JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2682 - 2691
https://publishoa.com
ISSN: 1309-3452

The Topp-Leone G- Rayleigh Rayleigh Distribution: Properties and Applications

Shoroq A. Al Sabah. and Kanaan A. A. Al Quraishy

Statistics, College of Administration and Economics, Iraq.
Correspondence should be addressed to Kanaan Adnan; kanaan.a@s.uokerbala.edu.iq

ABSTRACT

The (Topp Leone) distribution as well as the (Rayleigh) distribution are among the continuous distributions that are widely used in the analysis of survival and failure functions. This study used rule called the (Topp Leone Generator family) to make a new probability distribution it was called the (Topp Leon G-Rayleigh Rayleigh (TLG-RR)) distribution. By adding one of the shape parameters for an existing probability distribution, it called(Expanding Rayleigh distribution).This study will extract the basic properties of the distribution and estimate its parameters using estimation methodslike method of greatest possibility (MLE), method of weighted least squares (WLS) and method of Shrinkage to estimate parameters and reliability function. The study concluded that the proposed distribution(TLG-RR) is the best probability distribution in representing the real data used in the study.And best way to estimate the reliability function it is shrinkage method.

Keywords: Topp Leone Generator family, TLG- RR Distribution, Maximum likelihood estimation (MLE), Expansion Rayleigh Distribution, weighted least squares (WLS), Shrinkage.

1. Introduction

The (Topp Leone) distribution, which first discovered in 1955 by the two scientists (C.W. Topp and F.C. Leone) [1] as well as the (Rayleigh) distribution, in 1880 by the English scientist (The Lord Rayleigh) [2]. Researchers over time have developed new distributions, which has high flexibility and accuracy in representing data to study the different real phenomenon. By expanding existing distributions such as (T-X Family) by Alzaatreh et al. [6]. Or performing different expansions of the probability distributions by adding one or more parameters of the shape to the original distribution Adeyemi, S. & Adebanji, T. [10], to produce a distribution new probability that is more flexible in representing data, than the original distributions. In this study we will use the Top Leone's rule by Nadarajah and Kotz [3] with a goal to propose a new distribution named (the Topp Leone G-Rayleigh Rayleigh distribution (TLG-RR)), by developing the original distribution (Rayleigh extension distribution) by Ateeq et al. [4]. As suggested to fit the values of the variables to failure times within the period $(0, \infty)$. With the statement of its basic properties in order to estimate the reliability function. Because it is of great importance in measuring and extending the life of various devices. In this study, we will use the method of the greatest probability (MLE), [5] the method of weighted least squares (WLS) [7] and the method of Contraction [8] to estimate the parameters and reliability function, and we use simulation [9] to compare which is the best estimation method**.**

2. Methodology

This sector presents the method used to derive the cdf and pdf of the new family of distributions named Topp-Leone G-RR Distribution**.**

2.1. Topp Leone distribution

The (Topp Leone) distribution is one of the continuous statistical distributions, which are widely used in the analysis of survival and failure functions, which was first discovered by the two scientists (C.W. Topp and F.C. Leone)Buckley.

[1], but the first to use it as a generating function for distributions are the two scientists (Nadarajah and Kotz). Year (2003)Buckley. [3].If ((y is a random variable with a y~TL(α,β) for the period [1,0], then pdf and cdf are, respectively:

$$f_{TL}(y,\theta) = 2\theta y^{\theta-1}(1-y)(2-y)^{\theta-1} \quad ;0 > y > 1\,, \theta > 0 \tag{1}$$

$$F_{TL}(y,\theta) = y^{\theta}(2-y)^{\theta} \quad ;0 > y > 1\,, \theta > 0 \tag{2}$$

2.2.Topp Leone – G Familyrule

The TLG-F base did not receive much attention until researchers revived it in 2003 by Nadarajah and Kotz. They used this method to build new distributions. The (Topp-Leone G- family) is given as follows:Buckley. [3]

$$F_{TLG}(x) = G(x)^{\theta}\left(2 - G(x)\right)^{\theta} \tag{3}$$

$$f_{TLG}(x) = 2\alpha g(x)\left(1 - G(x)\right)G(x)^{\theta-1}(2 - G(x))^{\theta-1} \tag{4}$$

Where $g(x) = \frac{dG(x)}{dx}$

2.3. An Extension of Rayleigh distribution

The researchers Ateeq et al. called a Rayleigh distribution Buckley. [4] If x ~ RR (α, β), it has a pdf and cdf are give sequentially as follows:

$$g(x,\alpha,\beta) = \frac{x^3}{2\beta^4\alpha^2}\boldsymbol{e}^{-\frac{\boldsymbol{x^4}}{\boldsymbol{8\beta^4\alpha^2}}} \quad ;x > 0\,;\alpha,\beta > 0 \tag{5}$$

$$\boldsymbol{G}(x,\alpha,\beta) = 1 - \boldsymbol{e}^{-\frac{\boldsymbol{x^4}}{\boldsymbol{8\beta^4\alpha^2}}} \quad ;x > 0\,;\alpha,\beta > 0 \tag{6}$$

