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Operations on Interval-valued Binary Fuzzy Graphs

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ABSTRACT

In this paper we define Cartesian Product of Interval – valued binary fuzzy graphs and composition of Interval – valued binary fuzzy graphs. We investigate some of their properties.

KEYWORDS: Cartesian product, Composition, Interval – valued binary fuzzy graphs.

1. INTRODUCTION

The notion of fuzzy sets was introduced by L.A. Zadeh in 1965. It involves the concept of membership function defined on a universal set. The value of the membership function lies in $[0,1]$.This concept has found applications in computer science, artificial intelligence, decision analysis, information science, pattern recognition, operation research and robotics. The fuzzy relations between fuzzy sets were also considered by Rosenfeld (1975) who developed the structure of fuzzy graphs. Later on Bhattacharya gave some remarks on fuzzy graphs. The operations of union, join, Cartesian Product and composition on two fuzzy graphs were defined by Moderson J.N. and Peng. C.S.In this paper we study about the binary fuzzy graphs we define the operations of Cartesian product, composition on Interval – valued binary fuzzy graphs. Also we obtained some results related on Cartesian product on IVBFG, Composition of IVBFG.

Definition 1.1

A fuzzy subset μon a set X is a map $\mu: X \to [0,1]$. A map $\mathcal{B}: X \times X \to [0,1]$ is fuzzy relation on X if $\mathcal{B}(\alpha, \beta) \leq \mu(x) \wedge \mu(y)$ for all $\alpha, \beta \in \mathcal{B}$ is a symmetric fuzzy relation if $\mathcal{B}(\alpha, \beta) = \mathcal{B}(\beta, \alpha)$ for all $\alpha, \beta \in X$.

Definition 1.2

Let X be a non-empty regular set. A binary fuzzy set B in X is an objective having the practice $B = \left\{\left(\alpha, \mu^{\rho(\alpha)}, \mu^{\eta}(\alpha)\right) / \alpha \in X\right\}$where$\mu^{\rho} : X \to [0,1]$ and$\mu^{\eta} : X \to [0,1]$ are mappings.

Definition 1.3

A binary fuzzy graph of $BG^* = (V, E)$is a pair $BG = (A, B)$ where $A = \left(\mu_A^{\rho}, \mu_A^{\eta}\right)$ is a binary fuzzy set in V and $B = \left(\mu_B^{\rho}, \mu_B^{\eta}\right)$ is a binary fuzzy set in $V \times V$ such that $(\mu_B^{\rho}(\alpha\beta) \leq \mu_A^{\rho}(\alpha) \wedge \mu_A^{\rho}(\beta)$ for all $\alpha, \beta \in V \times V$, $(\mu_B^{\eta}(\alpha\beta) \leq \mu_A^{\eta}(\alpha) \wedge \mu_A^{\eta}(\beta)$for all$\alpha, \beta \in V \times V$, and$\mu_B^{\eta}(\alpha\beta) = \mu_B^{\rho}(\alpha\beta) = 0$ for all $\alpha, \beta \in V \times V - E$.

Definition 1.4

By an interval – valued binary fuzzy graph of $BG^* = (V, E)$ is a pair $BG = (A, B)$ where $A = \left[\left(\mu_{AIB}^{\rho N}, \mu_{AIB}^{\eta N}\right), \left(\mu_{AIB}^{\rho P}, \mu_{AIB}^{\eta P}\right)\right]$ is an interval – valued fuzzy set on V and $B = \left[\left(\mu_{BIB}^{\rho N}, \mu_{BIB}^{\eta N}\right), \left(\mu_{AIB}^{\rho P}, \mu_{BIB}^{\eta P}\right)\right]$ is an interval – valued fuzzy relation on E such that

$$\begin{cases} \mu_{BIB}^{\rho N}(xy) \leq \mu_{AIB}^{\rho N}(x) \wedge \mu_{AIB}^{\rho N}(y) \\ \mu_{BIB}^{\eta P}(xy) \leq \mu_{BIB}^{\eta P}(x) \wedge \mu_{BIB}^{\eta P}(y) \end{cases} for\ all\ x, y\ \in V \times V$$

$$\begin{cases} \mu_{BIB}^{\eta N}(xy) \geq \mu_{AIB}^{\eta N}(x) \vee \mu_{AIB}^{\eta N}(y) \\ \mu_{BIB}^{\eta P}(xy) \geq \mu_{BIB}^{\eta P}(x) \vee \mu_{BIB}^{\eta P}(y) \end{cases} for\ all\ x, y\ \in V \times V$$

and

$$\begin{cases} \mu_{BIB}^{\eta N}(xy) = \left(\mu_{BIB}^{\rho N}(xy)\right) = 0 \\ \mu_{BIB}^{\eta P}(xy) = \left(\mu_{BIB}^{\eta P}(xy)\right) = 0 \end{cases} for\ all\ x, y\ \in V \times V - E$$

