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A Comparative Study of Two Methods of Fuzzy Ranking

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ABSTRACT

Ranking of fuzzy numbers is one of the important studies in the development of fuzzy set theory and decision making process. Wang and Lee [1] while highlighting the drawback of the method - ranking fuzzy numbers with an area between the centroid point and origin point method, proposed by Chu and Tsao [2000] presented a revised method. In this paper these two methods are compared with another method RANFUW: **RAN**king **FU**zzy **W**eights proposed by Anand Raj [12] where a maximizing set, minimizing set and total utility concepts were proposed for the ranking of fuzzy numbers. For the comparative study the fuzzy numbers used by Chu and Tsao were considered. In addition, ranking of some special types of fuzzy numbers are also tried to compare the results. From this study, it is observed that RANFUW method is as simple as the other methods. It gives better results in both normalized and non-normalized fuzzy numbers without checking further the intermediate steps or requirements as required in the other methods.

Key Words: Fuzzy Numbers, Fuzzy Weights, Maximising Set, Minimising Set, Ranking Methods

Introduction

Different methods of fuzzy ranking have been proposed by many authors in the last three decades. A method of maximizing set is being proposed by Jain [5] for ordering the alternatives. Bass and Kwakernaak [6] proposed the concept of membership level. Baldwin and Guild [7] pointed out some drawbacks of comparing the alternatives and have presented some difficulties with above two methods. Bortlan and Degani [8] studied and presented some alternative methods of ranking of fuzzy numbers. Fuzzy multiple attribute decision making was discussed by Chen and Hwang [9]. Wang and Kerre discussed about the classification and dependencies of ordering methods [10]. Anand Raj and Nagesh Kumar [12] presented a method of ranking alternatives with fuzzy weights using maximizing and minimizing sets. They employed ranking multiple alternatives in multicriterion environment employing multiple experts' opinions using fuzzy numbers and linguistic variables. A detailed discussion of the drawbacks of the earlier methods was presented by Anand Raj [3] while justifying the advantages of the method proposed. The method was successfully applied to multicriterion river basin planning alternatives [3, 4]. Chu and Tsao [2] proposed method of ranking fuzzy numbers with an area between the centroid point and origin point. Wang and Lee [1] proposed the revised method of fuzzy ranking pointing out some drawbacks of above method proposed by Chu and Tsao [2].
A. Nagoor Gani, V.N. Mohamed[13] solved a fuzzy assignment problem by using a new ranking method which transforms crisp assignment problem in the linear programming problem form. Phani Bushan and Rao Peddia [14] proposes a novel approach of ranking fuzzy numbers using defuzzification which comprises of maximizing and minimizing set on the triangular fuzzy numbers from generalized trapezoidal fuzzy numbers. Nasseri et al., [15] demonstrated that the method proposed by Rao and Shankar[16] failed to rank effectively the generalized fuzzy number. Saini et al. [17] proposed new parametric entropy in α cut/(α, β) cut based distance measures and implemented for various conceivable estimations of parameters.
In this paper, it is proposed to study the comparison of two methods namely, revised method of Chu and Tsao as presented by Wang and Lee [1] and RANFUW and discusses the merits and demerits of these methods with some examples.

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The later part of this paper is organized in four sections. Next section gives the definitions of fuzzy numbers and their membership functions. Second section briefly describes the methods of ranking fuzzy numbers - RANFUW and Wang and Lee method. The third section presents the comparison of the two methods with the fuzzy numbers provided by Chu and Tsao [2]. The forth section gives the conclusions of the present study.

Definitions

Definition 1: Let **F** denote a universal set. Then the membership function $\mu_{\tilde{A}}(x)$ by which a fuzzy set A is defined has the form

$$\mu_{\tilde{A}}(x): \mathbf{F} \rightarrow [0, 1] \qquad (1)$$

Where [0, 1] denotes the interval of real numbers between 0 to 1, both inclusive.