2.3. Suggested Probability Distribution Topp Leone G-Rayleigh Rayleigh (TLG-RR)

By using the (Topp Leone G-family) baseBuckley. [3]We get The CDF of Suggested distribution (TLG-RR)as follows:

$$F(x,\theta) = \left(G(x)\right)^{\theta}(2 - G(x))^{\theta} = (1 - e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta}[2 - (1 - e^{-\frac{x^4}{8\beta^4\alpha^2}})]^{\theta} \tag{7}$$

$$\mathrm{F_{TLGRR}}(\mathrm{x},\alpha,\beta,\theta) = (1 - \mathrm{e}^{-\frac{\mathrm{x}^4}{8\beta^4\alpha^2}})^{\theta} \quad (1 + \mathrm{e}^{-\frac{\mathrm{x}^4}{8\beta^4\alpha^2}})^{\theta} \quad ;\mathrm{x} > 0\,;\alpha,\beta > 0 \tag{8}$$

The pdf of (TLG-RR) given as:

$$f_{TLGRR}(x,\alpha,\beta,\theta) = \begin{cases} \theta\dfrac{x^3}{\beta^4\alpha^2}e^{-\frac{x^4}{4\beta^4\alpha^2}} \; (1 - e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta-1} \; (1 + e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta-1} & ;x > \alpha,\beta,\theta > 0 \\ 0 & ; \quad \mathrm{e.w} \end{cases} \tag{9}$$

Where, β,θ are the shape parameters and α is scale parameter.
The pdf and cdf plots of the TLG-RR distribution are display in Fig. 1

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2682 - 2691
https://publishoa.com
ISSN: 1309-3452

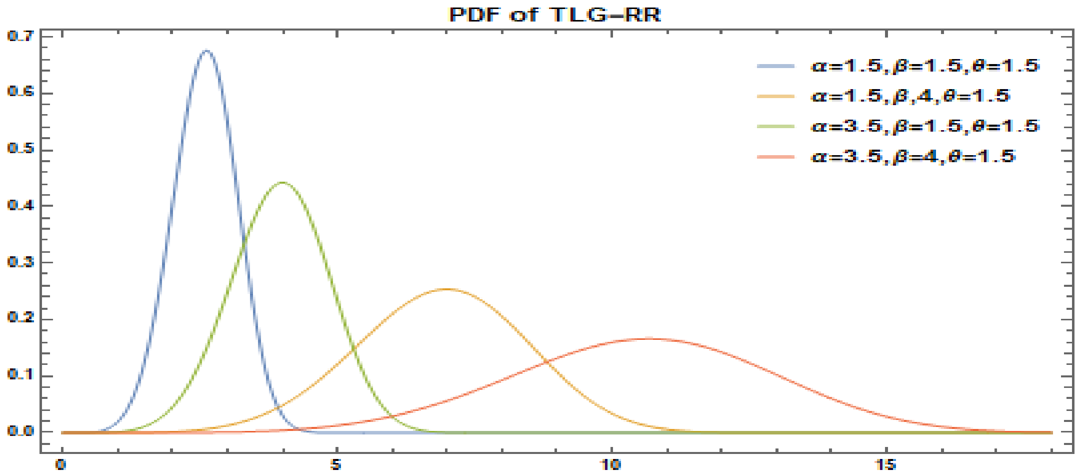


Figure (1) represents the pdf of (TLG-RR)

The cdf plots of the TLPG distribution are display in Fig. 2.

