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A Comparison of Domian Decomposition Method and Homotopy Perturb ation Method for The Solution of The Second Order Hyperbolic Telegraph Equation

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ABSTRACT

For the second-order linear and nonlinear hyperbolic Telegraph Equation, we develop a comparison study of Adomian decomposition method and Homotopy perturbation approach in this research. The HPM is a powerful mathematical tool despite its simplicity and conciseness. Its comparison with the ADM demonstrates this. We've included some real-world examples to show how capable and dependable the methods are. The sample computations demonstrate how HPM is simpler and requires less manipulation than ADM. In order to verify the mathematical model, 3-dimensional graphical representations of the second-order nonlinear hyperbolic Telegraph Equation are also provided The telegraph equation's 3- and 2-dimensional graphical solutions are represented and analyzed using MATLAB software.

Keywords: ADM, HPM, Telegraph equations, MATLAB software

1. Introduction:

More of late, obviously the message condition is a superior fit for depicting response dissemination than the overall dispersion condition. There are two associated fractional differential conditions that determine the flow and voltage on an electric transmission line at a given time and distance, known as the telegrapher's condition or just the message condition. Broadcast communications, computerized picture handling, signs and frameworks use broadcast conditions to send electrical signs by means of a message line, (Alonso, et. al; (1999), Kumar and Singh (2009).

The following is the usual form of the second-order hyperbolic telegraph equation:

$$\frac{\partial^2 u}{\partial t^2}+\alpha\frac{\partial u}{\partial t}+\beta u=\gamma\frac{\partial^2 u}{\partial x^2}+h(x,t)\,,\quad 0<x<L, 0<t\leq T \tag{1}$$

$$u(x,0)=g_1(x),\ \ 0<x<L \tag{2}$$

$$\frac{\partial u}{\partial t}(x,0)=g_2(x),\ 0<x<L \tag{3}$$

There are three constant coefficients in this equation: α, β, γ. The known functions g_1, g_2 and h are also constant coefficients, whereas the unknown function $u(x,t)$ can be either current or voltage flowing through the wire at time t and location x. L is the coil's inductance.

$$u_{tt}+a_1u_t=a_2u_{xx}+f(u)+h(x,t) \tag{4}$$

$$f(u)=\alpha u^3+\beta u^2+\gamma u \tag{5}$$

Where a_1, a_2, α, β, γ, are known constants coefficient.

Utilizing introducing scaling capacities, Lakestani and Saray (2010) formulated a mathematical technique for tackling a second-request transmit condition. while Kansa's methodology utilizing outspread premise capacities was proposed by Su et al. (2013)

to tackle the nonlinear message issue, It was proposed by Lin et al. (2018) to join the two level time-venturing Crank-Nicolson conspire (CNS) with a regressive replacement technique (BSM) to settle the message condition meshlessly. The BSM, a meshless semi-scientific collocation strategy, is utilized to settle these conditions. For the arrangement of the one-dimensional exaggerated message issue, Hashemi et al. (2019) joined semi-discretization with a mathematical integrator, bunch safeguarding plan (GPS).

It was proposed by Chinese mathematician He's in 1999 and really applied to find in wave condition in (He's 2005) that the HPM be used to address the subsequent solicitation direct and non-straight overstated message condition. Yet again in 2006, he used HPM to settle a wide extent of cutoff regard issues. To find a sensible solution for the reaction dispersal (RD) condition, Kumar and Singh (2010) presented HPM. For the game plan of a mostly differential condition, George Adomian (1986) figured out the Adomian crumbling method (ADM), which he later improved for use in settling a hotness condition (1987).

When it comes to resolving the RD equation, Kumar and Singh (2011) introduced a mathematical model comparing ADM and HPM, while Singh and Kumar (2017) discussed the relative study of HPM and DTM of that equation, as well as comparing them with VIM in a number of examples and deliberating on their respective abilities. According to Singh et al. (2019), the HPM method was used to solve a non-linear Fisher equation, and the same author did the same thing in (2020). A NHPM was created by Maurya et al. (2019) to get analytical solutions to two types of equations: the RD Equation and the Burgers-Huxley Equation in the first study. Several examples were provided to demonstrate the author's competence and the method's reliability. Using the Adomian Decomposition Method, Sayed et al. (2021) investigated two types of Adomian polynomials: novel accelerated Adomian polynomials and Adomian polynomials for solving the telegraph equation. HPM can drastically reduce the amount of work required if properly implemented. Examples illustrate the comparison, and the HPM's relevant aspects are shown in the analysis. We have utilized MATLAB software to perform all of the calculations.