Throughout this paper BG^* is a crisp graph and BG is an interval – valued binary fuzzy graph.

2. CARTESIAN PRODUCT ON INTERVAL – VALUED BINARY FUZZY GRAPHS

Definition 2.1

The Cartesian product $G_{1IB} \times G_{2IB}$ of two interval – valued binary fuzzy graphs $G_{1IB} = (A_1, B_1)$ and $G_{2IB} = (A_2, B_2)$ of the graphs $G_{1IB}^* = (V_1, E_1)$ and $G_{2IB}^* = (V_2, E_2)$ is defined as a pair $(A_1 \times A_2, B_1 \times B_2)$ such that

$$(i) \begin{cases} \left(\mu_{A1IB}^{\rho N} \times \mu_{A2IB}^{\rho N}\right)(x_1, x_2) = \left(\mu_{A1IB}^{\rho N}(x_1) \wedge \mu_{A2IB}^{\rho N}(x_2)\right) \\ \left(\mu_{A1IB}^{\eta N} \times \mu_{A2IB}^{\eta N}\right)(x_1, x_2) = \left(\mu_{A1IB}^{\eta N}(x_1) \wedge \mu_{A2IB}^{\eta N}(x_2)\right) \\ \left(\mu_{A1IB}^{\rho P} \times \mu_{A2IB}^{\rho P}\right)(x_1, x_2) = \left(\mu_{A1IB}^{\rho P}(x_1) \wedge \mu_{A2IB}^{\rho P}(x_2)\right) \\ \left(\mu_{A1IB}^{\eta P} \times \mu_{A2IB}^{\eta P}\right)(x_1, x_2) = \left(\mu_{A1IB}^{\eta P}(x_1) \wedge \mu_{A2IB}^{\eta P}(x_2)\right) \end{cases}$$

for all $(x_1, x_2) \in V$

$$(ii) \begin{cases} \left(\mu_{B1IB}^{\rho N} \times \mu_{B2IB}^{\rho N}\right)(x, x_2)(x, y_2) = \left(\mu_{A1IB}^{\rho N}(x) \wedge \mu_{B2IB}^{\rho N}(x_2, y_2)\right) \\ \left(\mu_{B1IB}^{\eta N} \times \mu_{B2IB}^{\eta N}\right)(x, x_2)(x, y_2) = \left(\mu_{A1IB}^{\eta N}(x) \vee \mu_{BIB}^{\eta N}(x_2, y_2)\right) \\ \left(\mu_{B1IB}^{\rho P} \times \mu_{B2IB}^{\rho P}\right)(x, x_2)(x, y_2) = \left(\mu_{A1IB}^{\rho P}(x) \wedge \mu_{B2IB}^{\rho P}(x_2, y_2)\right) \\ \left(\mu_{B1IB}^{\eta P} \times \mu_{B2IB}^{\eta P}\right)(x, x_2)(x, y_2) = \left(\mu_{A1IB}^{\eta P}(x) \vee \mu_{BIB}^{\eta P}(x_2, y_2)\right) \end{cases}$$

$$(iii) \begin{cases} \left(\mu_{B1IB}^{\rho N} \times \mu_{B2IB}^{\rho N}\right)(x_1, z)(x_1, z) = \left(\mu_{BIB}^{\rho N}(x_2, y_2) \wedge \mu_{A2IB}^{\rho N}(z)\right) \\ \left(\mu_{B1IB}^{\eta N} \times \mu_{B2IB}^{\eta N}\right)(x_1, z)(x_1, z) = \left(\mu_{A1IB}^{\eta N}(x_1, y_1) \vee \mu_{BIB}^{\eta N}(z)\right) \\ \left(\mu_{B1IB}^{\rho P} \times \mu_{B2IB}^{\rho P}\right)(x_1, z)(x_1, z) = \left(\mu_{BIB}^{\rho P}(x_2, y_2) \wedge \mu_{BIB}^{\rho P}(z)\right) \\ \left(\mu_{B1IB}^{\eta P} \times \mu_{B2IB}^{\eta P}\right)(x_1, z)(x_1, z) = \left(\mu_{A1IB}^{\eta P}(x_1, y_1) \vee \mu_{BIB}^{\eta P}(z)\right) \end{cases}$$