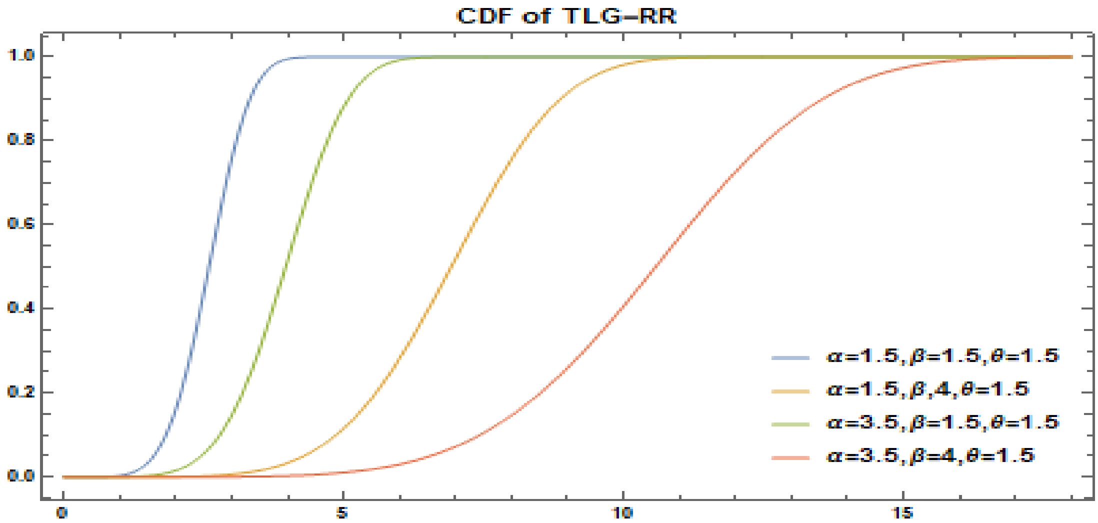


Figure (2) represents the cdf of (TLG-RR) distribution

2.4. Reliability

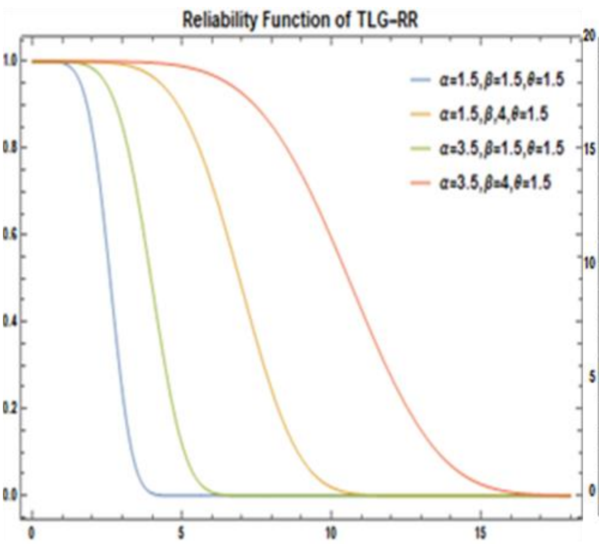
Reliability also defined as a function of the continuity of the work of any particular device, machine or machine for a specified time (t).The reliability function of the proposed distribution (TLG-RR) is the probability that the device or machine will not stop working.

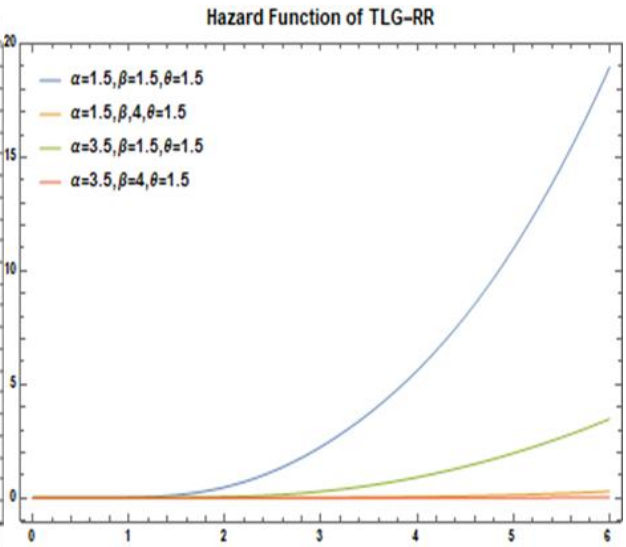
The reliability function and hazard rate of the (TLG-RR) distribution are give respectively by:Buckley. [11],[12]

$$R_{(t)} = 1 - [\,(1 + e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta}(1 - e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta}] \quad ; x > 0\,; \alpha,\beta,\theta > 0 \qquad (10)$$

$$h_{(x)} = \frac{\theta \frac{x^3}{\beta^4\alpha^2} e^{-\frac{x^4}{4\beta^4\alpha^2}}(1 + e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta-1}(1 - e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta-1}}{1 - \ (1 + e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta}(1 - e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta}} ; x > 0\,; \alpha,\beta,\theta > 0 \qquad (11)$$

The following graphs show that the shape of (TLG-RR) reliability, is skewed (Fig.3). And the (TLG-RR) hazard are shown in (Fig. 4).





FIGUARE 3. Reliability function of the TLG-RR .FIGURE 4.Hazard Function The TLG-RR

3. Statistical Properties of (TLG-RR) distribution

3. 1. Decentralized Moment:

Let x a random variable follows TLG-P Distribution then r^{th} order moment about origin of μ_r is:

$$\boldsymbol{\mu_r} = E(x^r) = \int_0^\infty x^r \theta \frac{x^3}{\beta^4\alpha^2} e^{-\frac{x^4}{4\beta^4\alpha^2}} (1 + e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta-1} (1 - e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta-1} dx \qquad (12)$$

By the binomial expansion, we will get the following expressions:

$$(1 + e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta-1} = \sum_{j=0}^{\theta-1} \binom{\theta-1}{j} e^{-j\frac{x^4}{8\beta^4\alpha^2}}$$

$$(1 - e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta-1} = \sum_{j=0}^{\theta-1} \binom{\theta-1}{k} (-1)^k e^{-k\frac{x^4}{8\beta^4\alpha^2}}$$

