JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2609 - 2618
https://publishoa.com
ISSN: 1309-3452

Type 3 Multi Fuzzy Sets

1. K. Hemabala, 2. B. Srinivasa Kumar
1. Assistant Professor, Department of Mathematics, Sree vidyanikethan Engineering college, Rangampet,Tirupathi , AP, India
2. Associate Professor, Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur , AP, India

ABSTRACT

In this paper, we present the initiative of type 3 multi fuzzy sets which is an extension of type3 fuzzy sets. we have attempted to propose an extension of type2 multi fuzzy sets into a type3 multi fuzzy sets. In the wake of characterizing type 3 multi fuzzy sets, we examine the algebraic properties of these sets including set- theoretic operation such as complement, inclusion, union, intersection with example. Also we illustrate some algebraic properties.

Keywords: MFS, T2FS, T2MFS, T3FS, T3MFS.

1.Introduction

L.A. Zadeh[4] defined a subset of a fuzzy set as one whose membership function is dominated by the containing set. Since membership functions are themselves usually imprecise in practice, fuzzy sets were extended to type 2 and higher order fuzzy sets by Zadeh[4]. For n>1, type n fuzzy sets are sets whose membership function take values in type (n-1) fuzzy sets on the unit interval. Such sets can model independent sources of uncertainty.

There has recently been renewed interest in applications of type n fuzzy set theory, although most work has focused on type 2 fuzzy sets. But sometimes single fuzzy set is not possible for some real life applications . Now, many others focused on type 1 and type 2 multi fuzzy sets. Yager [1] proposed a new and more generalized concepts of multi sets named as multi fuzzy sets(MFS) which can deal with many real life problems with some degree of ease. Sebastian and Rama Krishnan [2] also studied multi fuzzy sets and concluded that multi fuzzy set theory is an extension of Zadeh's fuzzy set theory, Atanassov's intuitionistic fuzzy set theory and L-fuzzy set theory. Recently, Mohuya B.kar, Bikashkoli Roy [16] proposed theory of type2 multi fuzzy sets and introduced complement, inclusion, union and intersection properties. In this paper, we introduce type3 multi fuzzy sets using Zadeh's extension principle.Type3 multi fuzzy sets will be an extension of the existing concepts and shall be helpful to deal with problems related to uncertainties.T3MFS is a type3 fuzzy set whose primary membership, secondary membership function has a sequence of tertiary membership values lying in the closed interval [0,1]. In this paper we first give definitions of MFS, T2FS, T2MFS.

2.Preliminaries

Before introducing the notion of type3 multi fuzzy sets, we first present some vital concepts of multi fuzzy sets, type2 fuzzy sets, type2 multi fuzzy sets.

2.1 Multi Fuzzy Set (MFS)

Let $\mho$ be a non empty set. A multi fuzzy set (MFS) A in$\mho$ is a set of non increasing ordered sequences

A = $\{(u, \mu^1{}_A(u), \mu^2{}_A(u),,\ldots\ldots\ \mu^m{}_A(u): u \in \mho\}$ where m is a positive integer.

Where $0 \le \mu^i{}_A(u) \le 1$ for i = 1,2,..m. is a multi membership function of the multi fuzzy set A. Mathematically A can be expressed as

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2609 - 2618
https://publishoa.com
ISSN: 1309-3452

$$A = \int_{u \in \mho}(\mu^1_A(u), \mu^2_A(u),, \ldots\ldots\ldots \mu^m_A(u))/ u$$

Where $\int$ denotes union over all admissible u . For a discrete universe of discourse, $\int$ is replaced by $\sum$.

2.1.1 Remarks:

1. If the sequences of the membership functions have only m- terms (finite number of terms), m is called the dimension of A.
2. The set of all multi – fuzzy sets in $\mho$ of dimension m is denoted by M^m FS($\mho$)
3. The multi fuzzy membership function A is a function from $\mho$ to $[0,1]^m$ such that for all u in $\mho$,
$A(u) =(\mu^1_A(u), \mu^2_A(u),,\ldots\ldots\ldots \mu^m_A(u))$

4. We denote the multi fuzzy set A as $A = (\mu^1_A, \mu^2_A,\ldots\ldots\ldots \mu^m_A)$ where $\mu^1_A(u) \geq \mu^2_A(u) \geq\ldots\ldots\ldots \mu^m_A(u)$.

