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Eigenvalue and Eigenvector of Picture Fuzzy Matrix

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ABSTRACT

Eigen values and Eigen vectors of matrices have many applications in both Engineering and science. For example Vibration analysis, Control theory, Quantum Mechanics and an analytical hierarchy technique is utilized for decision-making, etc. all create issues with eigenvalue problems.

In this article, we studied about Eigen value and Eigen vector of some particular types of Picture Fuzzy Matrices(PicFMs) together with suitable examples.

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1. Introduction

Fuzzy set theory was introduced by Lotfi A. Zadeh in 1965[13] to deal with uncertainty in real life situation, which is a generalization of classical set theory. After the concept of Fuzzy Matrix was introduced by Hashimoto in 1983. The Intuitionistic Fuzzy Set(IFS) was given by Atanassov in 1986 [1] which is a generalization of Fuzzy set and it is characterized by two function expressing the degree of membership and degree of non-membership. In Various fields of the social sciences and medical sciences, it was identified that two factors are not enough to denote certain kinds of data. In such situations, one more component is essential to represent the data completely. So the concept of PicFS was introduced by Cuong and Kreinovich in 2013 [2] as a generalization of IFS. Recently, in 2020 $PicFM$ and its application was studied by Shovan Dogra and Madhumangal Pal[11].

Eigenvalue problems play a significant role in in many fields of science as well as many authors studied the Eigenvalue and Eigenvector of Fuzzy matrices [3,4]. The similarity relations, invertibility and eigenvalues of matrices over intuitionistic fuzzy concept was studied by Sanjib Mondal and Madhumangal Pal in 2013 [10], Murugadas.P Studied some implication operation on PicFMs (2021) [6].

In this paper, we find the Eigenvalue and Eigenvector of some particular types of PicFMs.

2. Preliminaries

Throughout the article $\mathcal{P}_{jk}$ denotes PicFMs of order $j \times k$ and $\mathcal{P}_j$ denotes PicFMs of order $j \times j$.

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For a fundamental understanding about PicFS and $PicFM$ see (2,11)

Definition 2.1 [6] Suppose a $n \times n\ PicFM,\ I$ has diagonal entries $\langle \varepsilon_1,\ \varepsilon_2, 0\rangle$ and non diagonal entries as $\langle 0,0,\ \varepsilon_3\rangle$ where $\varepsilon_1 + \varepsilon_2 = 1,\ \varepsilon_2 + \varepsilon_3 = 1$ and $A = (\langle p_{nm\mu},, p_{nm\eta}, p_{nm\nu}\rangle$ an $n \times n$ PicFM such that $p_{nm\mu} \in [0, \varepsilon_1], p_{nm\eta} \in [0, \varepsilon_2]$ and $p_{nm\nu} \in [0, \varepsilon_3]$, then $IA = AI = A$.

Further $\langle 0,0,\varepsilon_3\rangle \vee \langle p_{nm\mu}, p_{nm\eta}, p_{nm\nu}\rangle = \langle p_{nm\mu}, p_{nm\eta}, p_{nm\nu}\rangle = \langle p_{nm\mu}, p_{nm\eta}, p_{nm\nu}\rangle \vee \langle 0,0,\varepsilon_3\rangle$ and $\langle \varepsilon_1, \varepsilon_2, 0\rangle \wedge \langle p_{nm\mu}, p_{nm\eta}, p_{nm\nu}\rangle = \langle p_{nm\mu}, p_{nm\eta}, p_{nm\nu}\rangle = \langle p_{nm\mu}, p_{nm\eta}, p_{nm\nu}\rangle \wedge \langle \varepsilon_1, \varepsilon_2, 0\rangle$ for $p_{nm\mu} \in [0, \varepsilon_1], p_{nm\eta} \in [0, \varepsilon_2]$ and $p_{nm\nu} \in [0, \varepsilon_3]$

3 .Operations on PicFSs and PicFMs

Definition 3.1 [12] For $a = \langle x',\ x''\ \ x'''\rangle$, $b = \langle \kappa',\ \kappa''\ \ \kappa'''\rangle \in PicFS,$
we define Joint (∨) and meet (∧) operations as,