Definition 2: A fuzzy subset A of Universal set **F** is normal iff

$$\sup_{x \varepsilon F} \mu_{\tilde{A}}(x) = 1 \qquad (2)$$

Where 'sup' is a supreme value.

Definition 3: A real fuzzy number described by ã is a fuzzy subset of the real line **R** represented as

$$\tilde{a} = (\alpha/\beta, \gamma/\delta) \qquad (3)$$

where α, β, γ, δ are the real numbers and are the parameters of the fuzzy number ã.
Also $\alpha \leq \beta \leq \gamma \leq \delta \in \mathbf{L}$ (1, 2…L), where **L** is the scale of preference structure to be used by the experts.

Definition 4:
Let the membership function $\mu_{\tilde{a}}(x)$ of the fuzzy number ã be given by

i) a continuous mapping from **R** to a closed interval [0, v], $0 < v \leq 1$;
ii) constant (zero) on (-∞, α]: $\mu_{\tilde{a}}(x) = 0$ for x when $-\infty < x \leq \alpha$;
iii) strictly increasing in the interval [α, β];
iv) a constant (v) in the interval [β, γ]: $\mu_{\tilde{a}}(x) = v$ for x when $\beta \leq x \leq \gamma$;
v) strictly decreasing in the interval [γ, δ]; and
vi) a constant (zero) in the interval [δ, ∞): $\mu_{\tilde{a}}(x) = 0$ for x when $\delta \leq x \leq \infty$

We call fuzzy number with such membership function a generalized triangular fuzzy number with trapezoidal membership function. This membership function can be represented as

$$\mu_{\tilde{a}}(x) = v \begin{cases} 0 & x \leq \alpha \\ v(x-\alpha)/(\beta-\alpha) & \alpha \leq x \leq \beta \\ & \beta \leq x \leq \gamma \\ v(\delta-x)/(\delta-\gamma) & \gamma \leq x \leq \delta \\ 0 & x \geq \delta \end{cases} \qquad (4)$$

For triangular fuzzy number β = γ. The membership function $\mu_{\tilde{a}}$ of the generalized fuzzy number is described by

$$\begin{matrix} \mu_{\tilde{a}}^{L}(x) \\ \mu_{\tilde{a}} = \end{matrix} \quad v \begin{cases} & \alpha \leq x \leq \beta \\ & \beta \leq x \leq \gamma \end{cases} \qquad (5)$$

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$\mu_{\tilde{a}}^{R}(x) \qquad\qquad \gamma \le x \le \delta$

$0 \qquad\qquad\qquad \text{otherwise}$

The graphical representation of the membership function of a fuzzy number is shown in Figure 1.

FIGURE 1

A BRIEF REVIEW OF THE METHODS:

RANFUW

Problem

To rank m alternatives (A_i; i = 1, 2, …, m) by a Decision Maker (DM) with the help of information supplied by n experts (E_j; j = 1, 2, …, n) about the alternatives for each of k criteria (C_k; k = 1, 2, …, K) and also the relative importance of each criteria with respect to some overall objective.

Let $\tilde{a}_{ij}^{k}$ be the fuzzy number assigned to alternatives A_i by an expert E_j for criteria C_k and let $\tilde{c}_{kj}$ be the fuzzy number assigned to criteria C_k by expert E_j. Then fuzzy numbers, a subset of **F**, are described by

$$\tilde{a}_{ij}^{k} = (\alpha_{ij}^{k} / \beta_{ij}^{k}, \gamma_{ij}^{k} / \delta_{ij}^{k}) \text{ and } \tilde{c}_{kj} = (\varepsilon_{kj} / \zeta_{kj}, \eta_{kj} / \theta_{kj}) \qquad (6)$$

where $\alpha < \beta < \gamma < \delta$ and $\varepsilon < \zeta < \eta < \theta \in$ **L** (1,2, …, L).

