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A Study on Bipolar Cubic Fuzzy Transition Matrices

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ABSTRACT

In this paper we introduce the concepts of internal cubic fuzzy graphs, external cubic fuzzy graphs and the notion of cubic bipolar fuzzy transition matrices, Internal and External bipolar fuzzy transition matrices. We discuss some results on internal cubic fuzzy graphs and external cubic fuzzy graphs and provide some results related with cubic bipolar fuzzy transition matrix.

Keywords: internal cubic fuzzy graphs,external cubic fuzzy graph, cubic bipolar fuzzy transition matrix.

INTRODUCTION

In 1975 Rosenfeld [16] introduced fuzzy graphs based on fuzzy set.Fuzzy graph theory plays essential roles in various discipines including information theory, neural networks,clustering problems and control theory,etc.Fuzzy models is more compatiable to the systemin compare with classical mode.Bhattacharya [6] gave some remarks on fuzzy graphs.Some operations on fuzzy graphs were introduced by mordeson and peng[9].Akram et al. has introduced several new concepts including bipolar fuzzy graphs,regular bipolar fuzzy graphs, irregular bipolar fuzzy graphs etc.Pal et al. [9,10] and Pramanik et al [12] added some useful results to the theory of interval – valued fuzzy graphs Parvathi et al [11] provided some different operations on intuitionistic fuzzy graphs and so on and pal [12] studied product of characteristic fuzzy graphs Jun et al [8] generalizations of fuzzy sets of cubic sets. They developed cubic set theory in many directions and for more detail about cubic sets one can see [7,8]. Kang and Kim [7] studied mappings of cubic sets. Sheikh Rashid, Naveed Yagoob, Muhammad Akram, Auhammad Gulistan studied certain concepts of cubic graphs. In this paper, we present new concepts like internal,external cubic fuzzy graph and bipolar cubic fuzzy transition matrix.An internal and external cubic bipolar fuzzy transition set are discussed. And also provides the cubic fuzzy graphs with results.

1. BASIC DEFINITIONS

Definition 1.1

A fuzzy graph with a non-empty finite set V as the underlying set is a pair $G^* = (\sigma, \mu)$, where $\sigma: V \to [0,1]$ is a fuzzy subset of $V, \mu: V \times V \to [0,1]$ is a symmetric fuzzy relation on the fuzzy subset σ, such that $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$, for all $u, v \in$V, where $\wedge$ indicates the minimum among $\sigma(u)$ and $\sigma(v)$.

Definition 1.2

Let $A_{2\times2}$be the set of all 2×2 transition matrices over the fuzzy algebra τ. The operations $(+,.)$ are defined on $A_{2\times2}$ as follows.

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If $C = (c_{ij})$ and $D = (d_{ij}) \in A_{2\times 2}$ with $c_{ij}, d_{ij} \in M$.

Then,

(i) $C + D = \begin{cases} c_{ij} + d_{ij}, & \text{if } c_{ij} + d_{ij} < 1 \\ c_{ij} + d_{ij} - 1, & \text{if } c_{ij} + d_{ij} > 1 \end{cases}$

(ii) $C + D = \begin{cases} [1\ 0]_{1\times 2}, \text{if } c_{ij} + d_{ij} = 1 \text{ in row } 1 \\ [0\ 1]_{1\times 2}, \text{if } c_{ij} + d_{ij} = 1 \text{ in row } 2 \end{cases}$

(iii) For any Scalar $\lambda \in (0,1), \lambda_A =$ fraction part of $(10\ \lambda c_{ij})$

The system $A_{2\times 2}$ together with these operations of component wise transition addition and multiplication is called as fuzzy transition matrix. Then the fuzzy transition matrix denoted by $(A_{2\times 2}, \tau)$.