1) $\langle x',\ x''\ \ x'''\rangle\ \vee \langle \kappa',\ \kappa''\ \ \kappa'''\rangle = \langle \max(x', \kappa'), \max(x'',\ \kappa''), \min(x''',\ \kappa''')\rangle = \langle (c',\ c\ \ c)\ \rangle$ if $c' + c'' + c''' \leq 1,$
otherwise find $\max\{c',\ c''\ \ c'''\}$ and replace $\max\{c',\ c''\ \ c'''\}$
by 1- (sum of the rest of the components)
2) $\langle x',\ x''\ \ x'''\rangle\ \wedge \langle \kappa',\ \kappa''\ \ \kappa'''\rangle = \langle \min(x', \kappa'),\ \ \min(x'',\ \kappa'')\ ,\ \ \max(x''',\ \kappa''')\rangle$
3) $a^c = \langle x''',\ x''\ \ x'\rangle$

Definition 3.2 For PicFMs , $M1 = (\langle m1'_{ij}, m1''_{ij}, m1'''_{ij}\rangle)_{m\times n},$
$M2 = (\langle m2'_{ij}, m2''_{ij}, m2'''_{ij}\rangle)_{m\times n}$

Define

1) $M1 \vee M2 = (\langle m1'_{ij} \vee m2'_{ij}, m1''_{ij} \vee m2''_{ij}, m1'''_{ij} \wedge m2'''_{ij}\rangle)$
2) $M1 \wedge M2 = (\langle m1'_{ij} \wedge m2'_{ij}, m1''_{ij} \wedge m2''_{ij}, m1'''_{ij} \vee m2'''_{ij}\rangle)$
3) $M1 \times M2 = (\langle \vee (m1'_{ij} \wedge m2'_{ij}), \vee (m1''_{ij} \wedge m2''_{ij}), \wedge (m1'''_{ij} \vee m2'''_{ij})\rangle)$
4) $M1^T = (\langle m1'_{ji}, m1''_{ji}, m1'''_{ji}\rangle)$ ($M1^T$ is transpose of $M1$)
5) $M1 \leq M2$ iff $m1'_{ij} \leq m2'_{ij},\ m1''_{ij} \leq m2''_{ij},\ m1'''_{ij} \geq m2'''_{ij}.$
6) $M1^c = (\langle m1'''_{ij}, m1''_{ij}, m1'_{ij}\rangle)$
7) $M1 \oplus M2 = (\langle m1'_{ij} \vee m2'_{ij}, m1''_{ij} \wedge m2''_{ij}, m1'''_{ij} \wedge m2'''_{ij}\rangle)$
8) $M1 \odot M2 = (\langle m1'_{ij} \wedge m2'_{ij}, m1''_{ij} \wedge m2''_{ij}, m1'''_{ij} \vee m2'''_{ij}\rangle)$

Here after *M1M2* means $M1 \times M2$.

4. Eigenvalue and Eigenvector of PicFM

Definition 4.1 Let $M \in \mathcal{P}_n$ and a scalar $\lambda = \langle \lambda_\mu,\ \lambda_\eta,\ \lambda_\nu\rangle \in P$ be said to be characteristic root(eigenvalue) of M and a vector not equal to zero Y be said to be a row(column) characteristic vector(eigenvector) related to the characteristic root (eigenvalue) λ of M if $YM = \lambda Y (MY\ = \lambda Y)$.

Theorem 4.2 If $M = (m_{ij}) = (\langle m_{ij\mu}, m_{ij\eta}, m_{nm\nu}\rangle) \in \mathcal{P}_n$ such that $m_{1i} = m_{2i} = \cdots = m_{i-1,i} = m_{i+1,i} = \cdots = m_{ni} = \psi$ (say) where $i \in \{1,2,3\ n\}$, then m_{ii} is the characteristic root corresponding to the column characteristic vector $[\psi, \psi, \psi, \ldots, I, \ldots, \psi]^T$, where i^{th} entry is $I = \langle \varepsilon_1,\ \varepsilon_2, 0\rangle$.