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Theorem 2.2

The Cartesian product $G_{1IB} \times G_{2IB} = (A_1 \times A_2, B_1 \times B_2)$of two interval – valued binary fuzzy graphs of the graphs G_{1IB}^* and G_{2IB}^* is an interval – valued binary fuzzy graph of $G_{1IB}^* \times G_{2IB}^*$.

Proof

We verify only conditions for $B_{1IB} \times B_{2IB}$ because conditions for $A_{1IB} \times A_{2IB}$ are obvious.

Let $x \in V_1, x_2y_2 \in E_2$ then,

$$\left(\mu_{B1IB}^{\rho N} \times \mu_{B2IB}^{\rho N}\right)\left((x, x_1), (x, y_2)\right) = \min\left(\mu_{A1IB}^{\rho N}(x), \mu_{B2IB}^{\rho N}(x_2, y_2)\right)$$

$$\leq \min\left(\mu_{A1IB}^{\rho N}(x), \min\left(\mu_{A2IB}^{\rho N}(x_2), \mu_{A2IB}^{\rho N}(y_2)\right)\right)$$

$$= \min\left(\min\left(\mu_{A1IB}^{\rho N}(x), \mu_{A2IB}^{\rho N}(x_2)\right), \min\left(\mu_{A2IB}^{\rho N}(x), \mu_{A2IB}^{\rho N}(y_2)\right)\right)$$

$$= \min\left(\left(\mu_{A1IB}^{\rho N} \times \mu_{A2IB}^{\rho N}\right)(x, x_2), \left(\mu_{A1IB}^{\rho N} \times \mu_{A2IB}^{\rho N}\right)(x, y_2)\right)$$

$$\left(\mu_{B1IB}^{\eta N} \times \mu_{B2IB}^{\eta N}\right)\left((x, x_2), (x, y_2)\right) = \min\left(\mu_{A1IB}^{\eta N}(x), \mu_{B2IB}^{\eta N}(x_2, y_2)\right)$$

$$\leq \min\left(\mu_{A1IB}^{\eta N}(x), \min\left(\mu_{A2IB}^{\eta N}(x_2), \mu_{A2IB}^{\eta N}(y_2)\right)\right)$$

$$= \min\left(\min\left(\mu_{A1IB}^{\eta N}(x), \mu_{A2IB}^{\eta N}(x_2)\right), \min\left(\mu_{A2IB}^{\eta N}(x), \mu_{A2IB}^{\eta N}(y_2)\right)\right)$$

$$= \min\left(\left(\mu_{A1IB}^{\eta N} \times \mu_{A2IB}^{\eta N}\right)(x, x_2), \left(\mu_{A1IB}^{\eta N} \times \mu_{A2IB}^{\eta N}\right)(x, y_2)\right)$$

$$\left(\mu_{B1IB}^{\rho P} \times \mu_{B2IB}^{\rho P}\right)\left((x, x_2), (x, y_2)\right) = \min\left(\mu_{A1IB}^{\eta P}(x), \mu_{B2IB}^{\eta P}(x_2, y_2)\right)$$

$$\leq \min\left(\mu_{A1IB}^{\rho P}(x), \min\left(\mu_{A2IB}^{\rho P}(x_2), \mu_{A2IB}^{\rho P}(y_2)\right)\right)$$

$$= \min\left(\min\left(\mu_{A1IB}^{\rho P}(x), \mu_{A2IB}^{\rho P}(x_2)\right), \min\left(\mu_{A1IB}^{\rho P}(x), \mu_{A2IB}^{\rho P}(y_2)\right)\right)$$

$$= \min\left(\left(\mu_{A1IB}^{\rho P} \times \mu_{A2IB}^{\rho P}\right)(x, x_2), \left(\mu_{A1IB}^{\rho P} \times \mu_{A2IB}^{\rho P}\right)(x, y_2)\right)$$