2.Fundamental thoughts of ADT transmit condition:

The Adomian decomposition method generates approximation series solutions to a wide range of equations efficiently and conveniently.

Eq. (1) may be expressed as Eq. (2) by defining the partial differential operators i.e. $I_{xx} = \frac{\partial^2}{\partial x^2}$, and and $I_{xx} = \frac{\partial^2}{\partial x^2}$

$$I_{tt}u + \alpha I_t u + \beta u = \gamma I_{xx}u + h(x,t), \tag{6}$$

With respect to the initial conditions of the boundary

$$\begin{cases} u(x,0) = g_1(x), & 0 < x < L \\ I_t u(x,0) = g_2(x), & 0 < x < L \end{cases} \tag{7}$$

Definition of left-inverse integral operators in formal terms $I_{tt}^{-1} = \int_0^t \int_0^t (.)dtdt$ and the result is obtained by using the inverse operator on both sides of (6), we get

$$u(x,t) = u(x,0) + t.I_t u(x,0) + I_{tt}^{-1}\big(\gamma I_{xx}u - \alpha I_t u - \beta u + h(x,t)\big), \tag{8}$$

If (7) is substituted for (8), we get the following:

$$u(x,t) = g_1(x)(x) + \varphi(x)t + I_{tt}^{-1}\big(\gamma I_{xx}u - \alpha I_t u - \beta u + h(x,t)\big), \tag{9}$$

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \quad (x,t) \in R^2 \tag{10}$$

$$\gamma I_{xx}u - \alpha I_t u - \beta u + h(x,t) = \sum_{n=0}^{\infty} P_n(u_1, u_2 , \dots \dots u_n) \tag{11}$$

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Specifically, $P_n(u_1, u_2, \dots \dots u_n)$ should be computed using what are known as Adomian polynomials. An elective calculation for processing Adomian polynomials was utilized to show up at the outcomes. Polynomials of adomian type. As a result, we can say:

$$P_n(u_1, u_2, \dots \dots u_n) = \gamma I_{xx} u_n - \alpha I_t u_n - \beta u_n + h(x,t), n = 0, 1, 2, \dots \quad (12)$$

For (9), substitute (10) and (12).

$$\sum_{n=0}^{\infty} u_n = g\_1(x) + \varphi(x)t + \sum_{n=0}^{\infty} I_{tt}^{-1}\left(\gamma I_{xx} u_n - \alpha I_t u_n - \beta u_n + h(x,t)\right), \quad (13)$$

where the elements $u_n (n \geq 0)$ satisfy the recursive connections,

$$u_0(x,t) = g_1(x) + g_2(x)t,$$

$$u_{n+1}(x,t) = I_{tt}^{-1}\left(\gamma I_{xx} u_n - \alpha I_t u_n - \beta u_n + h(x,t)\right), n = 0, 1, 2, \dots \quad (14)$$

Equation (12)'s exact solution is now known with certainty. Due to this, we must use an approximation of the solution from the shortened series of $f_n = \sum_{n=0}^{\infty} u_n$ with $\lim_{n \to \infty} f_n = u$ in order to complete the preparation.

3. Essential thoughts of He's HPM for Telegraph Equation:

For the purposes of this section, we will assume that a general notation for the telegraph equation in one dimension is

For the purposes of this section, we will assume that a general notation for the telegraph equation in one dimension is

$$\frac{\partial^2 u}{\partial t^2} + \alpha \frac{\partial u}{\partial t} + \beta u = \gamma \frac{\partial^2 u}{\partial x^2} + h(x,t) \quad 0 < t \leq T, 0 < x < L, \quad (15)$$

With respect to the initial conditions of the boundary

$$u(x,0) = g_1(x) 0 < x < L \ ; \frac{\partial u}{\partial t}(x,0) = g_2(x) 0 < x < L \quad (16)$$

Where h, g_1, g_2 and,, and α, β, γ are constants, respectively.

We made the accompanying homotopy to get the semi-scientific answer for condition (15) by HPM utilizing the essential circumstances:

$$\left(\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2}\right) + \left(\alpha \frac{\partial u}{\partial t} + \beta u - \gamma \frac{\partial^2 u}{\partial x^2} - h(x,t) + \frac{\partial^2 u_0}{\partial t^2}\right) p = 0 \quad (17)$$

or

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u_0}{\partial t^2} - \left(\alpha \frac{\partial u}{\partial t} + \beta u - \gamma \frac{\partial^2 u}{\partial x^2} - h(x,t) + \frac{\partial^2 u_0}{\partial t^2}\right) p \quad (18)$$