Proof: Let $= [\psi, \psi, \psi, \ldots, I, \ldots, \psi]^T = (y_{i1})$, then

$$MY = \begin{bmatrix} \sum_{k=1}^{n} m_{1k}\, y_{k1} \\ \sum_{k=1}^{n} m_{2k}\, y_{k2} \\ \sum_{k=1}^{n} m_{3k}\, y_{k3} \\ \vdots \\ \sum_{k=1}^{n} m_{nk}\, y_{kn} \end{bmatrix} \begin{bmatrix} \psi \\ \psi \\ \psi \\ \vdots \\ m_{ii} \\ \vdots \\ \psi \end{bmatrix} = m_{ii} \begin{bmatrix} \psi \\ \psi \\ \psi \\ \vdots \\ \mathrm{I} \\ \vdots \\ \psi \end{bmatrix}$$

Therefore $MY = m_{ii}Y$.

Hence m_{ii} is the characteristic root (eigen value) corresponding to the column characteristic vector(eigenvector) $Y = [\psi, \psi, \psi, \ldots\ I, \ldots, \psi]^T$

Example 4.3 Let $M = \begin{bmatrix} \langle 0.3,0.4,0.2\rangle & \langle 0,0,0.4\rangle & \langle 0.3,0.1,0.5\rangle \\ \langle 0.5,0.2,0.2\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.5,0.3,0.2\rangle \\ \langle 0.3,0.2,0.4\rangle & \langle 0,0,0.4\rangle & \langle 0.4,0.2,0.3\rangle \end{bmatrix}$ and

$Y = [\langle 0,0,0.4\rangle\ \langle 0.4, 0.6,\ 0\rangle\ \langle 0,0,0.4\rangle]^T$ then,

$$MY = \begin{bmatrix} \langle 0.3,0.4,0.2\rangle & \langle 0,0,0.4\rangle & \langle 0.3,0.1,0.5\rangle \\ \langle 0.5,0.2,0.2\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.5,0.3,0.2\rangle \\ \langle 0.3,0.2,0.4\rangle & \langle 0,0,0.4\rangle & \langle 0.4,0.2,0.3\rangle \end{bmatrix} \begin{bmatrix} \langle 0,0,0.4\rangle \\ \langle 0.4,0.6,0\rangle \\ \langle 0,0,0.4\rangle \end{bmatrix}$$

$$MY = \begin{bmatrix} \langle 0,0,0.4\rangle \\ \langle 0.4,0.3,0.2\rangle \\ \langle 0,0,0.4\rangle \end{bmatrix} = \langle 0.4,0.3,0.2\rangle \begin{bmatrix} \langle 0,0,0.4\rangle \\ \langle 0.4,0.6,0\rangle \\ \langle 0,0,0.4\rangle \end{bmatrix}$$

$MY = \langle 0.4,0.3,0.2\rangle Y$

Hence $\langle 0.4,\ 0.3,\ 0.2\rangle$ is the characteristic root of M corresponding to the column characteristic vector(eigen vector) Y.

Theorem 4.4 If $M = (m_{ij}) = (\langle m_{ij\mu}, m_{ij\eta}, m_{ij\nu}\rangle) \in \mathcal{P}_n$ such that $m_{1i} = m_{2i} = \cdots = m_{i-1,i} = m_{i+1,i} = \ = m_{ni} = \psi$, where $i \in \{1,2,3\ , n\}$, then m_{ii} is the characteristic root corresponding to the row characteristic vector $[\psi, \psi, \psi, \ldots, I, \ldots, \psi]$, where $I = \langle \varepsilon_1,\ \varepsilon_2, 0\rangle$ is the i^{th} entry.

Proof: Similar to the above theorem 4.2

Example 4.5 Let $M = \begin{bmatrix} \langle 0.4,0.3,0.2\rangle & \langle 0.5,0.2,0.1\rangle & \langle 0.4,0.3,0.2\rangle \\ \langle 0,0,0.4\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0,0,0.4\rangle \\ \langle 0.3,0.2,0.4\rangle & \langle 0.5,0.3,0.1\rangle & \langle 0.4,0.3,0.2\rangle \end{bmatrix}$ and