Definition 1.3

Let $(A_{2\times 2}, \tau)$ be a fuzzy transition vector space. We take Inner product of $C = (c_{ij})$ and $D = (d_{ij}) \in A_{2\times 2}$ is defined as $< C, D > = \bigoplus < c_{ij}, d_{ij} >$ which satisfies the following four conditions:

(i) $< C, D > = < D, C >$
(ii) $< \lambda C, D > = \lambda < C, D >, \lambda \in (0,1)$
(iii) $< C + D, E > = < C, E > + < D, E >, E \in A_{2\times 2}$
(iv) $< C, C > = 0$, if and only if the rows are identical.

Then the system $A_{2\times 2}$ together with this inner Product is called as fuzzy transition inner product space.

Definition 1.4

Let $(A_{2\times 2}, \tau)$ be a fuzzy transition vector space. The norm for every element $C \in A_{2\times 2}$ is denoted by $||C|| = < C, C >$ which satisfies

(i) $0 \leq ||C|| \leq 1, ||C|| = 0$ if and only if the rows are identical.
(ii) $||\lambda C|| = \lambda ||C||, \lambda \in (0,1)$
(iii) $||C + D|| = ||C|| + ||D||, D \in A_{2\times 2}$.

Then the system $A_{2\times 2}$ together with this norm is called fuzzy transition Normed linear space.

2. BIPOLAR CUBIC FUZZY TRANSITION

Definition 2.1

Let V be a non-empty set. By a cubic bipolar fuzzy transition set $C = < [C^P, \lambda^P], [C^N, \lambda^N] >$ in V is said to be an Internal cubic bipolar fuzzy transition set if

$$c_{11}\{< [S_\sigma^{Px}(u), S_\sigma^{Py}(u)], [S_\sigma^{Nx}(u), S_\sigma^{Ny}(u)], S_\sigma^P(u), S_\sigma^N(u) >\}$$

$$\leq \lambda\{< [S_\sigma^{Px}(u), S_\sigma^{Py}(u)], [S_\sigma^{Nx}(u), S_\sigma^{Ny}(u)], S_\sigma^P(u), S_\sigma^N(u) >\}$$

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$$\leq c_{22}\{< [S_\sigma^{Px}(u), S_\sigma^{Py}(u)], [S_\sigma^{Nx}(u), S_\sigma^{Ny}(u)], S_\sigma^{P}(u), S_\sigma^{N}(u) >\} \ \forall\ u \in V.$$

Definition 2.2

Let V be a non-empty set. By a cubic bipolar fuzzy transition set $C =< [C^P, \lambda^P], [C^N, \lambda^N] >$ in V is said to be an External cubic bipolar fuzzy transition set if

$\lambda\{< [S_\sigma^{Px}(u), S_\sigma^{Py}(u)], [S_\sigma^{Nx}(u), S_\sigma^{Ny}(u)], S_\sigma^{P}(u), S_\sigma^{N}(u) >\} \notin [c_{11}, c_{12}]$and

$$\lambda\{< [S_\sigma^{Px}(u), S_\sigma^{Py}(u)], [S_\sigma^{Nx}(u), S_\sigma^{Ny}(u)], S_\sigma^{P}(u), S_\sigma^{N}(u) >\} \notin [c_{21}, c_{22}] \forall\ u \in V.$$

Example 2.3

Let $M =$

$$\begin{bmatrix} < [0.1,0.1], [-0.1,-0.1], [0.1,-0.1] >< [0.9,0.9], [-0.9,-0.9], [0.9,-0.9] > \\ < [0.3,0.3], [-0.3,-0.3], [0.3,-0.3] >< [0.7,0.7], [-0.7,-0.7], [0.7,-0.7] > \end{bmatrix}$$

be the bipolar fuzzy transition matrix

$$||M|| =< M, M >$$

$$= \begin{bmatrix} < [0.1,0.1], [-0.1,-0.1], [0.1,-0.1] >< [0.9,0.9], [-0.9,-0.9], [0.9,-0.9] > \\ < [0.3,0.3], [-0.3,-0.3], [0.3,-0.3] >< [0.7,0.7], [-0.7,-0.7], [0.7,-0.7] > \end{bmatrix}$$