Equation (18) can be solved as

$$u(x,t) = u_0 + p u_1 + p^2 u_2 + p^3 u_3 + p^4 u_4 + \cdots \quad (19)$$

Condition (19) is switched over completely to condition (18), and the condition equivalent power p is as per the following: Equation p

$$p^0 : \ \frac{\partial^2 u_0}{\partial t^2} = \frac{\partial^2 u_0}{\partial t^2} \quad (20)$$

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$$p^1: \ \frac{\partial^2 u_1}{\partial t^2} = -\left(\alpha \frac{\partial u_0}{\partial t} + \beta u_0 - \gamma \frac{\partial^2 u_0}{\partial x^2} - h(x,t) + \frac{\partial^2 u_0}{\partial t^2}\right) \tag{21}$$

$$p^2: \ \frac{\partial^2 u_2}{\partial t^2} = -\left(\alpha \frac{\partial u_1}{\partial t} + \beta u_1 - \gamma \frac{\partial^2 u_1}{\partial x^2}\right) \tag{22}$$

$$p^3: \frac{\partial^2 u_3}{\partial t^2} = -\left(\alpha \frac{\partial u_2}{\partial t} + \beta u_2 - \gamma \frac{\partial^2 u_2}{\partial x^2}\right) \tag{23}$$

$$p^4: \frac{\partial^2 u_4}{\partial t^2} = -\left(\alpha \frac{\partial u_3}{\partial t} + \beta u_3 - \gamma \frac{\partial^2 u_3}{\partial x^2}\right) \tag{24}$$

⋮

Others have said the same thing.

With $u_0 = g_1(x) + t g_2(x)$, all the aforementioned linear equations may be solved quickly, and all of the solutions will be available. To find the answer to equation (15), simply substitute $p = 1$ into the equation (19).

$$u = u_0 + u_1 + u_2 + u_3 + u_4 + \cdots \tag{25}$$

Numerical Illustration:

Example 1: Consider equation (1), $\alpha = 2, \beta = 1, \gamma = 1, h(x,t) = 0$, (Biazar and Eslami (2010)),

$$u_{tt} + 2u_t + u = u_{xx} \tag{26}$$

In the presence of primary circumstances:

$$u(x,0) = e^x \ \text{ and } \quad u_t(x,0) = -2e^x \tag{27}$$

When applying ADM & HPM's primary conditions to construct a semi-analytical solution to equation (26), Equation (26) can be written as follows according to the ADM:

$$I_{tt}u + 2I_t u + u = I_{xx}u, \tag{28}$$

If equation (28) is solved as a decomposition series, then the answer is

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \quad (x,t) \in R^2 \tag{29}$$

where the components $u_n (n \geq 0)$ meet the recursive connections, and where

$$u_0(x,t) = u(x,0) + t\, u_t(x,0) = e^x - 2te^x, \tag{30}$$

$$u_{n+1}(x,t) = I_{tt}^{-1}(I_{xx}u_n - 2I_t u_n - u_n), n \geq 0 \tag{31}$$

Putting the value of $n = 0, 1, 2, 3, \ldots$

$$u_1(x,t) = I_{tt}^{-1}(I_{xx}u_0 - 2I_t u_0 - u_0) = 2t^2 e^x \tag{32}$$

$$u_2(x,t) = I_{tt}^{-1}(I_{xx}u_1 - 2I_t u_1 - u_1) = -\frac{4\,t^3}{3} e^x \tag{33}$$

$$u_3(x,t) = I_{tt}^{-1}(I_{xx}u_2 - 2I_t u_2 - u_2) = \frac{2t^4}{3} e^x \tag{34}$$

$$u_4(x,t) = I_{tt}^{-1}(I_{xx}u_3 - 2I_t u_3 - u_3) = -\frac{4t^5}{5} e^x \tag{35}$$

⋮

The series form of the solution to the equation $u_n(x,t)$ is

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots$$

$$u(x,t) = e^x - 2te^x + 2t^2 e^x - \frac{4\,t^3}{3} e^x + \frac{2t^4}{3} e^x - \frac{4t^5}{5} e^x + \cdots$$

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$$u(x,t) = e^{x-2t} \tag{36}$$

We established the following homotopy in accordance with the HPM:

$$\left(\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2}\right) + \left(2\frac{\partial u}{\partial t} + u - \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u_0}{\partial t^2}\right)p \ = 0 \tag{37}$$

or

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u_0}{\partial t^2} - p\left(2\frac{\partial u}{\partial t} + u - \frac{\partial^2 u}{\partial x^2} + + \frac{\partial^2 u_0}{\partial t^2}\right) \tag{38}$$