$Y = [\langle 0,0,0.4\rangle\ \langle 0.4, 0.6,\ 0\rangle\ \langle 0,0,0.4\rangle]$ then,

$$YM = [\langle 0,0,0.\ 4\rangle\ \langle 0.4,0.6,0\rangle\ \langle 0,0,0.4\rangle]\begin{bmatrix}\langle 0.4,0.3,0.2\rangle & \langle 0.5,0.2,0.1\rangle & \langle 0.4,0.3,0.2\rangle\\ \langle 0,0,0.4\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0,0,0.4\rangle\\ \langle 0.3,0.2,0.4\rangle & \langle 0.5,0.3,0.1\rangle & \langle 0.4,0.3,0.2\rangle\end{bmatrix}$$

$YM = [\langle 0,0,0.4\rangle\ \langle 0.4, 0.2, 0.3\rangle\ \langle 0,0,0.4\rangle]$

$YM = \langle 0.4,0.2,0.3\rangle[\langle 0,0,0.4\rangle\ \langle 0.4, 0.6,\ 0\rangle\ \langle 0,0,0.4\rangle]$

$YM = \langle 0.4,0.2,0.3\rangle Y$

Hence $\langle 0.4,\ 0.2,\ 0.3\rangle$ is the characteristic root(eigenvalue) of M corresponding to the row characteristic vector(eigen vector) Y.

Theorem 4.6 If $M = (m_{ij}) = (\langle m_{ij\mu}, m_{ij\eta}, m_{ij\nu}\rangle) \in \mathcal{P}_n$ such that $m_{1i} = m_{2i} = \cdots = m_{ni} = \lambda \geq m_{ij}$ for fixed i and for all i,j belong to $\{1,2,3,\ldots,n\}$, then λ is the characteristic root corresponding to the column characteristic vector $[I, I, I, \ldots, I, \ldots., I]^T$

Proof: Since $m_{1i} = m_{2i} = \ = m_{ni} = \lambda \geq m_{ij}$ for all i,j belongs $to\{1,2,3,\ldots,n\}$.

$\therefore \sum_{j=1}^{n} m_{ij} = \lambda$ and $Y = [I, I, I, \ldots, I, \ldots, I]^T$, then

$$MY = \begin{bmatrix}\sum_{j=1}^{n} m_{1j}\, I\\ \sum_{j=1}^{n} m_{2j}\, I\\ \sum_{j=1}^{n} m_{3j}\, I\\ \sum_{j=1}^{n} m_{nj}\, I\end{bmatrix} = \begin{bmatrix}\sum_{j=1}^{n} m_{1j}\\ \sum_{j=1}^{n} m_{2j}\\ \sum_{j=1}^{n} m_{3j}\\ \sum_{j=1}^{n} m_{nj}\end{bmatrix} = \begin{bmatrix}\lambda\\ \lambda\\ \lambda\\ \vdots\\ \lambda\end{bmatrix} = \lambda\begin{bmatrix}I\\ I\\ I\\ \vdots\\ I\end{bmatrix} = \lambda Y.$$

Hence, λ is the characteristic root of M corresponding to the column characteristic vector Y.

Example 4.7 Let $M = \begin{bmatrix}\langle 0.4,0.3,0.2\rangle & \langle 0.5,0.4,0.1\rangle & \langle 0.3,0.4,0.2\rangle\\ \langle 0.4,0.2,0.3\rangle & \langle 0.5,0.4,0.1\rangle & \langle 0.4,0.3,0.2\rangle\\ \langle 0.4,0.3,0.2\rangle & \langle 0.5,0.4,0.1\rangle & \langle 0.3,0.4,0.3\rangle\end{bmatrix}$ and