$$\begin{bmatrix} < [0.1,0.1], [-0.1,-0.1], [0.1,-0.1] >< [0.9,0.9], [-0.9,-0.9], [0.9,-0.9] > \\ < [0.3,0.3], [-0.3,-0.3], [0.3,-0.3] >< [0.7,0.7], [-0.7,-0.7], [0.7,-0.7] > \end{bmatrix}$$

$$=< [0.1,0.1], [-0.1,-0.1], [0.1,-0.1] > \oplus$$

$$< [0.7,0.7], [-0.7,-0.7], [0.7,-0.7] > \oplus$$

$$< [0.7,0.7], [-0.7,-0.7], [0.7,-0.7] > \oplus$$

$$< [0.9,0.9], [-0.9,-0.9], [0.9,-0.9] >$$

$$= (< [0.4,0.4], [-0.4,-0.4], [0.4,-0.4] >) \in$$

$$(< [0.1,0.1], [-0.1,-0.1], [0.1,-0.1] >< [0.9,0.9], [-0.9,-0.9], [0.9,-0.9] >)$$

$$= (< [0.4,0.4], [-0.4,-0.4], [0.4,-0.4] >) \in$$

$$(< [0.3,0.3], [-0.3,-0.3], [0.3,-0.3] >< [0.7,0.7], [-0.7,-0.7], [0.7,-0.7] >)$$

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Example 2.4

Let $M =$

$$\begin{bmatrix} < [0.1,0.1], [-0.1,-0.1], [0.1,-0.1] >< [0.9,0.9], [-0.9,-0.9], [0.9,-0.9] > \\ < [0.3,0.3], [-0.3,-0.3], [0.3,-0.3] >< [0.7,0.7], [-0.7,-0.7], [0.7,-0.7] > \end{bmatrix}$$

be the bipolar fuzzy transition matrix

$$\left||M|\right| =< M, M >$$

$$= \begin{bmatrix} < [0.1,0.1], [-0.1,-0.1], [0.1,-0.1] >< [0.9,0.9], [-0.9,-0.9], [0.9,-0.9] > \\ < [0.3,0.3], [-0.3,-0.3], [0.3,-0.3] >< [0.7,0.7], [-0.7,-0.7], [0.7,-0.7] > \end{bmatrix}$$

$$\begin{bmatrix} < [0.1,0.1], [-0.1,-0.1], [0.1,-0.1] >< [0.9,0.9], [-0.9,-0.9], [0.9,-0.9] > \\ < [0.3,0.3], [-0.3,-0.3], [0.3,-0.3] >< [0.7,0.7], [-0.7,-0.7], [0.7,-0.7] > \end{bmatrix}$$

$$=< [0.9,0.9], [-0.9,-0.9], [0.9,-0.9] > \oplus$$

$$< [0.8,0.8], [-0.8,-0.8], [0.8,-0.8] > \oplus$$

$$< [0.8,0.8], [-0.8,-0.8], [0.8,-0.8] > \oplus$$

$$< [0.6,0.6], [-0.6,-0.6], [0.6,-0.6] >$$

$$=< [0.1,0.1], [-0.1,-0.1], [0.1,-0.1] >$$

$$= \lambda\left\{< \left[S_\sigma^{Px}(u), S_\sigma^{Py}(u)\right], \left[S_\sigma^{Nx}(u), S_\sigma^{Ny}(u)\right], S_\sigma^{P}(u), S_\sigma^{N}(u) >\right\}$$

Here,

$$< [0.1,0.1], [-0.1,-0.1], [0.1,-0.1] > \notin (< [0.3,0.3], [-0.3,-0.3], [0.3,-0.3] >< [0.7,0.7], [-0.7,-0.7], [0.7,-0.7] >)$$

and

$$[0.1,0.1], [-0.1,-0.1], [0.1,-0.1] > \notin (< [0.4,0.4], [-0.4,-0.4], [0.4,-0.4] >< [0.6,0.6], [-0.6,-0.6], [0.6,-0.6] >)$$

Thus M is an External cubic bipolar fuzzy transition matrix.