$$\left(\mu_{B1IB}^{\eta P} \times \mu_{B2IB}^{\eta P}\right)\left((x, x_2), (x, y_2)\right) = \min\left(\mu_{A1IB}^{\eta P}(x), \mu_{B2IB}^{\eta P}(x_2, y_2)\right)$$

$$\leq \min\left(\mu_{A1IB}^{\eta P}(x), \min\left(\mu_{A2IB}^{\eta P}(x_2), \mu_{A2IB}^{\rho P}(y_2)\right)\right)$$

$$= \min\left(\min\left(\mu_{A1IB}^{\eta P}(x), \mu_{A2IB}^{\eta P}(x_2)\right), \min\left(\mu_{A1IB}^{\eta P}(x), \mu_{A2IB}^{\eta P}(y_2)\right)\right)$$

$$= \min\left(\left(\mu_{A1IB}^{\eta P} \times \mu_{A2IB}^{\eta P}\right)(x, x_2), \left(\mu_{A1IB}^{\eta P} \times \mu_{A2IB}^{\eta P}\right)(x, y_2)\right)$$

Similarly for $z \in V_2$ and $x_1y_1 \in E_1$ we have

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$$\left(\mu_{B1IB}^{\rho N}\times\mu_{B2IB}^{\rho N}\right)\left((x_1,z),(y_1,z)\right)=\min\left(\mu_{B1IB}^{\rho N}(x_1,y_1),\mu_{A2IB}^{\rho N}(z)\right)$$

$$\leq\min\left(\min\left(\mu_{A1IB}^{\rho N}(x_1),\mu_{A2IB}^{\rho N}(y_1)\right),\mu_{A2IB}^{\rho N}(z),\right)$$

$$=\min\left(\min\left(\mu_{A1IB}^{\rho N}(x_1),\mu_{A2IB}^{\rho N}(z)\right),\min\left(\mu_{A1IB}^{\rho N}(y_1),\mu_{A2IB}^{\rho N}(z)\right)\right)$$

$$=\min\left(\left(\mu_{A1IB}^{\rho N}\times\mu_{A2IB}^{\rho N}\right)(x_1,z),\left(\mu_{A1IB}^{\rho N}\times\mu_{A2IB}^{\rho N}\right)(y_1,z)\right)$$

$$\left(\mu_{B1IB}^{\eta N}\times\mu_{B2IB}^{\eta N}\right)\left((x_1,z),(y_1,z)\right)=\min\left(\mu_{B1IB}^{\eta N}(x,y_1),\mu_{A2IB}^{\eta N}(z)\right)$$

$$\leq\min\left(\min\left(\mu_{A1IB}^{\eta N}(x_1),\mu_{A1IB}^{\eta N}(y_1)\right),\mu_{A2IB}^{\eta N}(z),\right)$$

$$=\min\left(\min\left(\mu_{A1IB}^{\eta N}(x_1),\mu_{A2IB}^{\eta N}(z)\right),\min\left(\mu_{A1IB}^{\eta N}(y_1),\mu_{A2IB}^{\eta N}(z)\right)\right)$$

$$=\min\left(\left(\mu_{A1IB}^{\eta N}\times\mu_{A2IB}^{\eta N}\right)(x_1,z),\left(\mu_{A1IB}^{\eta N}\times\mu_{A2IB}^{\eta N}\right)(y_1,z)\right)$$

$$\left(\mu_{B1IB}^{\rho P}\times\mu_{B2IB}^{\rho P}\right)\left((x_1,z),(y_1,z)\right)=\min\left(\mu_{B1IB}^{\rho P}(x_1,y_1),\mu_{A2IB}^{\rho P}(z)\right)$$

$$\leq\min\left(\min\left(\mu_{A1IB}^{\rho P}(x_1),\mu_{A1IB}^{\rho P}(y_1)\right),\mu_{A2IB}^{\rho P}(z),\right)$$

$$=\min\left(\min\left(\mu_{A1IB}^{\rho P}(x_1),\mu_{A2IB}^{\rho P}(z)\right),\min\left(\mu_{A1IB}^{\rho P}(y_1),\mu_{A2IB}^{\rho P}(z)\right)\right)$$

$$=\min\left(\left(\mu_{A1IB}^{\rho P}\times\mu_{A2IB}^{\rho P}\right)(x_1,z),\left(\mu_{A1IB}^{\rho P}\times\mu_{A2IB}^{\rho P}\right)(y_1,z)\right)$$