2.1.2 Operations on MFS

Let m be a positive integer and let A and B in M^m FS($\mho$),

Where $A = (\mu^1_A, \mu^2_A,\ldots\ldots\ldots \mu^m_A)$ and $B = (\mu^1_B, \mu^2_B,\ldots\ldots\ldots \mu^m_B)$

then we have the following relations and operations

1. **Inclusion** : $A \subseteq B$ if and only if $\mu^i_A \leq \mu^i_B$ for all i=1,2……m
2. **Equality:** A = B if and only if $\mu^i_A = \mu^i_B$ for all i=1,2,…….m
3. **Union :** $A \cup B = \{(u, \max (\mu^1_A (u), \mu^1_B (u)),\ldots\ldots\ldots\ldots\max (\mu^m_A (u), \mu^m_B (u))): u \in \mho\}$
4. **Intersection** : $A \cap B = \{(u, \min (\mu^1_A (u), \mu^1_B (u))\ldots\ldots\ldots\min (\mu^m_A (u), \mu^m_B (u))): u \in \mho\}$

a. **Complement :**The multi – fuzzy complement of multi fuzzy set A is $A^c = \{ u, C(\mu^1_A (u)), C(\mu^2_A (u)),\ldots\ldots\ldots C(\mu^m_A (u)): u \in \mho\}$ where $C(\mu^i_A (u))$ is the complement of $\mu^i_A (u)$ for all i = 1 to m
$C(\mu^i_A (u)) = 1- \mu^i_A (u)$ for all i = 1 to m

Example 1: Let us consider fuzzy set A as follows $A = \{(u ,0.5),(u, 0.4),(u, 0.1),(v,0.9),(v,0.7),(v,0.6),(w,0.6),(w,0.4)\}$ of the universal set $\mho = \{u, v, w\}$. From this fuzzy set, we see that the element u occurs three times with membership values 0.5, 0.4 and 0.1 respectively; the element v occurs three times with membership values 0.9,0.7,0.6 respectively and the element w occurs two times with a membership value 0.6 and 0.4 . Thus, the set A can be rewritten in the form as $A= \{(u,0.5, 0.4, 0.1) ,(v ,0.9,0.7,0.6) ,(w ,0.6,0.4) \}$ which essentially is a multi-fuzzy set.

That is, $A= \{(0.5, 0.4, 0.1)/ u+(0.9,0.7,0.6)/ v) +(0.6,0.4) / w \}$

2.2 Type-2 Fuzzy Set (T2FS)

A type-2 fuzzy set is a fuzzy set whose membership degree includes uncertainty i.e., membership degree is a type-1 fuzzy set. A T2FS introduces a third dimension to the membership function via the second membership grades. A T2FS A is mathematically expressed as follows according to (Mendel and John [14])

$A= \{((u,\theta), \mu_A(u,\theta)): \forall u \in \mho \ , \forall J_u \subseteq [0,1]\}$
Where $0 \leq \mu_A(u,\theta) \leq 1$ is the secondary membership function and J_u is the primary membership of $u \in \mho$, which is the domain of $\mu_A(u,\theta)$. A can be expressed as

$$A = \int_{u \in \mho} (\int_{\theta \in J_u} \mu_A(u,\theta)/ \theta)/ u \ , J_u \subseteq [0,1]$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2609 - 2618
https://publishoa.com
ISSN: 1309-3452

Where $\iint$ denotes union over all admissible u and θ. For a discrete universe of discourse, $\int$ is replaced by $\sum$.

Example2: A = {((0.4/ 0.1) + (0.5.0.2) + (0.6/0.5))/ u+((0.9/0.7) + (1.0/0.6))/ v) +(0.6/0.4) / w }

2.3 Type2 multi fuzzy sets (T2MFS)

Let $\mho$ be the universe of discourse. Let A be a type 2 fuzzy set defined on $\mho$ and $\theta \in J_u \subseteq [0,1]$

be a primary membership value of an element $u \in \mho$. Then A is said to be a type2 multi fuzzy set if it has membership function of type1 defined on [0,1]. AT2MFS can be expressed according to (Mohuya B. Kar, Bikashkoli Roy [16])

For each $u \in \mho$, the secondary membership sequence is defined in non increasing order as

$\mu^1{}_A(u,\theta) \geq \mu^2{}_A(u,\theta) \geq \ldots\ldots \mu^m{}_A(u,\theta)$ and is denoted by $(\mu^1{}_A(u,\theta), \mu^2{}_A(u,\theta), \ldots\ldots \mu^m{}_A(u,\theta))$

Then the set A can be expressed mathematically as

$A = \{((u,\theta), (\mu_A^1(u,\theta), \mu_A^2(u,\theta), \ldots\ldots \mu_A^m(u,\theta))): \forall u \in \mho \ , \forall J_u \subseteq [0,1]\}$

Where $0 \leq \mu_A^i(u,\theta) \leq 1$, where i = 1,2,…m is the sequence of secondary membership function and J_u is the primary membership of $u \in \mho$ then the set A can be expressed as

$A = \int_{u \in \mho} (\int_{\theta \in J_u} (\mu_A^1(u,\theta), \mu_A^2(u,\theta), \ldots\ldots \mu_A^m(u,\theta))/\theta)/u \ , J_u \subseteq [0,1]$

wheres $\iint$ denotes union over all admissible u and θ. For a discrete universe of discourse, $\int$ is replaced by $\sum$.