Theorem 2.5

A family of Internal cubic fuzzy graphs is $\{G_i =< \sigma_i , \mu_i >/ \ i \in I\}$ .Then$_{\mathrm{i}\in \mathrm{I}}^{\cup}$ $X \ G_i$ is aninternal cubic fuzzy graph.
Proof:

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Let G_i is an internal cubic fuzzy graph.So we have

$$< \left(S_{A\sigma}{}^{x^N}(v), S_{A\sigma}{}^{y^N}(v)\right), S_{A\sigma}^N(v) > \leq < \left(S_{B\sigma}(v),\ S_{B\sigma}(v)\right), S_{B\sigma}(v) >$$

$\leq< \left((S_{A\sigma}{}^{x^p}(v), S_{A\sigma}{}^{y^p}(v)\right)\ ,\ S_{A\sigma}^P(v)\ >$And

$$< \left(S_{A\mu}{}^{x^N}(e), S_{A\mu}{}^{y^N}(e)\right), S_{A\mu}^N(e) > \leq < \left(S_{B\mu}(e), S_{B\mu}(e)\right), S_{B\mu}(e) >$$

$$\leq< \left(S_{A\mu}{}^{x^p}(e), S_{A\mu}{}^{y^p}(e)\right), S_{A\mu}^P(e) >$$

Which implies that

$(\cup_{i\in I}< \left(S_{A\sigma}{}^{x^N}(v), S_{A\sigma}{}^{y^N}(v)\right), S_{A\sigma}^N(v) > \left(\vee_{i\epsilon I} < \left(S_{B\sigma}(v), S_{B\sigma}(v)\right), S_{B\sigma}(v) >\right)\ \leq \Big(\Big(U_{i\in I} <$

$(S_{A\sigma}{}^{x^p}(v), S_{A\sigma}{}^{y^p}(v)\Big), S_{A\sigma}^P(v) >\Big)$And

$$\left(U_{i\in I} < \left(S_{A\mu}{}^{x^N}(e), S_{A\mu}{}^{y^N}(e)\right), S_{A\mu}^N(e) >\right) \leq \left(\bigvee_{i\epsilon I} < (S_{B\mu}(v), S_{B\mu}(v)), S_{B\mu}(v) >\right)$$

$$\leq\ \left(\cup_{i\in I}< \left(S_{A\mu}{}^{x^P}(e), S_{A\mu}{}^{y^P}(e)\right), S_{A\mu}^P(e) >\right)$$

Hence $\underset{i\in I}{\cup} X\ G_i$ is an internal cubic fuzzy graph.

Theorem 2.6

Let$G\ =\ (\sigma, \mu\ )$be a cubic fuzzy graph ofΩ^*If$G\ =\ (\sigma, \mu\ )$is both an internal cubic fuzzy graph and an external cubic fuzzy graph.

Then

$$\left(\left(S_{B\sigma}^x(v_i), S_{B\sigma}^y(v_i)\right), S_{B\sigma}(v_i)\right) \in\ U\left\{< \left(s_{A\sigma}{}^x(v_i\ ), s_{A\sigma}{}^y(v_i\ )\right), S_{A\sigma}(v_i) >\right\} \cup L\left\{< \left(s_{A\sigma}{}^x(v_i\ ), s_{A\sigma}{}^y(v_i\ )\right), S_{A\sigma}(v_i) >\right\}$$

And

$\left(\left(S_{B\mu}^x(e_i), S_{B\mu}^y(e_i)\right), S_{B\mu}(e_i)\right) \in\ U\ \left\{< \left(s_{B\mu}{}^x(e_i\ ), s_{B\mu}{}^y(e_i\ )\right), S_{B\mu}(e_i) >\right\} \cup L\ \left\{< \left(s_{B\mu}{}^x(e_i\ ), s_{B\mu}{}^y(e_i\ )\right), S_{B\mu}(e_i) >\right\}$for all $v_i \in V$and $e_i \in E\ \subseteq\ V \times V$.