$Y = [\langle 0.5,0.5,0\rangle\ \langle 0.5, 0.5,\ 0\rangle\ \langle 0.5, 0.5,\ 0\rangle]^T$ then,

$$MY = \begin{bmatrix}\langle 0.4,0.3,0.2\rangle & \langle 0.5,0.4,0.1\rangle & \langle 0.3,0.4,0.2\rangle\\ \langle 0.4,0.2,0.3\rangle & \langle 0.5,0.4,0.1\rangle & \langle 0.4,0.3,0.2\rangle\\ \langle 0.4,0.3,0.2\rangle & \langle 0.5,0.4,0.1\rangle & \langle 0.3,0.4,0.3\rangle\end{bmatrix}[\langle 0.5,0.5,0\rangle\ \langle 0.5,0.5,0\rangle\ \langle 0.5,0.5,0\rangle]$$

$$MY = \begin{bmatrix}\langle 0.5,0.4,0.1\rangle\\ \langle 0.5,0.4,0.1\rangle\\ \langle 0.5,0.4,0.1\rangle\end{bmatrix} = \langle 0.5,0.4,0.1\rangle\begin{bmatrix}\langle 0.5,0.5,0\rangle\\ \langle 0.5,0.5,0\rangle\\ \langle 0.5,0.5,0\rangle\end{bmatrix}$$

$MY = \langle 0.5,0.4,0.1\rangle Y.$

Thus $\langle 0.5,\ 0.4,\ 0.\ 1\rangle$ is the characteristic root of M with respect to the column characteristic vector(eigen vector) Y.

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Theorem 4.8 If $M = (m_{ij}) = (\langle m_{ij\mu}, m_{ij\eta}, m_{ijv}\rangle) \in \mathcal{P}_n$ such that $m_{i1} = m_{i2} = \cdots = m_{in} = \lambda \geq m_{ij}$ for fixed i and for all i,j belongs to $\{1, 2, 3,\ldots,n\}$, then λ is the characteristic root(eigenvalue) with respect to the row characteristic vector $[I, I, I, \ldots, I, \ldots. I]$.

Proof: Similar to previous Theorem 4.6

Example 4.9 Let $M = \begin{bmatrix} \langle 0.4,0.3,0.2\rangle & \langle 0.3,0.4,0.2\rangle & \langle 0.5,0.4,0.2\rangle \\ \langle 0.4,0.2,0.3\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.4,0.3,0.2\rangle \\ \langle 0.5,0.4,0.1\rangle & \langle 0.5,0.4,0.1\rangle & \langle 0.5,0.4,0.1\rangle \end{bmatrix}$ and

$Y = [\langle 0.5,0.5,0\rangle\ \langle 0.5, 0.5, 0\rangle\ \langle 0.5, 0.5, 0\rangle]$ then,

$$YM = [\langle 0.5,0.5,0\rangle\ \langle 0.5,0.5,0\rangle\ \langle 0.5,0.5,0\rangle] \begin{bmatrix} \langle 0.4,0.3,0.2\rangle & \langle 0.3,0.4,0.2\rangle & \langle 0.5,0.4,0.2\rangle \\ \langle 0.4,0.2,0.3\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.4,0.3,0.2\rangle \\ \langle 0.5,0.4,0.1\rangle & \langle 0.5,0.4,0.1\rangle & \langle 0.5,0.4,0.1\rangle \end{bmatrix}$$

$YM = [\langle 0.5,0.4,0.1\rangle\ \langle 0.5, 0.4, 0.1\rangle\ \langle 0.5, 0.4, 0.1\rangle]$

$YM = \langle 0.5,0.4,0.1\rangle[\langle 0.5,0.5,0\rangle\ \langle 0.5, 0.5, 0\rangle\ \langle 0.5, 0.5, 0\rangle]$

$YM = \langle 0.5,0.4,0.1\rangle Y.$

$\therefore \langle 0.5, 0.4, 0.1\rangle$ is the characteristic root of M with respect to the row characteristic vector(eigen vector) Y.

Definition 4.10 Let $M = (m_{ij}) = (\langle m_{ij\mu}, m_{ij\eta}, m_{nmv}\rangle) \in \mathcal{P}_n$ be a PicFM, then M is said to be row diagonally dominant if $m_{ii} \geq \sum_{j\neq i,j=1}^{n} m_{ij}$ and M is said to be column diagonally dominant if $m_{ii} \geq \sum_{i\neq j,i=1}^{n} m_{ij}$. Also M is said to be diagonally dominant if it is both row as well as column diagonally dominant.