Therefore

$$\mu_r = E(x^r) = \frac{\theta}{4\beta^4\alpha^2} \sum_{j=0}^{\theta-1}\sum_{k=0}^{\theta-1} \binom{\theta-1}{k} \frac{\Gamma(\theta)}{\Gamma(\theta-j)\, j!} (-1)^k \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^{\frac{r+4}{4}} \Gamma\left(\frac{r+4}{4}\right) \quad (13)$$

r=1,2,3,4,.,.,……,

When we substitute the values of (r) into equation (13), we get the first and second arithmetic mean, and so on.
Then the variance of the Suggested distribution (TLG-RR) given as follows:-

$$Var(x) = \frac{\theta}{4\beta^4\alpha^2} \sum_{j=0}^{\theta-1}\sum_{k=0}^{\theta-1} \binom{\theta-1}{k} \frac{\Gamma(\theta)}{\Gamma(\theta-j)\, j!} (-1)^k \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^{\frac{6}{4}} \Gamma\left(\frac{6}{4}\right) - \left(\frac{\theta}{4\beta^4\alpha^2} \sum_{j=0}^{\theta-1}\sum_{k=0}^{\theta-1} \binom{\theta-1}{k} \frac{\Gamma(\theta)}{\Gamma(\theta-j)\, j!} (-1)^k \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^{\frac{5}{4}} \Gamma\left(\frac{5}{4}\right)\right)^2 \qquad (14)$$

3.2. Central moments:

The central moments or the so-called moments about the arithmetic mean and their formula given as follows:

$$E(x-\mu)^r = \int_0^\infty (x-\mu)^r \theta \frac{x^3}{\beta^4\alpha^2} e^{-\frac{x^4}{4\beta^4\alpha^2}} (1 - e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta-1} (1 - e^{-\frac{x^4}{8\beta^4\alpha^2}})^{\theta-1} dx \qquad (15)$$

And the same way

$$(x-\mu)^r = \sum_{i=1}^{r} \binom{r}{i} (-1)^r \mu^r x^{r-i} \qquad (16)$$

$$E(x-\mu)^r = \frac{\theta}{4\beta^4\alpha^2} \sum_{i=1}^{r} \sum_{k,j=0}^{\theta-1} \binom{2}{i} \binom{\theta-1}{k} \binom{\theta-1}{j} (-1)^{2+k} \mu^2 \left\lceil \left(\frac{r-i+4}{4}\right) \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^{\frac{r-i+4}{4}} \right. \quad (17)$$

r=1,2,3,4,.,.,……,

3.3. COEFFICIENTS OF VARIATION (CV)

$$\mathrm{C.V} = \frac{\sigma}{\mu} \times 100$$

$$\mathrm{C.V} = \frac{\sqrt{\boldsymbol{\frac{\theta}{4\beta^4\alpha^2} \sum_{j,k=0}^{\theta-1} \binom{\theta-1}{k} \frac{\lceil(\theta)}{\lceil(\theta-j)\, j!} (-1)^k \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^{\frac{6}{4}} \left\lceil\left(\frac{6}{4}\right) - \left[\frac{\theta}{4\beta^4\alpha^2} \sum_{j,k=0}^{\theta-1} \binom{\theta-1}{k} \frac{\lceil(\theta)}{\lceil(\theta-j)\, j!} (-1)^k \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^{\frac{5}{4}} \left\lceil\left(\frac{5}{4}\right)\right]^2\right.\right.}}}{\boldsymbol{\frac{\theta}{4\beta^4\alpha^2} \sum_{j,k=0}^{\theta-1} \binom{\theta-1}{k} \frac{\lceil(\theta)}{\lceil(\theta-j)\, j!} (-1)^k \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^{\frac{5}{4}} \left\lceil\left(\frac{5}{4}\right)\right.}}$$

$\times\, 100$ (19)

3.4. COEFFICIENT OF SKEWEDNESS (C.S)

$$\mathrm{C.S} = \frac{\mathrm{E}(\mathrm{x}-\mu)^3}{\sigma^3}$$

$$\mathrm{C.S} = \frac{\frac{\theta}{4\beta^4\alpha^2} \sum_{j=0}^{\theta-1} \sum_{k=0}^{\theta-1} \binom{\theta-1}{k} \frac{\lceil(\theta)}{\lceil(\theta-j)\, j!} (-1)^k \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^{\frac{7}{4}} \left\lceil\left(\frac{7}{4}\right)\right.}{\left(\frac{\theta}{4\beta^4\alpha^2} \sum_{j=0}^{\theta-1} \sum_{k=0}^{\theta-1} \binom{\theta-1}{k} \frac{\lceil(\theta)}{\lceil(\theta-j)\, j!} (-1)^k \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^{\frac{6}{4}} \left\lceil\left(\frac{6}{4}\right)\right.\right)^{\frac{3}{2}}} \qquad (18)$$