Let $\mu_{Ai}(x)$ and $\mu_{Ck}(x)$ be the membership function of $\tilde{a}_{ij}^{k}$ and $\tilde{c}_{kj}$ respectively. Then the membership function can be defined as in definition 4 from above section.
All this data can be summed up in following matrices

$$R_k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} E_1 \;\; E_2 \; \dots \; E_n \\ \left(\begin{array}{c} \\ \mu_{Ai}(x) = \tilde{a}_{ij}^{k} \in \mathbf{L} \\ \\ \end{array} \right) \end{array} \qquad (7a)$$

$$R = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_k \end{array} \begin{array}{c} E_1 \;\; E_2 \; \dots \; E_n \\ \left(\begin{array}{c} \\ \mu_{Ck}(x) = \tilde{c}_{kj} \in \mathbf{L} \\ \\ \end{array} \right) \end{array} \qquad (7b)$$

Fuzzy weights
Given the data of R and R_k, the DM computes the fuzzy weights ($\tilde{w}_i$; i = 1, 2, ..., m) of all the alternatives using

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$$\tilde{w}_i = (1/KL) \square\ [(m_{i1} \square\ n_1) \oplus (m_{i2} \square\ n_2) \oplus ...(m_{ik} \square\ n_k)] \quad (8)$$

Where

$$\tilde{m}_{ik} = 1/n \square\ [a_{i1}^k \oplus a_{i2}^k \oplus ...a_{in}^k] \text{ and } \tilde{n}_k = 1/n \square\ [c_{k1} \oplus c_{k2} \oplus ...c_{kn}] \quad (9)$$

$\oplus$ represents fuzzy addition, $\square$ represents fuzzy multiplication. $\tilde{m}_{ik}$ and $\tilde{n}_k$ are simply the row averages of the matrices in Eqs. (7a) and (7b) respectively. The following equations are defined for this purpose.

$$\alpha_{ik} = \sum \alpha_{ij}^k / \text{n and } \varepsilon_k = \sum \varepsilon_{kj} / n \quad \text{where j = 1, 2, ..., n} \quad (10)$$

Similar expression can be written for β_{ik}, $\gamma_{ik}, \delta_{ik}, \zeta_k, \eta_k$ and θ_k. Then the fuzzy weight $\tilde{w}_i$ can be described as

$$\tilde{w}_i = (\alpha_i [L_{i1}, L_{i2}] / \beta_i, \gamma_i / \delta_i [U_{i1}, U_{i2}]), \quad (11)$$

Where

$$\alpha_i = \sum \alpha_{ik} \varepsilon_k / KL, \quad \beta_i = \sum \beta_{ik} \zeta_k / KL, \quad \gamma_i = \sum \gamma_{ik} \eta_k / KL, \quad \delta_i = \sum \delta_{ik} \theta_k / KL \quad (12a)$$

k = 1, 2, ..., K.

$$L_{i1} = \sum (\beta_{ik} - \alpha_{ik})(\zeta_k - \varepsilon_k) / KL, \quad L_{i2} = \sum \varepsilon_k (\beta_{ik} - \alpha_{ik}) + \alpha_{ik} (\zeta_k - \varepsilon_k) / KL \quad (12b)$$

$$U_{i1} = \sum (\delta_{ik} - \gamma_{ik})(\theta_k - \eta_k) / KL, \quad U_{i2} = -\sum \theta_k (\delta_{ik} - \gamma_{ik}) + \delta_{ik} (\theta_k - \eta_k) / KL \quad (12c)$$

k = 1, 2, ..., K.