Example 3: A = ((0.7,0.5,0.3)/0.8+ (0.5,0.7)/0.9)/ u + ((0.5,0.4)/0.2+0.7/0.5+ (0.4,0.2,0.1)/0.7)/ v.

2.3.1 Operations on T2MFS

Let A and B be two T2MFS over some universe $\mho$ then we have the following arithmetic operations

1. **Inclusion:** :A $\subseteq$ B if and only if $\theta \leq \eta$, $\mu_A^i(u,\theta) \leq \mu_B^i(u,\eta)$ for all i=1,2……m
2. **Equality:** A = B if and only if $\theta = \eta$, $\mu_A^i(u,\theta) = \mu_B^i(u,\eta)$ for all i=1,2……m
3. **Union :** AUB = max(A(x),B(x))={ λ , $\mu_{AUB}(u,\lambda)/ \lambda=\max(\theta,\eta)$ }

   Where $\mu_{AuB}(u,\lambda) = sup_{u \in \mho}\{min_{\lambda \in J_u \subseteq [0,1]}(\mu_A^i(u,\theta), \mu_B^i(u,\eta)): \lambda=\max(\theta,\eta)\}$, i= 1,2,…m
4. Intersection : A∩B = min(A(x),B(x))={ λ , $\mu_{A\cap B}(u,\lambda)/ \lambda=\min(\theta,\eta)$ }

   Where $\mu_{A\cap B}(u,\lambda) = sup_{u \in \mho}\{min_{\lambda \in J_u \subseteq [0,1]}(\mu_A^i(u,\theta), \mu_B^i(u,\eta)): \lambda=\min(\theta,\eta)\}$, i= 1,2,…m
5. Complement: $A^c = \sum_{u \in \Omega}(\sum_{\zeta \in J_u \subseteq [0,1]}(\mu_A^1(u,\zeta), (\mu_A^2(u,\zeta) \ldots\ldots (\mu_A^m(u,\zeta))/\zeta)/u$

Where $\zeta = 1 - \theta$ and θ is the primary membership function of A

2.4 Type3 Fuzzy sets (T3FS):

A type3 fuzzy set is a fuzzy set whose membership degree includes un certainty i.e membership degree is a type2 fuzzy set. A type3 fuzzy set introduces a fourth dimension to the membership function via the 3rd membership grades. A type3 fuzzy sets A is mathematically expressed as follows

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2609 - 2618
https://publishoa.com
ISSN: 1309-3452

A = {((u,θ, θ'), μ_A (u,θ, θ')): ∀ $u \in \mho$ and $\theta \in J_u \subseteq[0,1]$, $\theta' \in J_\theta \subseteq[0,1]$

Where $0 \leq \mu_A (u,\theta, \theta') \leq 1$is the tertiary membership function, J_θ is the secondary membership,J_u is the primary membership of $u \in \mho$. Then A can be expressed as

A = $\int_{u \in \mho}(\int_{\theta \in J_u}(\int_{\theta' \in J_\theta} \mu_A (u,\theta, \theta')/ \theta')/ \theta)/ u, J_u,J_\theta \subseteq[0,1]$

Where$\iiint$ denotes the union overall admissible u,θ, θ' For a discrete universe of discourse $\int$ is replaced by $\sum$

Example 4: The white color might be modeled as a type3 fuzzy set on the set of colors samples. Let the whiteness be the primary membership function of A and the degree of whiteness be the secondary membership function and the degree of fabric on which the white is printed is the tertiary membership function.

Example 5:LetΩ = { u , v}

A = ((0.8/0.9 + 0.6/0.8)/0.7 + 0.7/0.6/0.5)/ u + ((0.4/0.3+0.4/0.5)/0.8+(0.6/0.8+0.5/1.0)/0.7)/v

3 .Type 3 Multi fuzzy sets (T3MFS):

Let Ω be the universe of discourse. Let A be a type3 fuzzy set defined on Ω and $\theta \in J_u \subseteq[0,1]$ be a primary membership value of an element $u \in \Omega$ and $\theta' \in J_\theta \subseteq[0,1]$ be the secondary membership value of $\theta \in J_u$ then for each $u \in \Omega$, the tertiary membership sequence is defined in non increasing order as