3.5. COEFFICIENT OF KURTOSIS (KS)

$$\mathrm{K.S} = \frac{\mathrm{E}(\mathrm{x}-\mu)^4}{\sigma^4}$$

$$\mathrm{K.S} = \frac{\frac{\theta}{4\beta^4\alpha^2} \sum_{j=0}^{\theta-1} \sum_{k=0}^{\theta-1} \binom{\theta-1}{k} \frac{\lceil(\theta)}{\lceil(\theta-j)\, j!} (-1)^k \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^2 \lceil 2}{\left(\frac{\theta}{4\beta^4\alpha^2} \sum_{j=0}^{\theta-1} \sum_{k=0}^{\theta-1} \binom{\theta-1}{k} \frac{\lceil(\theta)}{\lceil(\theta-j)\, j!} (-1)^k \left[\frac{1}{4\beta^4\alpha^2}\left(\frac{j}{2}+\frac{k}{2}+1\right)\right]^{\frac{6}{4}} \left\lceil\left(\frac{6}{4}\right)\right.\right)^2} - 3 \qquad (19)$$

4. Estimation

4.1. Maximum likelihood method

$$(x_1, x_2, x_3, \ldots \ldots, x_n, \alpha, \beta, \theta) = \prod_{i=1}^{n} f(x_i, \alpha, \beta, \theta) \qquad (20)$$

$$\boldsymbol{L} = \prod_{i=1}^{n} f(x_i, \theta, \beta, \alpha) = \prod_{i=1}^{n} \theta \frac{x_i^{\,3}}{\beta^4\alpha^2} e^{-\frac{x_i^4}{4\beta^4\alpha^2}} \left(1 - e^{-\frac{x_i^4}{8\beta^4\alpha^2}}\right)^{\theta-1} \left(1 + e^{-\frac{x_i^4}{8\beta^4\alpha^2}}\right)^{\theta-1} \quad (21)$$

$$lnL = n\, ln\, \theta - 4n\, ln\, \beta - 2n\, ln\, \alpha + 3n \sum_{i=1}^{n} ln x_i - \frac{\sum_{i=1}^{n} x_i^{\,4}}{4\beta^4\alpha^2} + (\theta-1)\, ln\left(1 - e^{-\frac{\sum_{i=1}^{n} x_i^{\,4}}{4\beta^4\alpha^2}}\right) \qquad (22)$$

By partially differentiating equation (22) for the parameter $(\alpha, \beta, \theta,)$ and setting it equal to zero, we get

$$\frac{\partial lnL}{\partial \theta} = \frac{n}{\theta} + ln\left(1 - e^{-\frac{\sum_{i=1}^{n} x_i^{\,4}}{4\beta^4\alpha^2}}\right) = 0$$

$$\hat{\theta} = -\frac{n}{ln\left(1 - e^{-\frac{\sum_{i=1}^{n} x_i^{\,4}}{4\beta^4\alpha^2}}\right)} \quad (23)$$

$$\hat{\beta} = \sqrt[4]{\frac{\sum_{i=1}^{n} x_i^{\,4} - (\theta-1) \sum_{i=1}^{n} x_i^{\,4} \left(1 - e^{-\frac{\sum_{i=1}^{n} x_i^{\,4}}{4\beta^4\alpha^2}}\right)}{4n\alpha^2}} \quad (24)$$

$$\hat{\alpha} = \sqrt{\frac{\sum_{i=1}^{n} x_i^{\,4} - (\theta - 1)\sum_{i=1}^{n} x_i^{\,4}\left(1 - e^{-\frac{\sum_{i=1}^{n} x_i^{\,4}}{4\beta^4\alpha^2}}\right)}{4n\beta^4}} \quad (25)$$

4.2. Least-Squares and Weighted Least-Squares Methods

Weighted Least Squares (WLS) method differs from Ordinary Least Squares (OLS) method by having weight (wi).The general formula for the weighted least squares is give as follows:

$$Q = \sum_{k=1}^{m} wi\left(F(x_k) - \frac{k}{m+1}\right)^2 \quad (26)$$

Where wi = 1 for LSE and $wi = \frac{(\bar{n}+1)^2(\bar{n}+2)}{i(\bar{n}-i+1)}$ for WLSE with respect to α,β and θ furthermore, by resolving the nonlinear equations follow:

$$\frac{\partial Q}{\partial \theta} = 2\sum_{k=1}^{m} w_k\left[F(x_k) - \frac{k}{m+1}\right)]\left[\frac{\partial F(x_k)}{\partial \theta}\right] = 0 \quad (27)$$

$$\sum_{k=1}^{m} \frac{(\bar{n}+1)^2(\bar{n}+2)}{i(\bar{n}-i+1)}\left[(1 - e^{-\frac{x^4}{4\beta^4\alpha^2}})^\theta - \frac{k}{m+1}\right)]\left[(1 - e^{-\frac{x^4}{4\beta^4\alpha^2}})^\theta \ln(1 - e^{-\frac{x^4}{4\beta^4\alpha^2}})\right] = 0 \quad (28)$$

$$\frac{\partial Q}{\partial \beta} = 2\sum_{k=1}^{m} w_k\left[F(x_k) - \frac{k}{m+1}\right)]\left[\frac{\partial F(x_k)}{\partial \beta}\right] = 0 \quad (29)$$