Theorem 4.11 If $M = (m_{ij}) = (\langle m_{ij\mu}, m_{ij\eta}, m_{nmv}\rangle) \in \mathcal{P}_n$ such that $m_{11} = m_{22} = \cdots = m_{nn} = d$ (say) if M is diagonally dominant, then d is the characteristic root with respect to the row(column) characteristic vector
$[I, I, I, \ldots, I, \ldots, I]$ $([I, I, I, \ldots, I, \ldots, I]^T)$.

Proof: Here $M = (m_{ij}) = (\langle m_{ij\mu}, m_{ij\eta}, m_{nmv}\rangle) \in \mathcal{P}_n$ is diagonally dominant,
there fore $\sum_{j=1}^{n} m_{ij} = m_{ii} = d$ and $\sum_{i=1}^{n} m_{ij} = m_{jj} = d$. Also $Y = [I, I, I, \ldots, I, \ldots\ I]^T$, then

$$MY = \begin{bmatrix} \sum_{j=1}^{n} m_{1j}\, I \\ \sum_{j=1}^{n} m_{2j}\, I \\ \sum_{j=1}^{n} m_{3j}\, I \\ \sum_{j=1}^{n} m_{nj}\, I \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} m_{1j} \\ \sum_{j=1}^{n} m_{2j} \\ \sum_{j=1}^{n} m_{3j} \\ \sum_{j=1}^{n} m_{nj} \end{bmatrix} = \begin{bmatrix} d \\ d \\ d \\ \vdots \\ d \end{bmatrix} = d \begin{bmatrix} I \\ I \\ I \\ \vdots \\ I \end{bmatrix} = dY.$$

Hence, d is the characteristic root of M with respect to the column characteristic vector(eigenvector) Y.
Similarly, the theorem can be proved for row characteristic vector (eigenvector).

Example 4.12 Let $M = \begin{bmatrix} \langle 0.4,0.5,0.1\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.4,0.3,0.2\rangle \\ \langle 0.3,0.4,0.2\rangle & \langle 0.4,0.5,0.1\rangle & \langle 0.3,0.4,0.2\rangle \\ \langle 0.4,0.3,0.2\rangle & \langle 0.3,0.4,0.2\rangle & \langle 0.4,0.5,0.1\rangle \end{bmatrix}$ and

$Y = [\langle 0.5,0.5,0\rangle\ \langle 0.5, 0.5, 0\rangle\ \langle 0.5, 0.5, 0\rangle]$ then,

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$$YM = [\langle 0.5,0.5,0\rangle \ \langle 0.5,0.5,0\rangle \ \langle 0.5,0.5,0\rangle]\begin{bmatrix}\langle 0.4,0.5,0.1\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.4,0.3,0.2\rangle\\ \langle 0.3,0.4,0.2\rangle & \langle 0.4,0.5,0.1\rangle & \langle 0.3,0.4,0.2\rangle\\ \langle 0.4,0.3,0.2\rangle & \langle 0.3,0.4,0.2\rangle & \langle 0.4,0.5,0.1\rangle\end{bmatrix}$$

$YM = [\langle 0.4,0.5,0.1\rangle \ \langle 0.4, 0.5, 0.1\rangle \ \langle 0.4, 0.5, 0.1\rangle]$

$YM = \langle 0.4,0.5,0.1\rangle[\langle 0.5,0.5,0\rangle \ \langle 0.5, 0.5, 0\rangle \ \langle 0.5, 0.5, 0\rangle]$

$YM = \langle 0.4,0.5,0.1\rangle Y$

Hence $\langle 0.4, 0.5, 0.1\rangle$ is the characteristic root(eigenvalue) of M corresponding to the row characteristic vector(eigen vector) Y.

Remark 4.13 If $M = (m_{ij}) = (\langle m_{ij\mu}, m_{ij\eta}, m_{ij\nu}\rangle) \in \mathcal{P}_n$ such that $\sum_{i=1}^{n} m_{i1} = \sum_{i=1}^{n} m_{i2} = \cdots = \sum_{i=1}^{n} m_{in} = d$, then d is the characteristic root of M corresponding to the row characteristic vector $[I, I, I, I, I]$.