Where

$$U\left\{< \left(s_{A\sigma}{}^x(v_i\ ), s_{A\sigma}{}^y(v_i\ )\right), S_{A\sigma}(v_i) >\right\} = \left\{< \left(S_{A\sigma}^{x^P}(v_i), S_{A\sigma}^{x^P}(v_i)\right), S_{A\sigma}^P(v_i) > \backslash v_i \in P\right\},$$

$$L\left\{< \left(s_{A\sigma}{}^x(v_i\ ), s_{A\sigma}{}^y(v_i\ )\right), S_{A\sigma}(v_i) >\right\}\ = \left\{< \left(S_{A\sigma}^{x^P}(v_i), S_{A\sigma}^{x^P}(v_i)\right),\ S_{A\sigma}^P(v_i) > \backslash v_i \in P\right\}$$

And

$$U\ \left\{< \left(s_{A\mu}{}^x(e_i\ ), s_{A\mu}{}^y(e_i\ )\right), S_{A\mu}(e_i) >\right\}\ =\ \left\{< \left(S_{A\mu}^{x^P}(e_i), S_{A\mu}^{x^P}(e_i)\right), S_{A\mu}^P(e_i) > \backslash e_i \in P\right\},$$

$$L\ \left\{< \left(s_{A\mu}{}^x(e_i\ ), s_{A\mu}{}^y(e_i\ )\right), S_{A\mu}(e_i) >\right\}\ =\ \left\{< \left(S_{A\mu}^{x^N}(e_i), S_{A\mu}^{x^N}(e_i)\right), S_{A\mu}^N(e_i) > \backslash e_i \in P\right\}$$

Proof

Assume that $G\ =\ (\sigma\ , \mu\ )$is both an internal cubic fuzzy graph and an external cubic fuzzy graph. Then by definition we have

$$\left(\left(S_{B\sigma}^{x}(v_i), S_{B\sigma}^{y}(v_i)\right), S_{B\sigma}(v_i)\right) \in \left\{< \left(S_{A\sigma}^{x^N}(v_i), S_{A\sigma}^{x^N}(v_i)\right), S_{A\sigma}^{N}(v_i) >, < \left(S_{A\sigma}^{x^N}(v_i), S_{A\sigma}^{x^N}(v_i)\right), S_{A\sigma}^{N}(v_i) >\right\},$$

$$\left(\left(S_{B\mu}^{x}(e_i), S_{B\mu}^{y}(e_i)\right), S_{B\mu}(e_i)\right) \in \left\{< \left(S_{A\mu}^{x^N}(e_i), S_{A\mu}^{x^N}(e_i)\right), S_{A\mu}^{N}(e_i) >, < \left(S_{A\mu}^{x^N}(e_i), S_{A\mu}^{x^N}(e_i)\right), S_{A\mu}^{N}(e_i) >\right\}$$

And

$$\left(\left(S_{B\sigma}^{x}(v_i), S_{B\sigma}^{y}(v_i)\right), S_{B\sigma}(v_i)\right) \notin \left\{< \left(S_{A\sigma}^{x^N}(v_i), S_{A\sigma}^{x^N}(v_i)\right), S_{A\sigma}^{N}(v_i) >, < \left(S_{A\sigma}^{x^N}(v_i), S_{A\sigma}^{x^N}(v_i)\right), S_{A\sigma}^{N}(v_i) >\right\},$$

$$\left(\left(S_{B\mu}^{x}(e_i), S_{B\mu}^{y}(e_i)\right), S_{B\mu}(e_i)\right) \notin \left\{< \left(S_{A\mu}^{x^N}(e_i), S_{A\mu}^{x^N}(e_i)\right), S_{A\mu}^{N}(e_i) >, < \left(S_{A\mu}^{x^N}(e_i), S_{A\mu}^{x^N}(e_i)\right), S_{A\mu}^{N}(e_i) >\right\},$$