Equation (18) can be solved by taking the following form:

$$u(x,t) \ = \ u_0 \ + \ pu_1 \ + \ p^2 u_2 \ + p^3 u_3 \ + p^4 u_4 \ + \cdots \tag{39}$$

Using equation (39) as a starting point, let's transform it to equation (38) and see what happens.

$$p^0 : \ \frac{\partial^2 u_0}{\partial t^2} = \frac{\partial^2 u_0}{\partial t^2} \tag{40}$$

$$p^1 : \ \frac{\partial^2 u_1}{\partial t^2} = -\left(2\frac{\partial u_0}{\partial t} + u_0 - \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial t^2}\right) \tag{41}$$

$$p^2 : \ \frac{\partial^2 u_2}{\partial t^2} = -\left(2\frac{\partial u_1}{\partial t} + u_1 - \frac{\partial^2 u_1}{\partial x^2}\right) \tag{42}$$

$$p^3 : \frac{\partial^2 u_3}{\partial t^2} = -\left(2\frac{\partial u_2}{\partial t} + u_2 - \frac{\partial^2 u_2}{\partial x^2}\right) \tag{43}$$

$$p^4 : \frac{\partial^2 u_4}{\partial t^2} = -\left(2\frac{\partial u_3}{\partial t} + u_3 - \frac{\partial^2 u_3}{\partial x^2}\right) \tag{44}$$

$\vdots$

Others have said the same thing.

It is possible to begin with this approximation by solving the equations $u_0 = u(x,0) + t\,u_t(x,0) = \ g_1(x) + tg_2(x) = e^x - 2te^x$,

$$u_1 = \frac{t^2}{2!}4e^x, u_2 = -\frac{t^3 2^3}{3!}e^x, u_3 = \frac{t^4}{4!}2^4 e^x, u_4 = -\frac{t^5}{5!}2^5 e^x, \ldots . u_n = \frac{(-2)^{n+1}t^{n+1}}{(n+1)!}e^x$$

Equation (26) has a solution of the form:

$$w(x,t) = e^x - 2te^x + \frac{t^2}{2!}4e^x - \frac{t^3 2^3}{3!}e^x + \frac{t^4}{4!}2^4 e^x - \cdots\cdots\cdots\cdots \tag{45}$$

So,

$$w(x,t) = e^x[1 - 2t + \frac{2^2 t^2}{2!} - \frac{2^3 t^3}{3!} + \frac{4^3 t^4}{4!} - \cdots\cdots\cdots\cdots]$$

$$w(x,t) = e^x . e^{-2t} = e^{x-2t} \tag{46}$$

Which is the same exact solution as the one discovered by DTM (Biazar and Eslami (2010)),

In Fig. 1, we can see the space-time chart for the message condition (26) arrangement utilizing MATLAB or the ADM or HPM condition (36, 46). The worth of t is taken, and the scope of x is from $= 0$ to $t = 4$ and $x = 0$ to $x = 4$. In Figures 2 and 3, a correlation with the arrangement of the message condition (26) is shown by embracing the improved upsides of reality t as being $= 0, t = 0.75$, $t = 1.5$ and $t = 2.5$ where x reaches from $x = 0$ to $x = 4$, separately. Figure 1 shows that as the worth of x ascents, the worth of t falls and the worth of $u(x,t)$increases, and as the worth of x falls, the worth of trises and the worth of u $u(x,t)$ declines. Fig. 2 shows that as the worth of x be rises, the worth of $u(x,t)$will likewise rise erroneously at $t = 0$, , instead of other t organizes.

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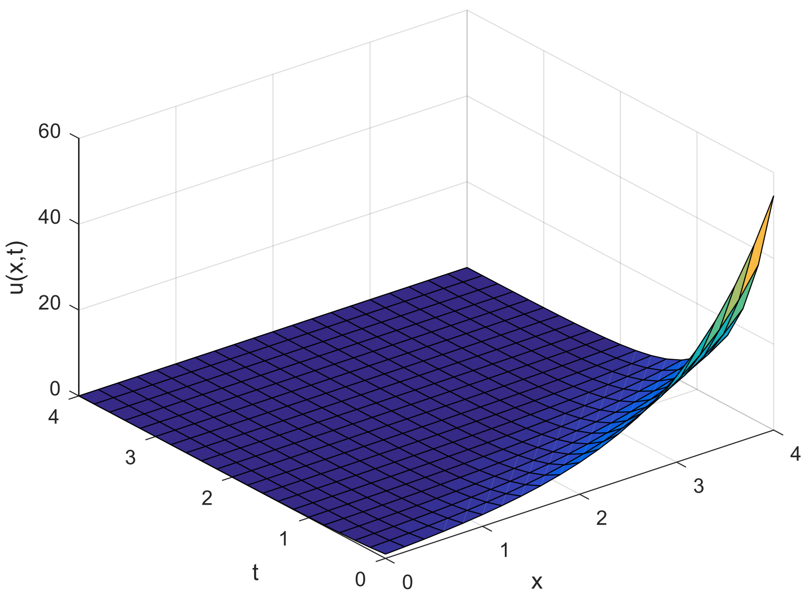

