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Q Fuzzy Regular Graphs

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ABSTRACT

In this paper, the notions of Q fuzzy graph are introduced, union, intersection of two Q fuzzy regular graphs are introduced, in this paper we introduced some operation on Q fuzzy graph, denoted QFG. We study the type of graph on operations Q fuzzy graph are established here and we study Q fuzzy depended of degree graph, regular graph of Q fuzzy graph and non-regular graph of Q fuzzy graph and complete Q fuzzy graph. If G is Q fuzzy graph the $G-\{e\}$ satisfied property of Q fuzzy graph. Also we apply QFG on removable vertex from graph in Q fuzzy graph, in other words if G is Q fuzzy graph then $G-\{v\}$ satisfied property of Q fuzzy graph.

Key words: Fuzzy graph, Regular fuzzy graph, non regular fuzzy graph, Operation of graphs.

Introduction:

In 1736, Euler first introduced the concept of graph Theory. The theory of graphs is extremely useful tool for solving combinatorial problems in different areas such that geometry, algebra, number theory, topology, operation research, optimization and computer science etc.

The first publications in fuzzy set theory by Zadeh [1965] and Goguen [1967, 1969] show the intention of the authors to generalize the classical notion of a set in [1975], Rosenfeld [1] introduced the concepts of fuzzy graphs there after many research have generalized they notions graph theory. In this paper our aim is to introduce the nation of Q fuzzy graph and some properties and operations, union of Q fuzzy graph, intersection of two Q fuzzy graph, we study of type of graph on Q fuzzy graph and properties removable edge and vertex on Q fuzz graph.

2. Preliminaries

Definition 2.1

Let V be an non empty finite set $P: V \to [0,1]$, let $Q: V \times V \to [0,1]$such that $Q(x,y) \leq P(x) \wedge P(y), \forall (x,y) \in V \times V$. A fuzzy graph $G=(P,Q)$.

Definition 2.2

Let $G_1=(P_1,Q_1)$ and $G_2=(P_2,Q_2)$ be two fuzzy graph over vertex set V, then the union of G_1 and $G_2$is fuzzy graph $G_3=(P_3,Q_3)$ over the set V, such that $P_3=(P_1 \vee P_2)$ and $Q_3=(Q_1 \vee Q_2)$, where

$$P_3(x)=\max\{P_1(x),P_2(x)\}, \forall x \in V, \text{and}$$

$$Q_3(x)=\max\{M_1(x,y),M_2(x,y)\}, \forall x,y \in V$$

Definition 2.3

Let $G_1=(P_1,Q_1)$ and $G_2=(P_2,Q_2)$ be two fuzzy graph over vertex set V, then the intersection of G_1 and $G_2$is fuzzy graph $G_3=(P_3,Q_3)$ over the set V, such that $P_3=(P_1 \wedge P_2)$ and $Q_3=(Q_1 \wedge Q_2)$, where

$$P_3(x)=\min\{P_1(x),P_2(x)\}, \forall x \in V, \text{and}$$

$$Q_3(x)=\min\{Q_1(x,y),Q_2(x,y)\}, \forall x,y \in V$$

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Q FUZZY REGULAR GRAPH

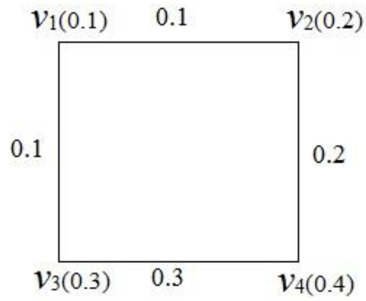
Definition 3.1

A fuzzy subset Q of G is called Q fuzzy Regular graph such that $Q(x,y) < p(x) + p(y) \forall x, y \in G$ and it is denoted by $QFG = (P,Q)$.

Example 3.2

Let $V = \{v_1, v_2, v_3, v_4\}$and $E = \{e_1, e_2, e_3, e_4\}$. Here the Q fuzzy regular graph.