$\mu^1{}_A(u, \theta, \theta') \geq \mu^2{}_A(u, \theta, \theta') \geq \ldots\ldots\ldots\ldots \mu^m{}_A(u, \theta, \theta')$ and is denoted by $(\mu^1{}_A(u, \theta, \theta'), \mu^2{}_A(u, \theta, \theta'), \ldots\ldots\ldots\ldots \mu^m{}_A(u, \theta, \theta'))$. Then type 3 multi fuzzy set A can be expressed mathematically as

A=$\sum_{u \in \Omega}(\sum_{\theta \in J_u \subseteq[0,1]}(\sum_{\theta' \in J_\theta \subseteq[0,1]}(\mu_A^1 (u, \theta, \theta'), \mu_A^2(u, \theta, \theta') ,\ldots\ldots\ldots \mu_A^m(u, \theta, \theta'))/ \theta')/ \theta)/ u$

If the universe is discrete , where as if Ω is a continuous universe, then A can be written as

A=$\int_{u \in \Omega}(\int_{\theta \in J_u \subseteq[0,1]}(\int_{\theta' \in J_\theta \subseteq[0,1]}(\mu_A^1 (u, \theta, \theta'), \mu_A^2(u, \theta, \theta') ,\ldots\ldots\ldots \mu_A^m(u, \theta, \theta'))/ \theta')/ \theta)/ u$

Example 6: Let us consider a type3 fuzzy set A defined in the universal set Ω= { u , v}

A= {((0.8,0.5,0.2)/0.6+(0.7,0.3)/0.9)/0.7+((0.7,0.6)/0.5+(0.5,0.4)/0.8)/0.8}/ u +{((0.7,0.6)/0.3+0.8/0.5)/0.6+((0.8,0.7)/0.4+(0.3,0.2,0.1)/0.7)/0.7}/ v

4.Operations on T3MFS

In this section we discuss four fundamental arithmetic operations inclusion, equality, complement, union, intersection. By using zadeh's extension principle we extend definition of T2MFS to T3MFS.
Let us denote the collection of all T3MFS over the universe x by T3MF(Ω)
Let A = $(\mu_A^1 (u, \theta, \theta'), \mu_A^2(u, \theta, \theta') ,\ldots\ldots\ldots \mu_A^m(u, \theta, \theta'))$
B = $(\mu_B^1 (u, \theta, \theta'), \mu_B^2(u, \theta, \theta') ,\ldots\ldots\ldots \mu_B^m(u, \theta, \theta'))$

4.1 Inclusion

Let A and B be two T3MFS over some universe Ω. Then we say that A⊆B if and only if
$\theta \leq \eta$, $\theta' \leq \eta'$, $\mu_A^i (u, \theta, \theta') \leq \mu_B^i (u,\eta,\eta')$ i=1,2,…m ,∀ $u \in \Omega$ where θ and η are primary membership functions of A and B, θ' *and* η^1 are the secondary membership functions of A and B, $\mu_A^i (u, \theta, \theta')$ and

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2609 - 2618
https://publishoa.com
ISSN: 1309-3452

μ_B^i (u,η,η') are the tertiary membership functions of A and B respectively. These sets are said to be equal if

$\theta = \eta$, $\theta' = \eta^1$, μ_A^i (u, θ, θ')= μ_B^i (u,η,η') i=1,2,…m ,∀ $u\epsilon\Omega$
Let us illustrate this with an example

Example 7: Let us consider two T3MFS, say A and B, where

A = (((0.6,0.5)/0.5+ (0.7,0.4,0.3,0.1)/0.5))/0.7)/ u + ((0.5,0.2)/0.5+0.1/0.4))/0.5)/ v)

B = (((0.7,0.5,0.4,0.1)/0.7+ (0.8,0.7,0.5,0.2)/0.6))/0.8))/ u +(((0.6,0.4,0.2)/0.8+ (0.8,0.3)/0.4))/0.6)/ v)

Then applying the above-mentioned definition, we can easily observe that A⊆ B.