Figure1. Space-time graph for the solution of telegraph equation (26) taking the reformed values of x and t

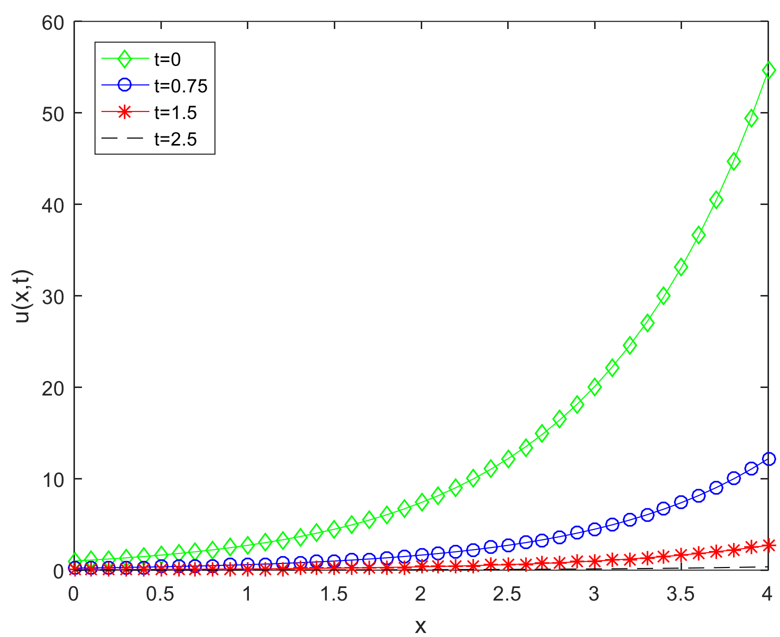

Figure2. Comparison the solution of telegraph equation (26) taking thereformed values of time $t = 0, t = 0.75$, $t = 1.5$ and $t = 2.5$

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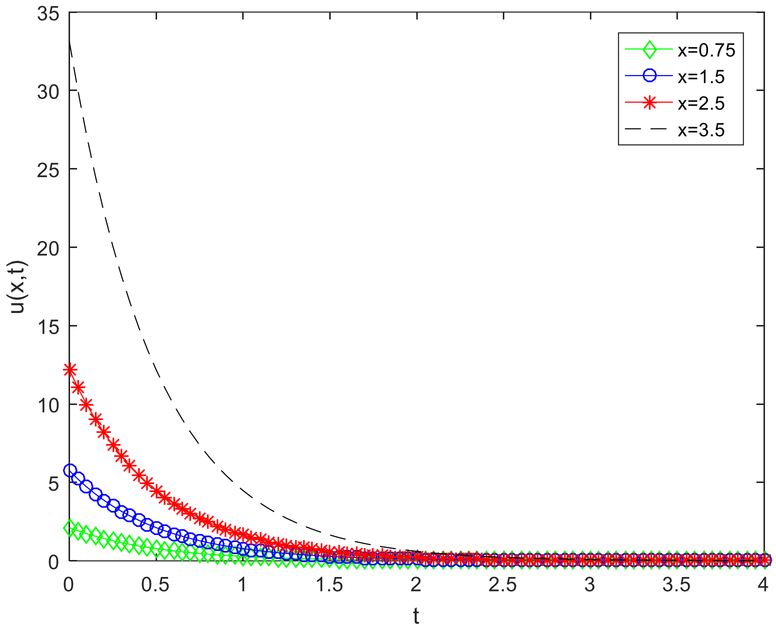

Figure3. Comparison the solution of telegraph equation (26) taking thereformed values of specs$x = 0.75$, $x = 1.5$, $x = 2.5$, and $x = 3.5$

While in Fig. 3, we see that the value of tbe rises and then falls falsely at the value $x = 3.5$, which is in contrast to the other values such as $x = 0.75$, $x = 1.5$, $x = 2.5$. Using the equations (36) and (46), ADM and HPM have found the same exact solution to the telegraph problem (26) by ADM and HPM as DTM (Biazar and Eslami (2010)) and the Galerkin Finite Element Method.

