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Wiener Index of Some Special Graphs Using Domination

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Abstract

Graph theory is more widely used in various of domains. Network Theory is the applications of graph-theoretic principles to the study of complex, dynamic interacting systems. Connectivity measures play an important part in the evolution of networks. The wiener index is one useful metric that a various application in chemistry and current strategies. This paper determines the wiener index of some special graphs by using Minimum Dominating Distance Matrix.

Keywords Graph Theory, Domination, Wiener Index.

1. Introduction

Topological indices are graph invariant numerical values that describe the geometry of a graph. The wiener index is connected to a vertex's intimate prominence in a graph, a measure inversely proportional to the sum of all distances between a given vertex and all other vertices that has been broadly utilized in many concepts. Best, Van Emde Boas, and Lenstra explored a class of graphs in conjunction with the Aanderaa-Karp-Rosenberg conjecture. A scorpion graph is a unique path of three vertices exists with one terminus connected to all remaining vertices and the other two path vertices having no other adjacencies [8]. The Moser spindle also known as Moser graph is an undirected graph with seven vertices and eleven edges in graph theory named the following mathematicians Leo Moser and his brother William [9]. The Krachhardt Kite graph is a simple graph with ten vertices and eighteen edges. The graph is named after David Krackhardt, a social network theory researcher. In 1990, Krackhardt developed the graph to identify between several conceptions of centrality [10].

2.Preliminaries

Definition 2.1[1]

A graph is represented by vertices as points and edges as line segments. A graph G = (V, E) consists of a nonempty set V of vertices and a set E of edges.

Definition 2.2[2]

A set D in a graph 'G' is a dominating set if each vertex is either in D or adjacent to a vertex in D. Any dominating set with minimum cardinality is called a minimum dominating set.

Definition 2.3[4,6,7]

Let D be a minimum dominating set of a graph G. The minimum dominating distance matrix of G is N x N matrix defined by $A_{Dd}(G) = (d_{ij})$ where $d_{ij} = \begin{cases} 1 & if \quad i = j \text{ and } v_i \in D. \\ d(v_i, v_j) & \text{otherwise} \end{cases}$

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Definition 2.4[3,5]

The Wiener number W and Wiener index W.I are computed by the definitions $W= 1/2\sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij}$ and Wiener Index = 2W/N(N-1) Where N is the number of vertices and d_{ij} is the distance between the vertices in terms of number of edges.

3.Main Results

In this section, it describes the wiener index of Scorpion graph, Moser graph and Krachhardt Kite graph using the minimum dominating distance matrix.

Scorpion graph consists of 12 vertices and 21 edges. Figure 3.1 represents the scorpion graph.

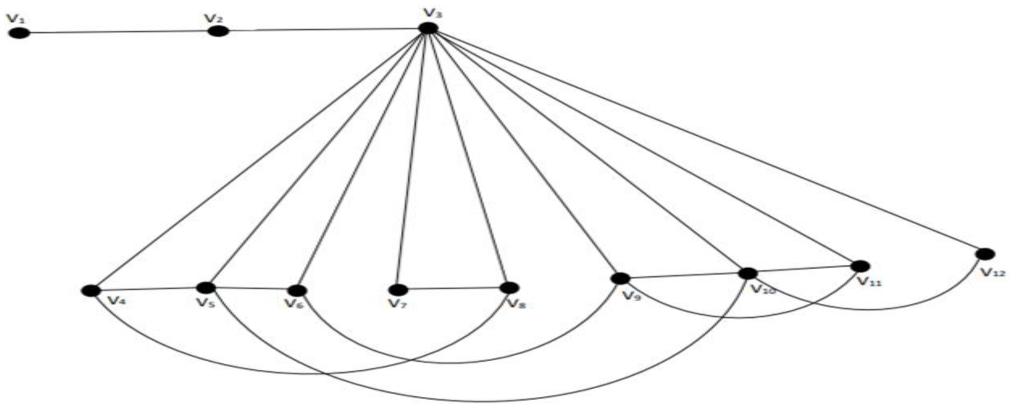

Figure 3.1 Scorpion graph

The following matrix is the minimum dominating distance matrix for Scorpion graph:

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}
V_1	1	1	2	3	3	3	3	3	3	3	3	3
V_2	1	0	1	2	2	2	2	2	2	2	2	2
V_3	2	1	1	1	1	1	1	1	1	1	1	1
V_4	3	2	1	0	1	2	2	1	2	2	2	2
V_5	3	2	1	1	0	1	2	2	2	1	2	2
V_6	3	2	1	2	1	0	2	2	1	2	2	2
V_7	3	2	1	2	2	2	0	1	2	2	2	2
V_8	3	2	1	1	2	2	1	0	2	2	2	2
V_9	3	2	1	2	2	1	2	2	0	1	1	2
V_{10}	3	2	1	2	1	2	2	2	1	0	1	1
V_{11}	3	2	1	2	2	2	2	2	1	1	0	2

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V_{12}	3	2	1	2	2	2	2	2	2	1	2	0

The Wiener Number of Scorpion graph by using minimum dominating distance matrix = $\frac{246}{2}$ =**123.**

The Wiener Index of Scorpion graph by using minimum dominating distance matrix $=\frac{2(123)}{12(12-1)}$

$= \frac{246}{12(11)}$ = **1.86.**

Moser graph consists of 7 vertices and 11 edges. Figure 3.2 illustrates the Moser graph.

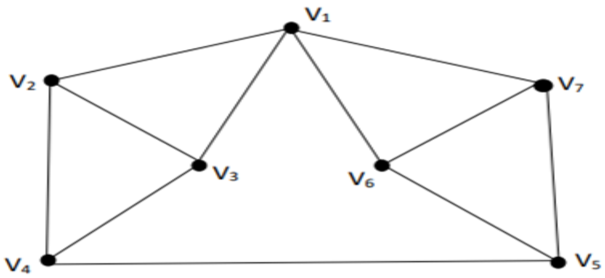

Figure 3.2 Moser graph

The following matrix is the minimum dominating distance matrix for Moser graph:

The Wiener Number of Moser graph by using minimum dominating distance matrix = $\frac{181}{2}$ =**90.5.**

The Wiener Index of Moser graph by using minimum dominating distance matrix $=\frac{2(90.5)}{10(10-1)}$

$= \frac{181}{10(9))}$ = **2.01.**

Krachhardt Kite graph consists of 10 vertices and 18 edges. Figure 3.3 shows the Krachhardt Kite graph.

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
V_1	0	1	2	3	3	4	4	4	4	4
V_2	1	1	1	2	2	3	3	3	3	3
V_3	2	1	0	1	1	2	2	2	2	2
V_4	3	2	1	0	1	1	1	2	1	2
V_5	3	2	1	1	0	2	1	1	2	1
V_6	4	3	2	1	2	0	1	2	1	2
V_7	4	3	2	1	1	1	1	1	1	1
V_8	4	3	2	2	1	2	1	0	2	1
V_9	4	3	2	1	2	1	1	2	1	1
V_{10}	4	3	2	2	1	2	1	1	1	0

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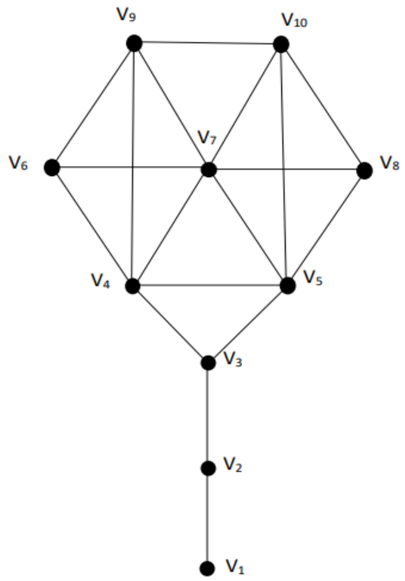

Figure 3.3 Krachhardt Kite graph

The following matrix is the minimum dominating distance matrix for Krachhardt Kite graph:

	V_1	V_2	V_3	V_4	V_5	V_6	V_7
V_1	1	1	1	2	2	1	1
V_2	1	0	1	1	2	2	2
V_3	1	1	0	1	2	2	2
V_4	2	1	1	0	1	2	2
V_5	2	2	2	1	1	1	1
V_6	1	2	3	2	1	0	1
V_7	1	2	3	2	1	1	0

The Wiener Number of Krachhardt Kite graph by using minimum dominating distance matrix = $\frac{66}{2}$ = **33.**

The Wiener Index of Krachhardt Kite graph by using minimum dominating distance matrix $= \frac{2(33)}{7(7-1)}$

$= \frac{66}{7(6)}$ = **1.57.**

5. Conclusion

This paper obtains the wiener index of some special graphs like Scorpion graph, Moser graph and Krachhardt Kite graph using Minimum Dominating Distance Matrix. In future, similar ideas are likely to applicable for complicated social network graphs.

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