$$\sum_{k=1}^{m} \frac{(\bar{n}+1)^2(\bar{n}+2)}{i(\bar{n}-i+1)}\left[(1 - e^{-\frac{x^4}{4\beta^4\alpha^2}})^\theta - \frac{k}{m+1}\right)]\left[\frac{\theta x^4 e^{-\frac{x^4}{4\beta^4\alpha^2}}(1 - e^{-\frac{x^4}{4\beta^4\alpha^2}})^{\theta-1}}{\beta^5\alpha^2}\right] = 0 \quad (30)$$

$$\frac{\partial Q}{\partial \alpha} = 2\sum_{k=1}^{m} w_k\left[F(x_k) - \frac{k}{m+1}\right)]\left[\frac{\partial F(x_k)}{\partial \alpha}\right] = 0 \quad (31)$$

$$\sum_{k=1}^{m} \frac{(\bar{n}+1)^2(\bar{n}+2)}{i(\bar{n}-i+1)}\left[(1 - e^{-\frac{x^4}{4\beta^4\alpha^2}})^\theta - \frac{k}{m+1}\right)]\left[\frac{\theta x^4 e^{-\frac{x^4}{4\beta^4\alpha^2}}(1 - e^{-\frac{x^4}{4\beta^4\alpha^2}})^{\theta-1}}{2\beta^4\alpha^3}\right] = 0 \quad (32)$$

After we solve the nonlinear equations (28,30,32) using the programMathmatica, we get estimated values for the parameters(α, β, θ) of the proposed distribution (TLG-RR).

4.3. Shrinkage Technique

This method summarizes that when there are two estimators for the parameters of the proposed model from two different methods, we can in this method form a third estimator that is a linear combination of the two known estimators. If we assume that $(\hat{\theta}_{1,}\hat{\beta}_{1,}\hat{\alpha}_1)$the estimators of the greatest possibility method, and $(\hat{\theta}_{2,}\hat{\beta}_{2,}\hat{\alpha}_2)$are weighted least squares estimators, the new estimators that are a mixture of the above estimators and can be symbolized by the symbol$(\hat{\theta}_{m,}\hat{\beta}_{m,}\hat{\alpha}_m)$.The new estimate byShrinkage Techniquegive as follows:

$$\hat{\theta}_m = \mathrm{P}\hat{\theta}_1 + (1 - P)\hat{\theta}_2 \quad (33)$$

Similar others parameter $(\hat{\beta}_{m,}\hat{\alpha}_m)$. Moreover,in this method we will solve numerical equations using the program to gate the value estimates$(\hat{\theta}_{m,}\hat{\beta}_{m,}\hat{\alpha}_m)$of (TLG-RR) distribution.

5. Simulation

In this section, we will the details of simulation experiments is presented in terms of taking the real values Of the parameters α,β and θ the generated sample sizes, and replicating them through the followingStages:

1. We take sample size (n=25, 50, 100 and 200) and replicate sample (N=1000).
2. We take several values for the parameters (α,β and θ), as shown in Table No. (1)

Table No. (1) Default values for parameters and suggested models

Table No. (1)

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2682 - 2691
https://publishoa.com
ISSN: 1309-3452

MODLE	Θ	β	α
1	1.5	1.5	1.5
2	1.5	1.5	3
3	1.5	4	1.5

3. At this point, we are generating random data using Mathematica version 12.2 software.
Then we find the estimators and reliability function, finding the criterion MSE. Table No. (2) It shows the real values of the reliability function and its estimators, the mean squared error (MSE) and the ranks for all the estimation methods used for the first model and according to the assumed sample sizes.