Thus

$$\left(\left(S_{B\sigma}^{x}(v_i), S_{B\sigma}^{y}(v_i)\right), S_{B\sigma}(v_i)\right) = \left\{< \left(S_{A\mu}^{x^N}(e_i), S_{A\mu}^{x^N}(e_i)\right), S_{A\mu}^{N}(e_i) >\right\}$$

Or

$$\left(\left(S_{B\sigma}^{x}(v_i), S_{B\sigma}^{y}(v_i)\right), S_{B\sigma}(v_i)\right) = \left\{< \left(S_{A\mu}^{x^P}(e_i), S_{A\mu}^{x^P}(e_i)\right), S_{A\mu}^{P}(e_i) >\right\}$$

And

$$\left(\left(S_{B\mu}^{x}(e_i), S_{B\mu}^{y}(e_i)\right), S_{B\mu}(e_i)\right) = \left\{< \left(S_{A\mu}^{x^N}(e_i), S_{A\mu}^{x^N}(e_i)\right), S_{A\mu}^{N}(e_i) >\right\}$$

Or

$$\left(\left(S_{B\mu}^{x}(e_i), S_{B\mu}^{y}(e_i)\right), S_{B\mu}(e_i)\right) = \left\{< \left(S_{A\mu}^{x^P}(e_i), S_{A\mu}^{x^P}(e_i)\right), S_{A\mu}^{P}(e_i) >\right\}.$$

Hence

$$\left(\left(S_{B\sigma}^{x}(v_i), S_{B\sigma}^{y}(v_i)\right), S_{B\sigma}(v_i)\right) \in U\left\{< \left(s_{A\sigma}{}^{x}(v_i), s_{A\sigma}{}^{y}(v_i)\right), S_{A\sigma}(v_i) >\right\} \cup L\left\{< \left(s_{A\sigma}{}^{x}(v_i), s_{A\sigma}{}^{y}(v_i)\right), S_{A\sigma}(v_i) >\right\}$$

And

$$\left(\left(S_{B\mu}^{x}(e_i), S_{B\mu}^{y}(e_i)\right), S_{B\mu}(e_i)\right) \in U\left\{< \left(s_{B\mu}{}^{x}(e_i), s_{B\mu}{}^{y}(e_i)\right), S_{B\mu}(e_i) >\right\} \cup L\left\{< \left(s_{B\mu}{}^{x}(e_i), s_{B\mu}{}^{y}(e_i)\right), S_{B\mu}(e_i) >\right\}$$

For all $v_i \in V$and $e_i \in E \subseteq V \times V$.

Consider two cubic fuzzy graphs $G_1 = < \sigma_1, \mu_1 >$ and $G_2 =< \sigma_2, \mu_2 >$in Ω^* If we exchange$\mu_{\sigma_1}$by $\mu_{\sigma_2}$and $\mu_{\mu_1}$by μ_{μ_2}we get the cubic fuzzy graph as$\widehat{G_1} = < \widehat{\sigma_1}, \widehat{\mu_1} >and\widehat{G_1} = < \widehat{\sigma_2}, \widehat{\mu_2} >$ respectively. For any two internal cubic fuzzy graph (or) external cubic fuzzy graph $G_1$and G_2, two cubic fuzzy graphs $\widehat{G_1}$and $\widehat{G_2}$may not be internal cubic fuzzy graph and external cubic fuzzy graph.

Theorem 2.7

If B is the bipolar fuzzy transition matrix.Then $\left||B|\right| = 0$ if and only if the rows are identical.

