.4451	0.3775

5. Conclusion

Pythagorean fuzzy sets are comparatively new mathematical technique in the fuzzy group of sets that has a higher ability to deal with imprecision in decision making. In this study, we delved deeper into the concept of IFS and PFS, were distinguished from IFS and also applied those concept of relationship of PFS using min-max-min principle for the career based placement problem. The decision could be either made by vertical or horizontal. Based on the aforementioned it is reasonable to infer that the min-max-min composition technique is appropriate and effective.

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