$$\left(\mu_{B1IB}^{\eta P}\times\mu_{B2IB}^{\eta P}\right)\left((x_1,z),(y_1,z)\right)=\min\left(\mu_{B1IB}^{\eta P}(x_1,y_1),\mu_{A2IB}^{\eta P}(z)\right)$$

$$\leq\min\left(\min\left(\mu_{A1IB}^{\eta P}(x_1),\mu_{A1IB}^{\eta P}(y_1)\right),\mu_{A2IB}^{\eta P}(z),\right)$$

$$=\min\left(\min\left(\mu_{A1IB}^{\eta P}(x_1),\mu_{A2IB}^{\eta P}(z)\right),\min\left(\mu_{A1IB}^{\eta P}(y_1),\mu_{A2IB}^{\eta P}(z)\right)\right)$$

$$=\min\left(\left(\mu_{A1IB}^{\eta P}\times\mu_{A2IB}^{\eta P}\right)(x_1,z),\left(\mu_{A1IB}^{\eta P}\times\mu_{A2IB}^{\eta P}\right)(y_1,z)\right)$$

3. COMPOSITION OF INTERVAL – VALUED BINARY FUZZY GRAPHS

Definition 3.1

The composition of $G_{1IB}[G_{2IB}]=(A_1\circ A_2,B_1\circ B_2)$ of two interval – valued binary fuzzy graphs G_{1IB} and $G_{2IB}$of the graphs G_{1IB}^{*} and G_{2IB}^{*} is defined as follows

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$$(i)\begin{cases}\left(\mu^{\rho N}_{A1IB}\circ\mu^{\rho N}_{A2IB}\right)(x_1,x_2)=\left(\mu^{\rho N}_{A1IB}(x_1)\wedge\mu^{\rho N}_{A2IB}(x_2)\right)\\ \left(\mu^{\eta N}_{A1IB}\circ\mu^{\eta N}_{A2IB}\right)(x_1,x_2)=\left(\mu^{\eta N}_{A1IB}(x_1)\wedge\mu^{\eta N}_{A2IB}(x_2)\right)\\ \left(\mu^{\rho P}_{A1IB}\circ\mu^{\rho P}_{A2IB}\right)(x_1,x_2)=\left(\mu^{\rho P}_{A1IB}(x_1)\wedge\mu^{\rho P}_{A2IB}(x_2)\right)\\ \left(\mu^{\eta P}_{A1IB}\circ\mu^{\eta P}_{A2IB}\right)(x_1,x_2)=\left(\mu^{\eta P}_{A1IB}(x_1)\wedge\mu^{\eta P}_{A2IB}(x_2)\right)\end{cases}$$

$$for\ all\ (x_1,x_2)\in V$$

$$(ii)\begin{cases}\left(\mu^{\rho N}_{B1IB}\circ\mu^{\rho N}_{B2IB}\right)(x_1,x_2)(x_1,y_2)=\left(\mu^{\rho N}_{A1IB}(x_1)\wedge\mu^{\rho N}_{B2IB}(x_2,y_2)\right)\\ \left(\mu^{\eta N}_{B1IB}\circ\mu^{\eta N}_{B2IB}\right)(x_1,x_2)(x_1,y_2)=\left(\mu^{\eta N}_{A1IB}(x_1)\vee\mu^{\eta N}_{BIB}(x_2,y_2)\right)\\ \left(\mu^{\rho P}_{B1IB}\circ\mu^{\rho P}_{B2IB}\right)(x_1,x_2)(x_1,y_2)=\left(\mu^{\rho P}_{A1IB}(x_1)\wedge\mu^{\rho P}_{B2IB}(x_2,y_2)\right)\\ \left(\mu^{\eta P}_{B1IB}\circ\mu^{\eta P}_{B2IB}\right)(x_1,x_2)(x_1,y_2)=\left(\mu^{\eta P}_{A1IB}(x_1)\vee\mu^{\eta P}_{B2IB}(x_2,y_2)\right)\end{cases}$$