Proof:

$$\left||B|\right| =< B, B >$$

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$$= \begin{bmatrix} < [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) > & < [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) > \\ < [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) > & < [S_{\sigma e}^{Px}(u), S_{\sigma e}^{Py}(u)], [S_{\sigma e}^{Nx}(u), S_{\sigma e}^{Ny}(u)], S_{\sigma e}^{P}(u), S_{\sigma e}^{N}(u) > \end{bmatrix}$$

$$\begin{bmatrix} < [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) > & < [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) > \\ < [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) > & < [S_{\sigma e}^{Px}(u), S_{\sigma e}^{Py}(u)], [S_{\sigma e}^{Nx}(u), S_{\sigma e}^{Ny}(u)], S_{\sigma e}^{P}(u), S_{\sigma e}^{N}(u) > \end{bmatrix}$$

$$\begin{aligned} &= \left(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)^2 \\ &\quad \oplus \Big(\big(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) \\ &\quad >\big)\big(< [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) >\big)\Big) \\ &\quad \oplus \Big(\big(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) \\ &\quad >\big)\big(< [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) >\big)\Big) \\ &\quad \oplus \left(< [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) >\right)^2 \end{aligned}$$

$$\begin{aligned} &= \left(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)^2 \\ &\quad \oplus \big(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) \\ &\quad >\big)\Big(1 - \big(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\big)\Big) \\ &\quad \oplus \big(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) \\ &\quad >\big)\Big(1 - \big(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\big)\Big) \\ &\quad \oplus \big(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) \\ &\quad >\big)\Big(1 - \big(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\big)\Big)^2 \end{aligned}$$

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$$= (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\left(1 - (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\right)^2$$
$$\oplus (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\left(1 - (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\right)$$
$$\ominus (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\left(1 - (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\right)^2$$
$$\oplus (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\left(1 - (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\right)$$
$$\ominus (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\left(1 - (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\right)^2 \oplus 1$$
$$\oplus (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\left(1 - (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\right)^2$$
$$\ominus 2(<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\left(1 - (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\right) = 0.$$

Theorem 2.8

If A is the bipolar fuzzy transition matrix. Then $||A||$ has the same value if the rows of A are interchanged.

Proof:

$$||A|| = < A, A >$$

$$= \begin{bmatrix} <[S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) > & <[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) > \\ <[S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) > & <[S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) > \end{bmatrix}$$

$$\begin{bmatrix} <[S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) > & <[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) > \\ <[S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) > & <[S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) > \end{bmatrix}$$

$$= (<[S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >)^2 \oplus$$

$$\left((<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)(<[S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) >)\right)$$
$$\oplus (<[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)(<[S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) >)$$
$$\oplus (<[S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >)^2$$

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$$= \left(< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)^2 \oplus 2(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)(< [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) >) \oplus \left(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >\right)^2$$

Now we interchange the rows of A

$$||A|| =< A, A >$$

$$= \begin{bmatrix} < [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) > & < [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) > \\ < [S_{\sigma e}^{Px}(u), S_{\sigma e}^{Py}(u)], [S_{\sigma e}^{Nx}(u), S_{\sigma e}^{Ny}(u)], S_{\sigma e}^{P}(u), S_{\sigma e}^{N}(u) > & < [S_{\sigma f}^{Px}(u), S_{\sigma f}^{Py}(u)], [S_{\sigma f}^{Nx}(u), S_{\sigma f}^{Ny}(u)], S_{\sigma f}^{P}(u), S_{\sigma f}^{N}(u) > \end{bmatrix}$$

$$\begin{bmatrix} < [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) > & < [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) > \\ < [S_{\sigma e}^{Px}(u), S_{\sigma e}^{Py}(u)], [S_{\sigma e}^{Nx}(u), S_{\sigma e}^{Ny}(u)], S_{\sigma e}^{P}(u), S_{\sigma e}^{N}(u) > & < [S_{\sigma f}^{Px}(u), S_{\sigma f}^{Py}(u)], [S_{\sigma f}^{Nx}(u), S_{\sigma f}^{Ny}(u)], S_{\sigma f}^{P}(u), S_{\sigma f}^{N}(u) > \end{bmatrix}$$

$$= \left(< [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) >\right)^2 \oplus \Big((< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >)(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >)\Big) \oplus \Big((< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >)(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >)\Big) \oplus \left(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)^2$$

$$= \left(< [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) >\right)^2 \oplus 2\Big((< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >)(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >)\Big) \oplus \left(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)^2$$