The membership function $\mu_{wi}(x)$ of $\tilde{w}_i$ is given by

$$\mu_{wi}(x) = \begin{cases} 0 & x < \alpha_i, \\ -L_{i2}/2L_{i1} + \{(L_{i2}/2L_{i1})^2 + (x-\alpha_i/L_{i1})\}^{1/2} & \alpha_i < x < L_{i1}y^2 + L_{i2}y + \alpha_i, \\ \omega_i & L_{i1}y^2 + L_{i2}y + \alpha_i < x < U_{i1}y^2 + U_{i2}y + \delta_i, \\ -U_{i2}/2U_{i1} - \{(-U_{i2}/2U_{i1})^2 + (x-\delta_i/U_{i1})\}^{1/2} & U_{i1}y^2 + U_{i2}y + \delta_i < \delta_i, \\ 0 & x > \delta_i \end{cases} \quad (13)$$

The membership function of maximizing set $\mu_M(x)$ and minimizing set $\mu_m(x)$ are, respectively, given by

$$\mu_M(x) = \begin{cases} v\{(x-x_{min})/(x_{max}-x_{min})\}^r & x_{min} < x < x_{max} \\ 0 & \text{otherwise,} \end{cases} \quad (14a)$$

$$\mu_m(x) = \begin{cases} v\{(x_{max} - x)/(x_{max}-x_{min})\}^r & x_{min} < x < x_{max} \\ 0 & \text{otherwise,} \end{cases} \quad (14b)$$

where $v = \min_{1<i<m}(\omega_i)$; $x_{max} = \sup_{1<i<m}(\delta_i)$;$x_{min} = \inf_{1<i<m}(\alpha_i)$; where inf refers to infimum (see figure 2).

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In case if r = 1 we get the linear membership function; if r = 2 we get convex curved (risk-prone) membership function and if r = $\frac{1}{2}$ we get concave curve (risk-averse) membership function (shown graphically in figure 2).

FIGURE 2

The total utility $U_T(i)$ of the membership function $\mu_{wi}(x)$ is then defined as

$$U_T(i) = \{U_M(i) + (1 - U_m(i))\} /2 \tag{15}$$

where $U_M(i) = \sup_x\{\mu_{wi}(x) \cap \mu_M(x)\}$ and $U_m(i) = \sup_x\{\mu_{wi}(x) \cap \mu_m(x)\}$.
Using $U_T(i)$ one can rank the alternatives. If two alternatives have the same utility values ($U_T(1) = U_T(2)$), one may use the vertices of the graphs of the corresponding membership functions to make the decision. That is, the vertex farther right is the largest, with decreasing size from right to left.

Chu and Tsao Method

Chu and Tsao [2] considered that the inverse function of $\mu_A^L(x)$ exists as $\mu_A^L(x) : [\alpha, \beta] \rightarrow [0, \omega]$ is continuous and strictly increasing , and the inverse function of $\mu_A^R(x)$ exists as $\mu_A^R(x)$: $[\gamma, \delta] \rightarrow [0, \omega]$ is continuous and strictly decreasing. Then Chu and Tsao proposed the inverse functions $g_A^L(x)$ and $g_A^R(x)$ of $\mu_A^L(x)$ and $\mu_A^R(x)$ respectively such that $g_A^L(x)$: $[0, \omega] \rightarrow [\alpha, \beta]$ and $g_A^R(x) : [0, \omega] \rightarrow [\gamma, \delta]$ are strictly increasing and strictly decreasing respectively over the range.

Chu and Tsao proposed a ranking method with an area between the centroid and the original points based on $\mu_A^L(x)$, $\mu_A^R(x)$, $g_A^L(x)$, and $g_A^R(x)$ for fuzzy numbers. The centroid point of a fuzzy number corresponded to a value $\bar{x}$ on the horizontal axis and a value $\bar{y}$ on the vertical axis. The centroid point ($\bar{x}$ (A), $\bar{y}$ (A)) of a fuzzy number A was defined as

$$\bar{x}(A) = \frac{\int_{\alpha}^{\beta} x\mu_A^L(x)dx + \int_{\beta}^{\gamma} xdx + \int_{\gamma}^{\delta} x\mu_A^R(x)dx}{\int_{\alpha}^{\beta} \mu_A^L(x)dx + \int_{\beta}^{\gamma} dx + \int_{\gamma}^{\delta} \mu_A^R(x)dx} \tag{16}$$