G:



In G_1 graph, we apply Q fuzzy regular graph

$Q(v_1v_2) = 0.1$ and $P(v_1) = 0.1, P(v_2) = 0.2$

such that $P(v_1) + P(v_2) = 0.1 + 0.2 = 0.3$

then$Q(v_1, v_2) < P(v_1) + P(v_2)$

Next

$Q(v_1v_3) = 0.1$and$P(v_1) = 0.1, P(v_3) = 0.3$

such that $P(v_1) + P(v_3) = 0.1 + 0.3 = 0.4$

then$Q(v_1, v_3) < P(v_1) + P(v_3)$

$Q(v_3v_4) = 0.3$and$P(v_3) = 0.3, P(v_4) = 0.4$

such that $P(v_3) + P(v_4) = 0.3 + 0.4 = 0.7$

then$Q(v_3, v_4) < P(v_3) + P(v_4)$

$Q(v_2v_4) = 0.2$and$P(v_2) = 0.2, P(v_4) = 0.4$

such that $P(v_2) + P(v_4) = 0.3 + 0.4 = 0.6$

then$Q(v_2, v_4) < P(v_2) + P(v_4)$

QFuzzy Regular graph depended of degree graph

Definition 3.3

Let $QFG_1 = (P_1, Q_1)$ and $QFG_2 = (P_2, Q_2)$ be two Q fuzzy regular graph over vertex set V, then the union of QFG_1 and $QFG_2$is fuzzy graph $QFG_3 = (P_3, Q_3)$ over the set V, such that $P_3 = (P_1 \vee P_2)$ and $Q_3(x,y) = (Q_1 \vee Q_2)$, where

$$P_3(x) = \max\{P_1(x), P_2(x)\}, \forall x \in V, \text{and}$$

$$Q_3(x,y) = \max\{Q_1(x,y), Q_2(x,y)\}, \forall x, y \in V$$

Definition 3.4

Let $QFG_1 = (P_1, Q_1)$ and $QFG_2 = (P_2, Q_2)$ be two Q fuzzy regular graph over the set V. Then the intersection of QFG_1 and $QFG_2$is a fuzzy graph $QFG_3 = (P_3, Q_3)$ over the set V, such that $P_3 = (P_1 \wedge P_2)$ and $Q_3 = (Q_1 \wedge Q_2)$, where

$$P_3(x) = \min\{P_1(x), P_2(x)\}, \forall x \in V, \text{and}$$

$$Q_3(x,y) = \max\{Q_1(x,y), Q_2(x,y)\}, \forall x, y \in V$$

Definition 3.5

Let G be a simple graph, A fuzzy subset of Q of G is called Q fuzzy regular graph such that $Q(x,y) < np(x)mp(y), \forall(x,y) \in G$. An Q fuzzy regular graph of $G = (P, Q)$ over the set V, such that depended of degree graph $\forall(x,y) \in G, n, m$ be numbers degree of P.

Theorem3.6

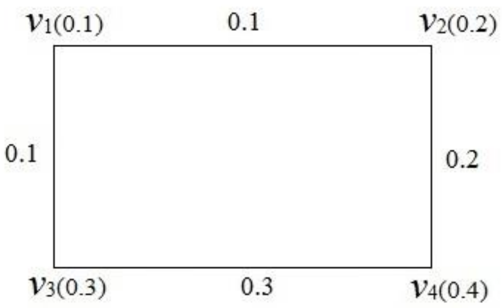
Let G be 2-regular graph, then G has Q fuzzy regular graph.

Proof

Suppose that G be 2-regular fuzzy graph, then $\sum d(v) = 2$ of each vertex G, we must prove G has Q fuzzy regular graph.

The condition of Q fuzzy regular graph.

$Q(x,y) < np(x) + mp(y)$such that $n, m = 2$ of the following graph G, degree 2.



Show the graph is 2-regular graph.

If $Q(v_1, v_2) = 0.1$ and $nP(v_1) + mP(v_2) = 2(0.1) + 2(0.2)$

$$= 0.2 + 0.4$$

$$= 0.6$$

Hence $Q(v_1v_2) < nP(v_1) + mP(v_2)$.

Hence G has Q fuzzy regular graph.

Theorem 3.7

Let G be not regular fuzzy graph then G has Q fuzzy regular graph.

Proof

Suppose that G be not regular graph in other word degree of each vertex is not equal.