4.2 Complement: L et A be a T3MFS over some universe Ω. Then, the complement of A denoted by A^c and is defined as

$A^c = \sum_{u\epsilon\Omega}(\sum_{\zeta \in J_u \subseteq [0,1]}(\sum_{\zeta'\epsilon J_\theta \subseteq [0,1]}(\mu_A^1(u,\zeta,\zeta'), (\mu_A^2(u,\zeta,\zeta'), \ldots\ldots.. (\mu_A^m(u,\zeta,\zeta')))/\zeta')/\zeta)/u$

Where ζ = 1- θ , ζ'= 1-η and θ is the primary membership function, η is the secondary membership function of A. Let us illustrate with an example

Example 8: A = {(0.3,0.2,0.1)/0.5+(0.8,0.7,0.6)/0.7)/0.6}/u

A^c = {(0.3,0.2,0.1)/0.5+(0.8,0.7,0.6)/0.3)/0.4}/u

4.3 Union : Let A and B are two T3MFS over the universe Ω. Then the union of A and B is defined as

AUB = max(A(x),B(x))={ λ, λ' , $\mu_{AUB}(\lambda,\lambda')$/ λ=max(θ,η) , λ'=max(θ',η')}

Where $\mu_{AuB}(\lambda,\lambda')$ = $sup_{\lambda'=\max(\theta',\eta')}$ $min_{u\epsilon\Omega}$ {μ_A^i (u,θ,θ'), μ_B^i (u,η,η^1)}: λ = max(θ,η)

Example 9 : A={(0.3,0.2,0.1)/0.5+(0.8,0.7,0.6)/0.7)/0.6}/u

B={(0.7,0.6,0.5)/0.8+(0.9,0.6)/0.6)/0.9}/ u

θ	η	λ=max(θ,η)	θ'	η'	λ'=max(θ',η')	μ_A^j	μ_B^j	Min(μ_A^j,μ_B^j)
0.6	0.9	0.9	0.5	0.8	0.8	0.3,0.2,0.1	0.7,0.6,0.5	0.3,0.2,0.1
0.6	0.9	0.9	0.5	0.6	0.6	0.3,0.2,0.1	0.9,0.6,0.0	0.3,0.2,0.0
0.6	0.9	0.9	0.7	0.8	0.8	0.8,0.7,0.6	0.7,0.6,0.5	0.7,0.6,0.5
0.6	0.9	0.9	0.7	0.6	0.7	0.8,0.7,0.6	0.9,0.6,0.0	0.8,0.6,0.0

AUB = {sup{((0.3,0.2,0.1),(0.7,0.6,0.5))/0.8} + (0.8,0.6,0.0)/0.7 + (0.3,0.2,0.0)/0.6}/ u

= ({(0.7,0.6,0.5)/0.8 + (0.8,0.6)/0.7 + (0.3,0.2)/0.6}/0.9)/ u

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2609 - 2618
https://publishoa.com
ISSN: 1309-3452

4.4 Intersection : Let A and B are two T3MFS over the universe Ω. Then the union of A and B is defined as

A∩B = min(A(x),B(x))={ λ, λ' , $\mu_{A\cap B}(\lambda, \lambda')$/ λ=min(θ, η) , λ'=min(θ', η')}

Where $\mu_{A\cap B}(\lambda, \lambda')$ = $sup_{\lambda'=\max(\theta', \eta')}$ $min_{u\epsilon\Omega}$ $\{\mu_A^i(u, \theta, \theta'), \mu_B^i(u, \eta, \eta^1)\}$: λ = max(θ, η)

Example 10 :

A={(0.7,0.6,0.4)/0.9+(0.5,0.4,0.2)/0.7)/0.8}/u

B={(0.4,0.3,0.2)/0.8+(0.7,0.6)/0.6)/0.4}/ u

θ	η	λ=min(θ, η)	θ'	η'	λ'=min(θ', η')	μ_A^j	μ_B^j	Min(μ_A^j, μ_B^j)
0.8	0.4	0.4	0.9	0.8	0.8	0.7,0.6,0.4	0.4,0.3,0.2	0.4,0.3,0.2
0.8	0.4	0.4	0.9	0.4	0.4	0.7,0.6,0.4	0.7,0.6,0.0	0.7,0.6,0.0
0.8	0.4	0.4	0.7	0.8	0.7	0.5,0.4,0.2	0.4,0.3,0.2	0.4,0.3,0.2
0.8	0.4	0.4	0.7	0.4	0.4	0.5,0.4,0.2	0.7,0.6,0.0	0.5,0.4,0.0

A∩B = ({(0.4,0.3,0.2)/0.8 + (0.4,0.3,0.2)/0.7 + sup{(0.7,0.6,0.0)+(0.5,0.4,0.0))/0.4}/0.4)/ u

= ({(0.4,0.3,0.2)/0.8 + (0.4,0.3,0.2)/0.7 + (0.7,0.6)/0.4}/0.4)/ u

5.Properties of T3MFS

In this section, we discuss four fundamental properties of T3MFS. Let A, B and C be three T3MFS over a universe Ω. Then the following relations hold:

(i) Idempotent Law : A∪ A = A, A∩ A = A.
(ii) De Morgan's law : (A∪ B)^c = A^c∩ B^c, (A∩ B)^c = A^c∪ B^c.
(iii) Commutative Law : A∪ B = B∪ A, A∩ B = B∩ A.
(iv) Associative Law : A∪ (B∪ C) = (A∪ B)∪ C, A∩ (B∩ C) = (A∩ B)∩ C.
(v) Distributive Law : (A∪ B)∩ C = (A∩ C) ∪ (B∩ C), (A∩ B) ∪ C = (A∪ C)∩ (B∪ C).