$$for\ all\ x_1\in V_2 and\ x_2,y_2\in E_2$$

$$(iii)\begin{cases}\left(\mu^{\rho N}_{B1IB}\circ\mu^{\rho N}_{B2IB}\right)(x_1,z)(x_1,z)=\left(\mu^{\rho N}_{BIB}(x_2,y_2)\wedge\mu^{\rho N}_{A1IB}(z)\right)\\ \left(\mu^{\eta N}_{B1IB}\circ\mu^{\eta N}_{B2IB}\right)(x_1,z)(y_1,z)=\left(\mu^{\eta N}_{A1IB}(x_1,y_1)\vee\mu^{\eta N}_{BIB}(z)\right)\\ \left(\mu^{\rho P}_{B1IB}\circ\mu^{\rho P}_{B2IB}\right)(x_1,z)(x_1,z)=\left(\mu^{\rho P}_{BIB}(x_2,y_2)\wedge\mu^{\rho P}_{2IB}(z)\right)\\ \left(\mu^{\eta P}_{B1IB}\circ\mu^{\eta P}_{B2IB}\right)(x_1,z)(y_1,z)=\left(\mu^{\eta P}_{A1IB}(x_1,y_1)\vee\mu^{\eta P}_{BIB}(z)\right)\end{cases}$$

$$for\ all\ z\in V_2 and\ x_1,y_1\in E_1$$

$$(iv)\begin{cases}\left(\mu^{\rho N}_{B1IB}\circ\mu^{\rho N}_{B2IB}\right)(x_1,z)(y_1,z)=\left(\mu^{\rho N}_{BIB}(x_2,y_2)\wedge\mu^{\rho N}_{A2IB}(z)\right)\\ \left(\mu^{\eta N}_{B1IB}\circ\mu^{\eta N}_{B2IB}\right)(x_1,x_2)(y_1,y_2)=\left(\mu^{\rho N}_{A1IB}(x_2)\wedge\mu^{\rho N}_{A2IB}(y_2)\wedge\mu^{\rho N}_{B2IB}(x_1,x_2)\right)\\ \left(\mu^{\rho P}_{B1IB}\circ\mu^{\rho P}_{B2IB}\right)(x_1,z)(x_1,z)=\left(\mu^{\rho P}_{BIB}(x_2,y_2)\wedge\mu^{\rho P}_{2IB}(z)\right)\\ \left(\mu^{\eta P}_{B1IB}\circ\mu^{\eta P}_{B2IB}\right)(x_1,x_2)(y_1,y_2)=\left(\mu^{\rho P}_{A1IB}(x_2)\wedge\mu^{\rho P}_{A2IB}(y_2)\wedge\mu^{\rho P}_{B2IB}(x_1,x_2)\right)\end{cases}$$

$$for\ all\ (x_1,x_2)(y_1,y_2)\in E^{\circ}-E.$$

$$(v)\begin{cases}\left(\mu^{\eta N}_{B1IB}\circ\mu^{\eta N}_{B2IB}\right)(x_1,x_2)(\beta_1,\beta_2)=\left(\mu^{\eta N}_{A1IB}(x_2)\vee\mu^{\eta N}_{A2IB}(y_2)\vee\mu^{\eta N}_{B1IB}(x_1,y_2)\right)\\ \left(\mu^{\eta P}_{B1IB}\circ\mu^{\eta P}_{B2IB}\right)(x_1,x_2)(\beta_1,\beta_2)=\left(\mu^{\eta P}_{A1IB}(x_2)\vee\mu^{\eta P}_{A2IB}(y_2)\vee\mu^{\eta P}_{B1IB}(x_1,x_2)\right)\end{cases}$$

$$for\ all\ (x_1,x_2)(y_1,y_2)\in E^{\circ}-E.$$

Theorem 3.2

The composition $G_{1IB}[G_{2IB}]$ of interval – valued binary fuzzy graphs G_{1IB} and G_{2IB} of G^*_{1IB} and G^*_{2IB} is an interval – valued binary fuzzy graph of $G^*_{1IB}[G^*_{2IB}]$.

Proof

Similarly as in the previous proof we verify the conditions for $B_{1IB}\circ B_{2IB}$ only.