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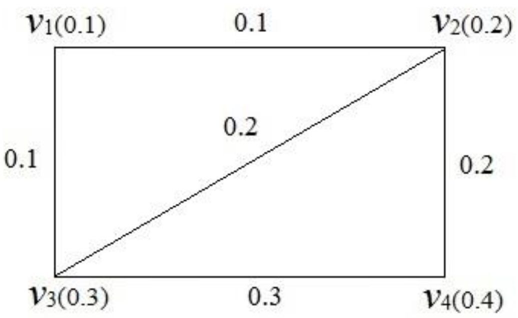
The following graph



$$d(v_1) = 2$$

$$d(v_2) = 3$$

$$d(v_3) = 3$$

$$d(v_4) = 2$$

$$Q(v_1v_2) = 0.1, P(v_1) = 0.1, P(v_2) = 0.2$$

$$n\big(P(v_1)\big) + m\big(P(v_2)\big) = 2(0.1) + 2(0.2)$$

$$= 0.2 + 0.4 = 0.6$$

Hence, $Q(v_1, v_2) < nP(v_1) + mP(v_2)$.

$$Q(v_2v_3) = 0.2, P(v_2) = 0.2, P(v_3) = 0.3$$

$$n\big(P(v_2)\big) + m\big(P(v_3)\big) = 3(0.2) + 3(0.3)$$

$$= 0.6 + 0.9 = 1.5$$

Hence, $Q(v_2v_3) < nP(v_2) + mP(v_3)$.

$$Q(v_1v_3) = 0.1, P(v_1) = 0.1, P(v_3) = 0.3$$

$$n\big(P(v_1)\big) + m\big(P(v_3)\big) = 2(0.1) + 3(0.3)$$

$$= 0.2 + 0.9 = 1.1$$

Hence, $Q(v_1v_3) < nP(v_1) + mP(v_3)$.

$$Q(v_2v_4) = 0.2, P(v_2) = 0.2, P(v_4) = 0.4$$

$$n\big(P(v_2)\big) + m\big(P(v_4)\big) = 3(0.2) + 2(0.4)$$

$$= 0.6 + 0.8 = 1.4$$

Hence, $Q(v_2v_4) < nP(v_2) + mP(v_4)$.

$$Q(v_3v_4) = 0.3, P(v_3) = 0.3, P(v_4) = 0.4$$

$$n\big(P(v_3)\big) + m\big(P(v_4)\big) = 3(0.3) + 2(0.4)$$

$$= 0.9 + 0.4 = 1.3$$

Hence, $Q(v_3v_4) < nP(v_3) + mP(v_4)$.

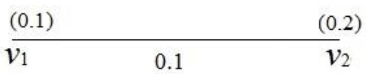
Hence, G has Q fuzzy regular graph.

Theorem 3.8

Let G be K_n complete fuzzy graph then G has Q fuzzy regular graph, $n = 2,3,4,5$.

Proof

Suppose that G be K_2 complete graph.



$$Q(v_1v_2) = 0.1,$$

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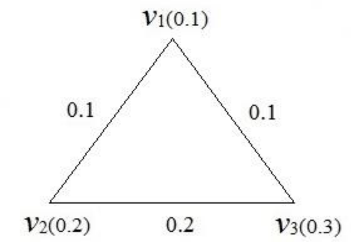
$P(v_1) = 0.1, P(v_2) = 0.2$

then$Q(v_1, v_2) < p(v_1) + P(v_2)$

Hence K_2 is Q fuzzy graph.

If G be K_3 complete graph.

Similarly we can prove that K_3 is Q fuzzy regular graph.



In same method with respect to the complete graph K_5 is a Q fuzzy regular graph.

Definition 3.7

If $G = (V, E)$ and V has at least two elements then for any vertex v of G, $G - \{v\}$ denotes the subgraph of G vertex set $V - \{v\}$ whose edges are all those of G which are not incident with v ie. $G - \{v\}$is obtained from G by removing v and all the edges of G which have v as an end. $G - \{v\}$is referred to a vertex deleted subgraph. If $G = \{V, E\}$ and e is an edge of G then $G - \{e\}$ denotes the subgraph of G having v as its vertex set.