Example 2: Here's an example of how to use it: Let $\alpha = 4, \beta = 2, \gamma = 1, h(x,t) = 0$ values in equation (1), (Dehghan, and Shokri, (2008))

$$u_{tt} + 4u_t + 2u = u_{xx} \tag{47}$$

When it comes to the most basic conditions:

$$u(x,0) = \sin(x) \text{ and } \quad u_t(x,0) = -\sin(x), \qquad 0 \le x \le \pi \tag{48}$$

Using ADM & HPM's primary conditions, find the semi-analytical solution to equation (47). Equation (47), according to the ADM, can be expressed as follows:

$$I_{tt}u + 4I_t u + 2u = I_{xx}u, \tag{49}$$

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \quad (x,t) \in R^2 \tag{50}$$

The components $u_n(n \ge 0\,))$ meet the recursive connections,

$$u_0(x,t) = u(x,0) + t\,u_t(x,0) = \sin(x) - t\sin(x) = \sin(x)\,(1-t), \tag{51}$$

$$u_{n+1}(x,t) = I_{tt}^{-1}(I_{xx}u_n - 4I_t u_n - 2u_n), n \ge 0 \tag{52}$$

Using the $n = 0, 1, 2, 3, \ldots$

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$$u_1(x,t) = I_{tt}^{-1}(I_{xx}u_0 - 4u_0 - 2u_0) = \sin(x)\left(\frac{t^2}{2} + \frac{t^3}{2}\right) \quad (53)$$
$$u_2(x,t) = I_{tt}^{-1}(I_{xx}u_1 - 4I_t u_1 - 2u_1) = -\sin(x)\left(\frac{2\,t^3}{3} + \frac{5t^4}{8} + \frac{3t^5}{40}\right) \quad (54)$$
$$u_3(x,t) = I_{tt}^{-1}(I_{xx}u_2 - 4I_t u_2 - 2u_2) = \sin(x)\left(\frac{2t^4}{3} + \frac{3t^5}{5} + \frac{9\,t^6}{80} + \frac{3\,t^7}{560}\right) \quad (55)$$

The series form of the solution to the equation $u_n(x,t)$

$$\vdots$$

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots$$

$$u(x,t) = \sin(x)\,(1-t) + \sin(x)\left(\frac{t^2}{2} + \frac{t^3}{2}\right) - \sin(x)\left(\frac{2\,t^3}{3} + \frac{5t^4}{8} + \frac{3t^5}{40}\right) + \sin(x)\left(\frac{2t^4}{3} + \frac{3t^5}{5} + \frac{9\,t^6}{80} + \frac{3\,t^7}{560}\right) + \cdots$$

$$u(x,t) = \sin(x)\,e^{-t} \quad (56)$$

We established the following homotopy in accordance with the HPM:

$$\left(\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2}\right) + \left(4\frac{\partial u}{\partial t} + 2u - \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u_0}{\partial t^2}\right)p = 0 \quad (57)$$

or

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u_0}{\partial t^2} - p\left(4\frac{\partial u}{\partial t} + 2u - \frac{\partial^2 u}{\partial x^2} + + \frac{\partial^2 u_0}{\partial t^2}\right) \quad (58)$$

In order to find the solution to equation (58), take the following form into consideration:

$$u(x,t) = u_0 + pu_1 + p^2u_2 + p^3u_3 + p^4u_4 + \cdots \quad (59)$$

Because of this change, the equation for equal power p is now written as follows:

$$p^0: \ \frac{\partial^2 u_0}{\partial t^2} = \frac{\partial^2 u_0}{\partial t^2} \quad (60)$$
$$p^1: \ \frac{\partial^2 u_1}{\partial t^2} = -\left(4\frac{\partial u_0}{\partial t} + 2u_0 - \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial t^2}\right) \quad (61)$$
$$p^2: \ \frac{\partial^2 u_2}{\partial t^2} = -\left(4\frac{\partial u_1}{\partial t} + 2u_1 - \frac{\partial^2 u_1}{\partial x^2}\right) \quad (62)$$
$$p^3: \frac{\partial^2 u_3}{\partial t^2} = -\left(4\frac{\partial u_2}{\partial t} + 2u_2 - \frac{\partial^2 u_2}{\partial x^2}\right) \quad (63)$$
$$p^4: \frac{\partial^2 u_4}{\partial t^2} = -\left(4\frac{\partial u_3}{\partial t} + 2u_3 - \frac{\partial^2 u_3}{\partial x^2}\right) \quad (64)$$

$\vdots$

Others have said the same thing.