Here,

$$\Big((< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >) - (< [S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)], [S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) >)\Big) = \Big((< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >) - (< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)\Big)$$

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$$== \Big((< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >) - (< [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >) \Big)^2 \oplus 2(< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >) \Big((< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >)(\delta < [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >) \Big) \oplus \Big((< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >) - \delta(< [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >) \Big)^2$$

$$= (< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >)^2 + (\delta < [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >)^2 + 2\Big((< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >)(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >) \Big) + (< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >)^2 + (\delta < [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >)^2 - 2(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >)(\delta < [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >)$$

$$= (< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >)^2 + (< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >)^2 + 2\Big((< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >)(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >) \Big) + 2(< [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >)^2 - 2\Big((< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >)(< [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >) \Big)$$

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$$= \left(< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)^2 + \left(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >\right)^2 + 2\left(\left(< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)\left(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)\right) + 2\left(< [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >\right)^2 - 2\left(\left(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >\right)\left(< [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >\right)\right)$$

$$= \left(< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)^2 + \left(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >\right)^2 + 2\left(\left(< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)\left(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)\right) + 2\left(< [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >\right)^2 + 2\left(\left(< [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >\right)\left(\left(< [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >\right) - \left(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >\right)\right)\right)$$

$$= \left(< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)^2 + \left(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >\right)^2 + 2\left(\left(< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)\left(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)\right) - 2\left(\left(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)\left(\delta < [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >\right)\right)$$

$$= \left(< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)^2 + \left(< [S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)], [S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >\right)^2 + 2\left(\left(< [S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)], [S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)\left(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)\right) - 2\left(\left(< [S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)], [S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)\left(\delta < [S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)], [S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >\right)\right)$$

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$$= \left(< \left[S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)\right], \left[S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)\right], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)^2 + \left(< \left[S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)\right], \left[S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)\right], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >\right)^2 + 2\left(\left(< \left[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)\right], \left[S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)\right], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >\right)\left(\left(< \left[S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)\right], \left[S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)\right], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >\right) - \left(\delta < \left[S_{\sigma}^{Px}(u), S_{\sigma}^{Py}(u)\right], \left[S_{\sigma}^{Nx}(u), S_{\sigma}^{Ny}(u)\right], S_{\sigma}^{P}(u), S_{\sigma}^{N}(u) >\right)\right)\right)$$

$$= \left(< \left[S_{\sigma a}^{Px}(u), S_{\sigma a}^{Py}(u)\right], \left[S_{\sigma a}^{Nx}(u), S_{\sigma a}^{Ny}(u)\right], S_{\sigma a}^{P}(u), S_{\sigma a}^{N}(u) >\right)^2 \oplus \left(< \left[S_{\sigma d}^{Px}(u), S_{\sigma d}^{Py}(u)\right], \left[S_{\sigma d}^{Nx}(u), S_{\sigma d}^{Ny}(u)\right], S_{\sigma d}^{P}(u), S_{\sigma d}^{N}(u) >\right)^2 \oplus \left(< \left[S_{\sigma b}^{Px}(u), S_{\sigma b}^{Py}(u)\right], \left[S_{\sigma b}^{Nx}(u), S_{\sigma b}^{Ny}(u)\right], S_{\sigma b}^{P}(u), S_{\sigma b}^{N}(u) >\right)\left(< \left[S_{\sigma c}^{Px}(u), S_{\sigma c}^{Py}(u)\right], \left[S_{\sigma c}^{Nx}(u), S_{\sigma c}^{Ny}(u)\right], S_{\sigma c}^{P}(u), S_{\sigma c}^{N}(u) >\right)$$

Conclusion:

In this paper,new concepts like internal cubic fuzzy graph and external cubic fuzzy graph are introduced. The family of internal cubic fuzzy graph is an internal cubic fuzzy graph is discussed.And cubic bipolar fuzzy transition sets, Internal and External transition set and interesting results on them are provide by means of examples and theorems.

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