Remark 4.14 If $M = (m_{ij}) = (\langle m_{ij\mu}, m_{ij\eta}, m_{nm\nu}\rangle) \in \mathcal{P}_n$ such that $\sum_{j=1}^{n} m_{1j} = \sum_{j=1}^{n} m_{2j} = \cdots = \sum_{j=1}^{n} m_{nj} = d$, then d is the characteristic root (eigenvalue) of M corresponding to the column characteristic vector(eigenvector) $[I, I, I, \ldots, I, \ldots, I]^T$

Theorem 4.15 Let $M = (m_{ij}) \in \mathcal{P}_n$ then M has zero column if and only if $\psi \in \sigma(M)$ Here $\sigma(M)$ denote set of all eigenvalue of M.

Proof: Neccessary condition: Let i^{th} column of M be zero, consider $Y = [\psi, \psi, \psi, \ldots, I, \ldots, \psi]^T$, then Y is a vector that is not zero which satisfying the equation $MY = \psi Y = \psi$.

Hence, Y is an column characteristic vector with respect to the characteristic root ψ.

Sufficient condition: Let $Y = [y_1, y_2, \ldots y_n]^T$ be a column characteristic vector corresponding to the characteristic root ψ, then $MY = \psi$. Consider $y_i \neq \psi$ for $i \in \{1, 2, 3, \ldots, n\}$then $MY = \psi$ implies that $\sum_{k=1}^{n} m_{jk}\, y_k = \psi$ for each $j = \{1,2,3, \ldots, n\}$.

Therefore $m_{jk} y_k = \psi$ for each i and k. Since $y_i \neq \psi$, $m_{ij} = \psi$, for each j.

Hence the i^{ij} column of M is zero.

Definition 4.16 Let $\sigma(M)$ be the collection of all of the eigenvalues of M, then $\delta(M)$ is said to be the spectral radius of M if $\delta(M) = \sup\{\lambda | \lambda \in \sigma(M)\}$

Theorem 4.17 Let $M = (m_{ij}) \in \mathcal{P}_n$ then $\delta(M)$ is either ψ or I.

Proof: If $\sigma(M) = \{\psi\}$, then $\delta(M) = \psi$, otherwise if there exists $\lambda \in \sigma(M)$ where $\lambda \neq \psi$, then there is an nonzero characteristic vector $Y \in V^n i.e$, set of all column vectors of M of order n such that $MY = \lambda Y$. We have for every α with $\lambda \leq \alpha \leq I$, $\alpha.\lambda = \lambda$ and $\lambda.\lambda = \lambda$.

Hence $\lambda Y = (\alpha.\lambda)Y = \alpha(\lambda Y)$ impiles that

$M(\lambda Y) = \lambda(MY) = \lambda(\lambda Y) = (\lambda.\lambda)Y = \lambda Y = \alpha(\lambda Y)$.

Therefore, $\in \sigma(M)$.

Since α is an arbitrary value, $\in \sigma(M)$. Hence $\delta(M) = I$.

Theorem 4.18 Let $M, N \in \mathcal{P}_n$ if $M \leq N$, then $\delta(M) \leq \delta(N)$.

Proof: From previous theorem, $\delta(M)$ is either ψ or I.

If $\delta(M) = \psi$, then $\delta(M) \leq \delta(N)$ holds good.

If $\delta(M) = I$, then we have to show $\delta(N) = I$.

Since $\delta(M) = I$, then by definition $I \in \sigma(M)$ and $MY = IY = Y$ for some column vector Y which is non zero.Let $e = [I, I, I, \dots, I]^T$, then $Y \leq e$.

Also $M^n Y = M^{n-1} MY = M^{n-1} Y = M^{n-2} Y = \ = M^2 Y = MY = Y,$

i.e $Y = M^n Y \leq M^n e \leq N^n e$. Since $Y \leq e$ and $M \leq N$. Y is non zero.

Hence $N^n e$ is non zero. If $Z = N^n e$, then $NZ = N^{n+1} e = N^n e = Z = IZ$. Hence $I \in \sigma(N)$. Thus, $\delta(N) = I$.

5. Conclusion

In this paper, we investigate the Eigenvalue and Eigenvector of some particular type of PicFMs with suitable examples. In future, We anticipate that the ongoing research will boost the range among the researchers for finding all the Eigenvalues and Eigenvectors of PicFMs.

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