Assuming $u_0 = u(x,0) + t\,u_t(x,0) = g_1(x) + tg_2(x) = (1-t)\sin(x)$, as a starting point, the following approximation is obtained by solving the equations above:

$u_1 = \sin(x)\left(\frac{t^2}{2!} + \frac{3\,t^3}{3!}\right)$, $u_2 = -\sin(x)\left(\frac{4\,t^3}{3!} + \frac{15t^4}{4!} + \frac{9t^5}{5!}\right)$,

$u_3 = \sin(x)\left(\frac{16t^4}{4!} + \frac{72t^5}{5!} + \frac{81\,t^6}{6!} + \frac{27\,t^7}{7!}\right)$, ….

Thus, the arrangement of condition (26) is as per the following:

$$u(x,t) = \sin(x)\,(1-t) + \sin(x)\left(\frac{t^2}{2!} + \frac{3\,t^3}{3!}\right) - \sin(x)\left(\frac{4\,t^3}{3!} + \frac{15t^4}{4!} + \frac{9t^5}{5!}\right)$$
$$+ \sin(x)\left(\frac{16t^4}{4!} + \frac{72t^5}{5!} + \frac{81\,t^6}{6!} + \frac{27\,t^7}{7!}\right) + \cdots$$

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$$u(x,t) = \sin(x)\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} - \cdots\right)$$

$$u(x,t) = \sin(x)\, e^{-t} \tag{65}$$

Which is the same exact solution as HAM (Hosseini et al., 2010) discovered.

Fig. 4 shows the space-time chart for the arrangement of the message issue (47) utilizing MATLAB of condition (56) or condition (65) by ADM or HPM, with t and x lying between $= 0$ to $t = 3$ and $x = 0$ to $x = \pi$. While figures 5 and 6 give a correlation of the arrangement of broadcast condition (47) utilizing improved upsides of time $t = 0, t = 0.75$, $t = 1.5$ and $t = 2.5$ when x is among 0 and π, and utilizing changed upsides of $x = 0.75, x = 1.5, x = 2.5$ and $x = 3.5$ when t is somewhere in the range of 0 and 3. At the point when the worth of x and t fills in fig. 4, the worth of $u(x,t)$ increases from the get go, then diminishes after some time. In fig. 5, the worth of $u(x,t)$ be increments when the worth of x is between $x = 0$ to $x = 1.5$, and diminishes when the worth of x is between $x = 1.5$ to $x = 3$ dishonestly at $t = 0$, contrasted with different upsides of $= 0.75$,$t = 1.5$ and $t = 2.5$.

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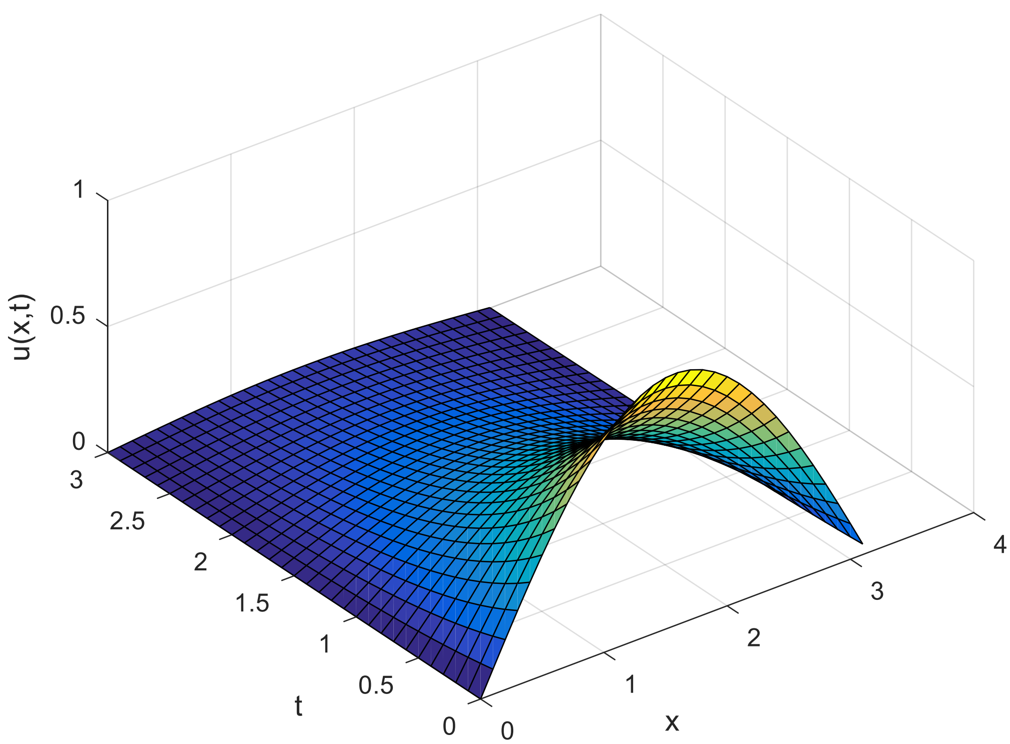