Let us illustrate with example

Example11: A={((0.3,0.2,0.1)/0.5+(0.8,0.7,0.6)/0.7)/0.6}/u

B={((0.7,0.6,0.5)/0.8+(0.9,0.6)/0.6)/0.9}/ u

C={((0.6,0.5,0.4)/0.4+(1.0,0.4)/0.8)/0.8}/ u

i) A∩ A = A

θ	η	λ=min(θ, η)	θ'	η'	λ'=min(θ', η')	μ_A^j	μ_A^j	Min(μ_A^j, μ_A^j)
0.6	0.6	0.6	0.5	0.5	0.5	0.3,0.2,0.1	0.3,0.2,0.1	0.3,0.2,0.1

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2609 - 2618
https://publishoa.com
ISSN: 1309-3452

0.6	0.6	0.6	0.5	0.7	0.5	0.3,0.2,0.1	0.8,0.7,0.6	0.3,0.2,0.1
0.6	0.6	0.6	0.7	0.5	0.5	0.8,0.7,0.6	0.3,0.2,0.1	0.3,0.2,0.1
0.6	0.6	0.6	0.7	0.7	0.7	0.8,0.7,0.6	0.8,0.7,0.6	0.8,0.7,0.6

A∩ A={{(sup((0.3,0.2,0.1) ,(0.3,0.2,0.1) ,(0.3,0.2,0.1)))/0.5 + (0.8,0.7,0.6)/0.7}/ 0.6}/ $\mathcal{U}$

= {((0.3,0.2,0.1)/0.5+(0.8,0.7,0.6)/0.7)/0.6}/$\mathcal{U}$ = A

A∩ A = A

Similarly we can prove the result AUA=A

ii) (A∪ B)^c = A^c∩ B^c

Finding AUB

θ	η	λ=max(θ,η)	θ'	η'	λ'=max(θ',η')	μ_A^j	μ_B^j	Min(μ_A^j,μ_B^j)
0.6	0.9	0.9	0.5	0.8	0.8	0.3,0.2,0.1	0.7,0.6,0.5	0.3,0.2,0.1
0.6	0.9	0.9	0.5	0.6	0.6	0.3,0.2,0.1	0.9,0.6,0.0	0.3,0.2,0.0
0.6	0.9	0.9	0.7	0.8	0.8	0.8,0.7,0.6	0.7,0.6,0.5	0.7,0.6,0.5
0.6	0.9	0.9	0.7	0.6	0.7	0.8,0.7,0.6	0.9,0.6,0.0	0.8,0.6,0.0

AUB={{(sup((0.3,0.2,0.1), (0.7,0.6,0.5)))/0.8+ (0.8,0.6,0.0)/0.7 + (0.3,0.2,0.0)/0.6}/0.9}/ $\mathcal{U}$

={((0.7,0.6,0.5)/0.8+(0.8,0.6)/0.7+(0.3,0.2)/0.6)/0.9}/ $\mathcal{U}$

(AUB)^c = {((0.7,0.6,0.5)/0.2+(0.8,0.6)/0.3+(0.3,0.2)/0.4)/0.1}/ $\mathcal{U}$

A^c={(0.3,0.2,0.1)/0.5+(0.8,0.7,0.6)/0.3)/0.4}/$\mathcal{U}$

B^c={0.7,0.6 ,0.5)/0.2+(0.9,0.6)/0.4)/0.1}/ $\mathcal{U}$

θ	η	λ=min(θ,η)	θ'	η'	λ'=min(θ',η')	μ_A^j	μ_B^j	Min(μ_A^j,μ_B^j)
0.4	0.1	0.1	0.5	0.2	0.2	0.3,0.2,0.1	0.7,0.6,0.5	0.3,0.2,0.1
0.4	0.1	0.1	0.5	0.4	0.4	0.3,0.2,0.1	0.9,0.6,0.0	0.3,0.2,0.0
0.4	0.1	0.1	0.3	0.2	0.2	0.8,0.7,0.6	0.7,0.6,0.5	0.7,0.6,0.5
0.4	0.1	0.1	0.3	0.4	0.3	0.8,0.7,0.6	0.9,0.6,0.0	0.8,0.6,0.0