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In the case $x \in V_1, x_2y_2 \in E_2$, according to (ii) we obtain

$$\left(\mu^{\rho N}_{B1IB} \circ \mu^{\rho N}_{B2IB}\right)\left((x, x_2)(x, y_2)\right) = \min\left(\mu^{\rho N}_{A1IB}(x), \mu^{\rho N}_{B2IB}(x_2, y_2)\right)$$

$$\leq \min\left(\mu^{\rho N}_{A1IB}(x), \min\left(\mu^{\rho N}_{A2IB}(x_2), \mu^{\rho N}_{A2IB}(y_2)\right)\right)$$

$$= \min\left(\min(\mu^{\rho N}_{A1IB}(x)\,, \mu^{\rho N}_{A2IB}(x_2)) \min\left(\mu^{\rho N}_{A2IB}(x), \mu^{\rho N}_{A2IB}(y_2)\right)\right)$$

$$= \min\left(\left(\mu^{\rho N}_{A1IB} \circ \mu^{\rho N}_{A2IB}\right)(x, x_2), \left(\mu^{\rho N}_{A1IB} \circ \mu^{\rho N}_{A2IB}\right)(x, y_2)\right)$$

$$\left(\mu^{\eta N}_{B1IB} \circ \mu^{\eta N}_{B2IB}\right)\left((x, x_2), (x, y_2)\right) = \min\left(\mu^{\eta N}_{A1IB}(x), \mu^{\eta N}_{B2IB}(x_2, y_2)\right)$$

$$\leq \min\left(\mu^{\eta N}_{A1IB}(x), \min\left(\mu^{\eta N}_{A2IB}(x_2), \mu^{\eta N}_{A2IB}(y_2)\right)\right)$$

$$= \min\left(\min\left(\mu^{\eta N}_{A1IB}(x), \mu^{\eta N}_{A2IB}(x_2)\right), \min\left(\mu^{\eta N}_{A1IB}(x), \mu^{\eta N}_{A2IB}(y_2)\right)\right)$$

$$= \min\left(\left(\mu^{\eta N}_{A1IB} \circ \mu^{\eta N}_{A2IB}\right)(x, x_2), \left(\mu^{\eta N}_{A1IB} \circ \mu^{\eta N}_{A2IB}\right)(x, y_2)\right)$$

$$\left(\mu^{\rho P}_{B1IB} \circ \mu^{\rho P}_{B2IB}\right)\left((x, x_2), (x, y_2)\right) = \min\left(\mu^{\rho P}_{A1IB}(x), \mu^{\rho P}_{B2IB}(x_2, y_2)\right)$$

$$\leq \min\left(\mu^{\rho P}_{A1IB}(x), \min\left(\mu^{\rho P}_{A2IB}(x_2), \mu^{\rho P}_{A2IB}(y_2)\right)\right)$$

$$= \min\left(\min\left(\mu^{\rho P}_{A1IB}(x), \mu^{\rho P}_{A2IB}(x_2)\right), \min\left(\mu^{\rho P}_{A1IB}(x), \mu^{\rho P}_{A2IB}(y_2)\right)\right)$$

$$= \min\left(\left(\mu^{\rho P}_{A1IB} \circ \mu^{\rho P}_{A2IB}\right)(x, x_2), \left(\mu^{\rho P}_{A1IB} \circ \mu^{\rho P}_{A2IB}\right)(x, y_2)\right)$$

$$\left(\mu^{\eta P}_{B1IB} \circ \mu^{\eta P}_{B2IB}\right)\left((x_1, x_2), (x, y_2)\right) = \min\left(\mu^{\eta P}_{A1IB}(x), \mu^{\eta P}_{B2IB}(x_2, y_2)\right)$$

$$\leq \min\left(\mu^{\eta P}_{A1IB}(x), \min\left(\mu^{\eta P}_{A2IB}(x_2), \mu^{\eta P}_{A2IB}(y_2)\right)\right)$$

$$= \min\left(\min\left(\mu^{\eta P}_{A1IB}(x), \mu^{\eta P}_{A2IB}(x_2)\right), \min\left(\mu^{\eta P}_{A2IB}(x), \mu^{\eta P}_{A2IB}(y_2)\right)\right)$$

$$= \min\left(\left(\mu^{\eta P}_{A1IB} \circ \mu^{\eta P}_{A2IB}\right)(x_1, x_2), \left(\mu^{\eta P}_{A1IB} \circ \mu^{\eta P}_{A2IB}\right)(x, y_2)\right)$$

In the case$z \in V_2, x_1y_1 \in E_1$ the proof is similar.

CONCLUSION

In this study we introduced the Cartesian product, Composition on Interval – valued Binary fuzzy graphs and some theorems are discussed.

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