Figure4. space-time graph for the solution of telegraph equation (47) taking the reformed values of x and t

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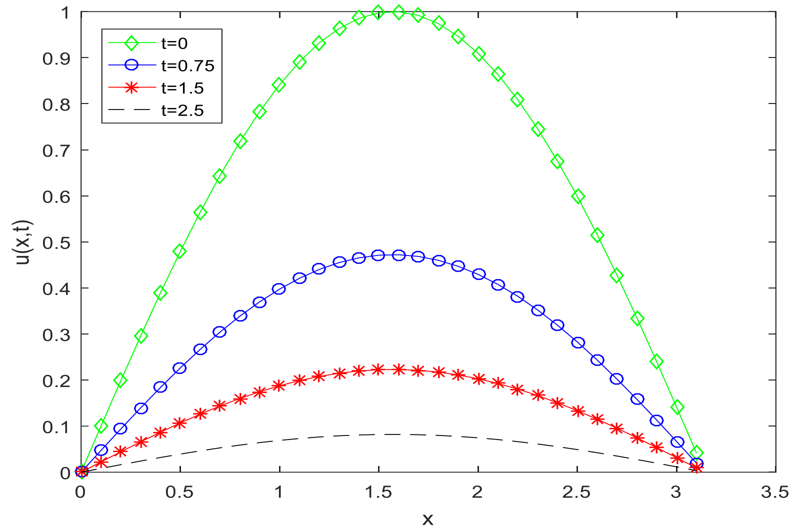

Figure5. Comparison the solution of telegraph equation (47) taking formed values of time $t = 0$, $t = 0.75$, $t = 1.5$ and $t = 2.5$

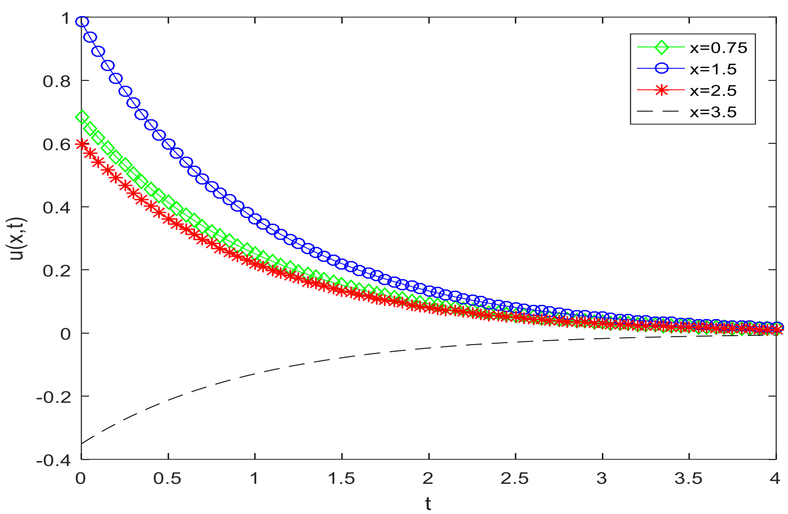

Figure6. Comparison the solution of telegraph equation (47) taking formed values of specs $x = 0.75$, $x = 1.5$, $x = 2.5$, and $x = 3.5$

Fig. 3 shows that when t increases, the value of $u(x, t)$ will increase at $x = 3.5$ and slowly drop at $x = 1.5$, as opposed to increasing at $x = 0.75$and decreasing at $x = 2.5$., respectively. For the telegraph equation (47) by ADM and HPM, the equations (56) and (65) reflect the exact solution
This is the very result as that found by HAM in their review. (Hashemi et. al. (2019)).

4. Conclusions

we utilized ADM and HPM to accomplish estimated or scientific arrangements in this article. The two procedures yield agreeable semi-scientific arrangements when contrasted with the other options. The HPM dodges the prerequisite to ascertain the Adomian polynomials, which may be troublesome in certain occurrences. This is a significant finding. Discovering exact and inexact answers for direct and nonlinear exaggerated message conditions utilizing the HPM is a solid numerical instrument. This strategy is likewise very reliable and fit when contrasted with others. MATLAB programming was utilized to process the series, which was gathered from the ADM and HPM, and to envision a few powerful message conditions in three measurements and two measurements by changing the upsides of different boundaries.

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