n	**ti**	$\boldsymbol{R.Real}$	$\boldsymbol{\hat{R}.Mle}$	**Mse**	$\boldsymbol{\hat{R}.Wls}$	**Mse**	$\boldsymbol{\hat{R}.Shr}$	**Mse**
	1.648	0.9421	0.9393	0.001283	0.9256	0.002048	0.9402	**0.0003944**
	15	87	07	8	65	0.004198	01	**05**
	1.926	0.8667	0.8640	0.003232	0.8473	68	0.8622	**0.0002013**
25	4	74	95	5	39	0.004919	86	**66**
	2.013	0.8334	0.8305	0.004014	0.8137	45	0.8279	**0.0003037**
	23	55	13	7	71	0.005916	43	**86**
	2.148	0.7716	0.7676	0.005259	0.7522	4	0.7644	**0.0005275**
	54	82	78	0.005729	33	0.006240	19	**15**
	2.203	0.7434	0.7387	5	0.7242	71	0.7354	**0.0006395**
	23	13	15	0.007182	62	0.006604	16	**12**
	2.637	0.4709	0.4574	2	0.4581	31	0.4583	**0.0005575**
	28	34	02	0.007020	3	0.006417	84	**04**
	2.691	0.4345	0.4201	0.005399	0.4230	34	0.4217	**0.0005627**
	27	49	27	6	45	0.005057	94	**04**
	2.950	0.2704	0.2552	0.005168	0.2663	64	0.2583	**0.0001470**
	32	41	2	6	38	0.004877	15	**42**
	2.977	0.2550	0.2401	0.003689	0.2517	31	0.2431	**0.0003418**
	31	75	39	5	9	0.003704	63	**91**
	3.138	0.1730	0.1609		0.1743	32	0.0162	**0.0003035**
	28	33	77		45		85	**62**
IMSE			0.004798		0.004998		**0.0003979**	
		Rank of method	2		3		**1**	
n	ti	$\boldsymbol{R.Real}$	$\hat{R}.Mle$	Mse	$\hat{R}.Wls$	Mse	$\hat{R}.Shr$	**Mse**
	1.648	0.9421	0.9416	0.000576	0.9360	0.000714	0.9496	**5.56542E-**
	15	87	98	262	94	672	47	**05**
	1.926	0.8667	0.8692	0.001385	0.8622	0.001527	0.8800	**0.0001771**
50	4	74	6	74	14	46	82	**12**
	2.013	0.8334	0.8371	0.001679	0.8300	0.001800	0.8487	**0.0002329**
	23	55	41	33	96	63	19	**92**
	2.148	0.7716	0.7772	0.002104	0.7708	0.002186	0.7898	**0.0003299**
	54	82	65	73	31	39	46	**24**
	2.203	0.7434	0.7497	0.002251	0.7437	0.002319	0.7626	**0.0003695**
	23	13	21	46	62	39	35	**18**
	2.637	0.4709	0.4806	0.002756	0.4821	0.002913	0.4937	**0.0005191**
	28	34	27	53	04	87	2	**99**
	2.691	0.4345	0.4443	0.002757	0.4469	0.002947	0.4570	**0.0005045**
	27	49	49	58	12	34	11	**24**

n	ti	$R.Real$	$\hat{R}.Mle$	Mse	$\hat{R}.Wls$	Mse	$\hat{R}.Shr$	Mse
	2.95032	0.270441	0.280162	0.00251105	0.286825	0.00285386	0.289144	**0.000349805**
	2.97731	0.255075	0.264744	0.00245275	0.271676	0.00280781	0.273221	**0.000329273**
	3.13828	0.173033	0.182229	0.00197472	0.190019	0.00237019	0.18751	**0.000209593**
IMSE			0.002045		0.002244		**0.000308**	
		Rank of method	2		3		**1**	
n	ti	$R.Real$	$\hat{R}.Mle$	Mse	$\hat{R}.Wls$	Mse	$\hat{R}.Shr$	**Mse**
100	1.64815	0.942187	0.941442	0.000262863	0.939692	0.000356274	0.944525	**5.46633E-06**
	1.9264	0.866774	0.866921	0.000591857	0.864884	0.000781485	0.87105	**0.000018291**
	2.01323	0.833455	0.833992	0.000704453	0.831993	0.000921582	0.838405	**2.45002E-05**
	2.14854	0.771682	0.772867	0.000866615	0.771069	0.0011134	0.777662	**3.57549E-05**
	2.20323	0.743413	0.744858	0.000924089	0.74319	0.00117669	0.749781	**4.05621E-05**
	2.63728	0.470934	0.473976	0.00116687	0.474012	0.00136649	0.478914	**6.36765E-05**
	2.69127	0.434549	0.437712	0.00116669	0.437986	0.00135805	0.442473	**6.27782E-05**
	2.95032	0.270441	0.273951	0.00102059	0.275173	0.00119035	0.277271	**4.66402E-05**
	2.97731	0.255075	0.258598	0.000990245	0.25989	0.00115822	0.261724	**4.42135E-05**
	3.13828	0.173033	0.176526	0.000763882	0.178095	0.000913412	0.178448	**2.93202E-05**
IMSE				0.000846	0.001034		**3.71E-05**	
		Rank of method		2	3		**1**	
n	ti	$R.Real$	$\hat{R}.Mle$	Mse	$\hat{R}.Wls$	Mse	$\hat{R}.Shr$	**Mse**
200	1.64815	0.942187	0.9411	0.000123638	0.940385	0.000135813	0.942542	**1.25733E-07**
	1.9264	0.866774	0.865515	0.000291481	0.864661	0.000309888	0.867277	**2.53542E-07**
	2.01323	0.833455	0.832177	0.000350277	0.831341	0.000368549	0.833984	**2.79232E-07**
	2.14854	0.771682	0.770401	0.000434935	0.769667	0.000451213	0.772217	**2.85713E-07**
	2.20323	0.743413	0.742138	0.000464443	0.741472	0.000479802	0.743937	**2.74817E-07**
	2.63728	0.470934	0.46992	0.000562044	0.470205	0.000601822	0.471144	**4.4056E-08**
	2.69127	0.434549	0.433605	0.000556591	0.434027	0.000604251	0.434704	**2.39587E-08**
	2.95032	0.270441	0.269966	0.000467538	0.270923	0.000546236	0.270362	**6.2259E-09**
	2.97731	0.255075	0.254657	0.000452157	0.255651	0.000532116	0.254978	**9.46188E-09**
	3.13828	0.173033	0.172963	0.000342849	0.174085	0.000419174	0.172859	**3.03743E-08**