A^c∩ B^c ={{(sup((0.3,0.2,0.1), (0.7,0.6,0.5)))/0.2+ (0.8,0.6,0.0)/0.3+ (0.3,0.2,0.0)/0.4}/0.1}/ $\mathcal{U}$

={((0.7,0.6,0.5)/0.2+(0.8,0.6)/0.3+(0.3,0.2)/0.4)/0.1}/ $\mathcal{U}$

(A∪ B)^c = A^c∩ B^c,

Similarly we can prove the result (A∩ B)^c = A^c∪ B^c.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2609 - 2618
https://publishoa.com
ISSN: 1309-3452

iii) A∪ B = B∪ A

Finding BUA

θ	η	λ=max(θ,η)	θ'	η'	λ'=max(θ',η')	μ_B^j	μ_A^j	Min(μ_B^j,μ_A^j)
0.9	0.6	0.9	0.8	0.5	0.8	0.7,0.6,0.5	0.3,0.2,0.1	0.3,0.2,0.1
0.9	0.6	0.9	0.6	0.5	0.6	0.9,0.6,0.0	0.3,0.2,0.1	0.3,0.2,0.0
0.9	0.6	0.9	0.8	0.7	0.8	0.7,0.6,0.5	0.8,0.7,0.6	0.7,0.6,0.5
0.9	0.6	0.9	0.6	0.7	0.7	0.9,0.6,0.0	0.8,0.7,0.6	0.8,0.6,0.0

BUA={{(sup((0.3,0.2,0.1), (0.7,0.6,0.5)))/0.8+ (0.8,0.6,0.0)/0.7 + (0.3,0.2,0.0)/0.6}/0.9}/ $\mathcal{U}$

={((0.7,0.6,0.5)/0.8+(0.8,0.6)/0.7+(0.3,0.2)/0.6)/0.9}/ $\mathcal{U}$ = AUB from (ii)

AUB=BUA

Similarly we can prove the result A∩ B = B∩ A

iv) A∪ (B∪ C) = (A∪ B)∪ C

AUB={((0.7,0.6,0.5)/0.8+(0.8,0.6)/0.7+(0.3,0.2)/0.6)/0.9}/ $\mathcal{U}$ from (ii)

Finding (AUB)UC

θ	η	λ=max(θ,η)	θ'	η'	λ'=max(θ',η')	μ_{AUB}^j	μ_C^j	Min($\mu_{\text{AUB}}^j,\mu_C^j$)
0.9	0.8	0.9	0.8	0.4	0.8	0.7,0.6,0.5	0.6,0.5,0.4	0.6,0.5,0.4
0.9	0.8	0.9	0.8	0.8	0.8	0.7,0.6,0.5	1.0,0.4,0.0	0.7,0.4,0.0
0.9	0.8	0.9	0.7	0.4	0.7	0.8,0.6,0.0	0.6,0.5,0.4	0.6,0.5,0.0
0.9	0.8	0.9	0.7	0.8	0.8	0.8,0.6,0.0	1.0,0.4,0.0	0.8,0.4,0.0
0.9	0.8	0.9	0.6	0.4	0.6	0.3,0.2,0.0	0.6,0.5,0.4	0.3,0.2,0.0
0.9	0.8	0.9	0.6	0.8	0.8	0.3,0.2,0.0	1.0,0.4,0.0	0.3,0.2,0.0

(AUB)UC={{(sup((0.6,0.5,0.4), (0.7,0.4,0.0) , (0.8,0.4,0.0) , (0.3,0.2,0.0))/0.8+ (0.6,0.5,0.0)/0.7 + (0.3,0.2,0.0)/0.6}/0.9}/ $\mathcal{U}$

={{(0.8,0.5,0.4)/0.8+(0.6,0.5)/0.7+(0.3,0.2)/0.6}/0.9}/ $\mathcal{U}$

Finding BUC

θ	η	λ=max(θ,η)	θ'	η'	λ'=max(θ',η')	μ_B^j	μ_C^j	Min(μ_B^j,μ_C^j)
0.9	0.8	0.9	0.8	0.4	0.8	0.7,0.6,0.5	0.6,0.5,0.4	0.6,0.5,0.4
0.9	0.8	0.9	0.8	0.8	0.8	0.7,0.6,0.5	1.0,0.4,0.0	0.7,0.4,0.0
0.9	0.8	0.9	0.6	0.4	0.6	0.9,0.6,0.0	0.6,0.5,0.4	0.6,0.5,0.0
0.9	0.8	0.9	0.6	0.8	0.8	0.9,0.6,0.0	1.0,0.4,0.0	0.9,0.4,0.0

BUC={{sup((0.6,0.5,0.4),(0.7,0.4,0.0),(0.9,0.4,0.0))/0.8+(0.6,0.5,0.0)/0.6}/ 0.9}/ $\mathcal{U}$