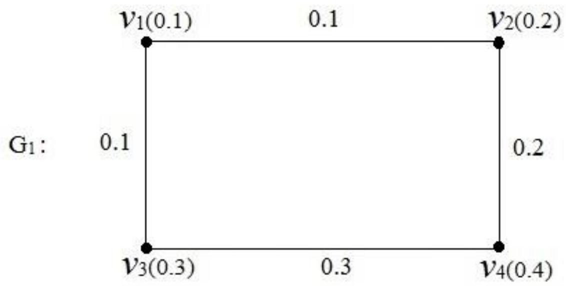
Theorem 3.10

Let G be Q fuzzy regular graph then $G - v$ is Q fuzzy graph.

Proof

Suppose that Gbe$Q$ fuzzy graph we remove vertex of graph G.

Now apply vertex 2-regular graph. Let G be 2-regular G-fuzzy graph.



So removable from G is $v_4$ie. $G - \{v_4\}$

$$Q(v_1 v_2) = 0.1,$$

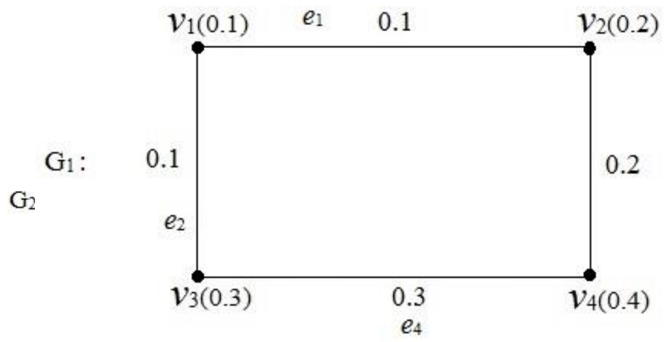
$$P(v_1) = 0.1, P(v_2) = 0.2$$

So, $Q(v_1, v_2) < p(v_1) + P(v_2)$

$$Q(v_1 v_3) = 0.1,$$

$$P(v_1) = 0.1, P(v_3) = 0.3,$$

So, $Q(v_1 v_3) < p(v_1) + P(v_3)$



Hence the removable graph G_2 has a Q fuzzy graph.

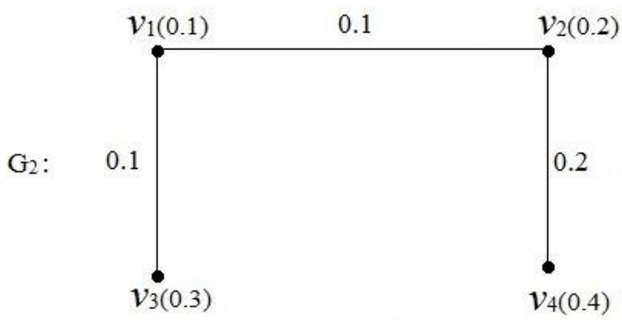
Theorem 3.11

Let G be Q fuzzy regular graph then $G - e$ is an$Q$ fuzzy graph.

Proof

Suppose that G be Q fuzzy regular graph.

We apply remove edge of 2-regular graph, removable ie. $G_2 - \{e_4\}$



$$Q(v_1, v_2) = 0.1$$

$$P(v_1) = 0.1, P(v_2) = 0.2$$

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$$\therefore \; Q(v_1, v_2) < p(v_1) + P(v_2)$$

Similarly

$$Q(v_1, v_3) = 0.1$$

$$P(v_1) + \; P(v_3) = 0.1 + 0.3 = 0.4$$

Hence $Q(v_1, v_3) < p(v_1) + P(v_3)$

Hence, G_2 is Q fuzzy graph.

CONCLUSION

We define Q fuzzy regular graph as following let G be simple graph A fuzzy subset Q of G is called Q fuzzy regular graph such that $Q(x, y) < P(x) + P(y) \forall (x, y) \in G$, and we study the operations union, intersection, degree of Q fuzzy regular graph, take some type of regular – non regular – complete graph. Finally we study removable vertex and removable edge from the Q fuzzy regular graph and remained the Q fuzzy graph.

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