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2682 - 2691
https://publishoa.com
ISSN: 1309-3452

IMSE	0.000405	0.000445	**1.33E-07**
Rank of method	2	3	**1**
Overall Rank	8_2	12_3	**4_1**

Table No. (2)

6. Application

This section of the application includes the real (real) data obtained by the researcher from the National Card Department in Karbala, which represents the working times of the printers until they stopped working for the period (2016) to (2021), see Table No. (2) in order to apply it in the form The proposed probability of estimating the reliability function, the following table No. (3) Represents the values offor the test criteria (AIC, AICc, BIC) the quality criteria of fit for the Rayleigh distribution and the proposed distribution (TL G-RR). And the table No. (3) Shows the values of the goodness of fit criteria for choosing the best distribution.

Table No. (3)

distribution	**Parameter estimation**			**AIC**	**AICc**	**BIC**
	α	β	θ			
TL G-RR	1.68	1.54	1.336	205.313	205.563	213.128
Rayleigh Rayleigh	1.51096	1.4367	-	208.003	208.127	213.214

7. Conclusions

1. Based on the foregoing, we can conclude from the simulation results and from the tables (2), that the shrinkage method is the best method in the process of estimating the reliability function at all sample sizes with respect to the proposed probability distribution (TLG-RR), as it has the partial and total rank. First and lowest percentage of total ranks.
2. We note from Table (3) above that the proposed distribution (TLG-RR), has a lower value for the test criteria (AIC, AICc, BIC) than the original distribution (Rayleigh Rayleigh), and thus the proposed distribution (TLG-RR), is the best distribution in representing the data of the research sample.

Availability of real data

Table (4) represents real data for the study sample, representing the period of operation of the printer until its failure, in years, drawn from the National Card Department in Karbala

Table (4)

1.5	**2**	**2.4**	**3.1**	**3.3**	**3.5**	**3.6**	**3.8**	**3.9**	**4.4**
1.5	2.1	2.5	3.2	3.4	3.5	3.6	3.8	3.9	4.4
1.5	2.1	2.6	3.2	3.4	3.5	3.6	3.8	4	4.5
1.5	2.2	2.7	3.2	3.4	3.5	3.6	3.8	4	4.6
1.6	2.3	2.7	3.2	3.4	3.5	3.7	3.8	4.1	4.6
1.7	2.3	2.8	3.3	3.4	3.5	3.7	3.8	4.1	4.6
1.8	2.3	2.9	3.3	3.4	3.6	3.8	3.8	4.1	4.6
1.9	2.3	3	3.3	3.4	3.6	3.8	3.8	4.2	4.7
1.9	2.4	3	3.3	3.4	3.6	3.8	3.8	4.3	4.7
2	2.4	3.1	3.3	3.4	3.6	3.8	3.8	4.3	4.7

please visit: https://www.nid-moi.gov.iq.com.

References

1- C. W., Topp, & F. C. Leone, **"A family of J-shaped frequency functions**," Journal of the American Statistical Association, 50(269), 209-219, (1955).

2- Dey, S., Dey, T., & Kundu, D. (2014),"**Two-parameter Rayleigh distribution: different methods of estimation**", American Journal of Mathematical and Management Sciences, 33(1), 55-74.

3- Nadarajah, S., &Kotz, S. (2003). **Moments of some J-shaped distributions. Journal of Applied Statistics**, 30(3), 311-317.

4- A.K., Qasim, T. B., & A. R. Alvi, "**An extension of Rayleigh distribution andapplications**". Cogent Mathematics & Statistics, 6(1), 1622191, (2019).

5- Lehmann, E. L., & Casella, G. (2006). "**Theory of point estimation*".*** Springer Science & Business Media**.**

6- J. M. Muntadher& K. Mushtaq,"**Cubic Transformation Burr XII Distribution with Practical Application**, "Karbala University,P:43. (2021).

7- J. J ,Swain., Venkatraman, S., & J. R. Wilson, **,"Least-squares estimation of distribution functions in Johnson's translation system**, "Journal of Statistical Computation and Simulation, 29(4), 271-297, (1988).

8- J. R. Thompson, **"Some shrinkage techniques for estimating the mean**, "Journal of the American Statistical Association, 63(321), 113-122, (1968).

9- J. A. N Al-obedy,"Comparison of **Bayes' Estimators for the Pareto Type-I Reliability Function Under Different Double Informative Priors Functions**," journal of Economics And Administrative Sciences, 25(113), (2019).

10- Adeyemi, S., & Adebanji, T. (2004). "**The exponentiated generalized Pareto distribution**". Ife Journal of Science, 6(2), 127-133.

11- Cheng, C. H. (1996). **Fuzzy repairable reliability based on fuzzy GERT**. Microelectronics Reliability, 36(10), 1557-1563.

12- Davis, D. J. F. (1952). **An analysis of some failure data. Journal of the American Statistical Association**, 47(258), 113-150.

13- https://www.nid-moi.gov.iq.com.