={{(0.9,0.5,0.4)/0.8++(0.6,0.5,0.0)/0.6}/0.9}/ $\mathcal{U}$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2609 - 2618
https://publishoa.com
ISSN: 1309-3452

Finding AU(BUC)

θ	η	λ=max(θ,η)	θ'	η'	λ'=max(θ',η')	μ_A^j	μ_{BUC}^j	Min(μ_A^j,μ_{BUC}^j)
0.6	0.9	0.9	0.5	0.8	0.8	0.3,0.2,0.1	0.9,0.5,0.4	0.3,0.2,0.1
0.6	0.9	0.9	0.5	0.6	0.6	0.3,0.2,0.1	0.6,0.5,0.0	0.3,0.2,0.0
0.6	0.9	0.9	0.7	0.8	0.8	0.8,0.7,0.6	0.9,0.5,0.4	0.8,0.5,0.4
0.6	0.9	0.9	0.7	0.6	0.7	0.8,0.7,0.6	0.6,0.5,0.0	0.6,0.5,0.0

AU(BUC) = {{sup((0.3,0.2,0.1),(0.8,0.5,0.4))/0.8+(0.6,0.5,0.0)/0.7 +(0.3,0.2,0.0)/0.6}/ 0.9}/ u

={{(0.8,0.5,0.4)/0.8+(0.6,0.5)/0.7+(0.3,0.2)/0.6}/0.9}/ u

A∪ (B∪ C) = (A∪ B)∪ C,

Similarly we can prove the result A∩ (B∩ C) = (A∩ B)∩ C.

v) Easily we can prove the result (A∪ B)∩ C = (A∩ C)∪ (B∩ C), (A∩ B)∪ C = (A∪ C)∩ (B∪ C)

6. Conclusion:

In this manuscript we extend the concepts MFS to T3MFS. The T3MFS may be useful to various real life applications. Also we established some algebraic properties. In the future, scholars may attempt to generalize this concept further by studying higher order multi fuzzy sets in abstract setting.

References:

1. Yager, R.R. On the theory of bags. Int. J. Gen. Syst. 1986, 13, 23–37.

2. Sebastian, S.; Ramakrishnan, T.V. Multi-fuzzy sets: An extension of fuzzy sets. Fuzzy Inf. Eng. 2011, 3, 35–43.

3. Zadeh, L.A. Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets Syst. 1978, 1, 3–28.

4.L. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning - I," Information Sciences, 8,3, 199-249, 1975.

5. Mizumoto, M.; Tanaka, K. Some properties of fuzzy sets of type-2. Inf. Control 1976, 31, 312–340

6. Mizumoto, M.; Tanaka, K. Fuzzy sets of type-2 under algebraic product and algebraic sum. Fuzzy Sets Syst. 1981, 5, 277–280.

7. Zadeh, L.A. Fuzzy sets. Inf. Control 1965, 8, 338–353.

8. Goguen, J.A. L-fuzzy sets. J. Math. Anal. Appl. 1967, 18, 145–174.

9. Sebastian, S.; Ramakrishnan, T.V. Multi-fuzzy Subgroups. Int. J. Contemp. Math. Sci. 2011, 6, 365–372.

10. Sebastian, S.; Ramakrishnan, T.V. Multi-fuzzy extension of crisp functions using bridge functions. Ann. Fuzzy Math. Inform. 2011, 2, 1–8.

11. Sebastian, S.; Ramakrishnan, T.V. Multi-fuzzy topology. Int. J. Appl. Math. 2011, 24, 117–129

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2609 - 2618
https://publishoa.com
ISSN: 1309-3452

12. Sebastian, S.; Ramakrishnan, T.V. Multi-fuzzy sets. Int. Math. Forum 2010, 5, 2471–2476.

13. Sebastian, S.; Ramakrishnan, T.V. Multi-fuzzy extensions of functions. Adv. Adapt. Data Anal. 2011, 3, 339–350.

14. Mendel, J.M.; John, R.I. Type-2 fuzzy sets made simple. IEEE Trans. Fuzzy Syst. 2002, 10, 307–315.

15. R. Muthuraj and S. Balamurugan, multi fuzzy group and its level sub groups, Gen. Math. Notes, Vol. 17, No.1, 2013, 74 – 81.

16.Mohuya B.Kar, Bikashkoli Roy, Type-2 Multi-Fuzzy Sets and Their Applications in Decision Making, symmetry 2019,11,170.

17.John T. Rickard, Janet Aisbett, Fuzzy Subsethood for Type-n Fuzzy Sets, NAFIPS, June 2008.