and

$$\bar{y}(A) = \frac{\int_0^{\omega} yg_A^L(y)dy + \int_0^{\omega} yg_A^R(y)dy}{\int_0^{\omega} g_A^L(y)dy + \int_0^{\omega} g_A^R(y)dy} \tag{17}$$

where $\mu_A^L(x)$ and $\mu_A^R(x)$ were the left and right membership functions of A respectively, and $g_A^L(x)$ and $g_A^R(x)$ were inverse functions of $\mu_A^L(x)$ and $\mu_A^R(x)$ respectively.
The area between the centroid point ($\bar{x}$ (A), $\bar{y}$ (A)) and original point (0, 0) of the fuzzy number A was defined as

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$$S(A) = \bar{x}(A) * \bar{y}(A) \quad (18)$$

where $\bar{x}$ (A) and $\bar{y}$ (A) indicate the distance values from the centroid point and the original point on horizontal axis and vertical axis for the fuzzy number A. They ranked fuzzy numbers according to the area covered. The larger the area, the larger is the fuzzy number. They proposed the following relations.

(a) If S(A) > S(B), then A > B.
(b) If S(A) < S(B), then A < B. (19)
(c) If S(A) = S(B), then A = B.

The Revised method proposed by Wang and Lee

The revised method proposed by Wang and Lee [1] is based on the method proposed by Chu and Tsao. Wang and Lee proposed that for the ranking the fuzzy number calculation of $\bar{x}$ will serve the purpose unless $\bar{x}$ are equal. If they are equal, then ranking is done with the comparison of $\bar{y}$ of the numbers. First ranking which is based on $\bar{x}$, they mentioned that greater the value of $\bar{x}$, that number should be ranked first. In the case when $\bar{x}$ is equal for the two numbers then their ranking will be done with the comparison of the $\bar{y}$ of the fuzzy numbers. The same greater principle is used for the ranking of the numbers.

Comparison of two methods

For the comparison the numbers are taken from Chu and Tsao [2]. The numbers are being ranked with these two methods and the ranking order is compared. Fuzzy numbers of special type are introduced for the critical comparison. The fuzzy numbers and their ranking with these two methods are presented in this section.

Example 1

The triangular fuzzy number B_1 (1.9, 2, 2.1) and B_2 (2.1, 3, 4) are taken for the comparison (see figure 3)

FIGURE 3

Ranking with the revised method is

$\bar{x}(B_1) = 2$ and $\bar{x}(B_2) = 3.033$
As the $\bar{x}$ are different therefore calculation of $\bar{y}$ is not necessary. Therefore $B_1 < B_2$
With the method of RANFUW the utilities of these two numbers are given as

$U_T(B_1) = 0.0705$ and $U_T(B_2) = 0.548$

The result is same as the revised method.

Example 2

In this example three numbers P_1 (0.2, 0.3, 0.5), P_2 (0.17, 0.36, 0.58) and P_3 (0.28, 0.4, 0.7) are taken for the evaluation (see figure 4)

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FIGURE 4

The result by revised method is

$\bar{x}$ (P_1) = 0.333 $\bar{x}$ (P_2) = 0.357 and $\bar{x}$ (P_3) = 0.450

Therefore $P_1 < P_2 < P_3$

RANFUW method gives

$U_T(P_1) = 0.329$ $U_T(P_2) = 0.38$ $U_T(P_3) = 0.488$ which leads to $P_1 < P_2 < P_3$

For this example also the ranking remains same.

Example 3

Another three numbers are taken for the assessment of two methods. A_1 (0.4, 0.5, 1), A_2 (0.4, 0.7, 1) and A_3 (0.4, 0.9, 1) are ranked with these methods (see figure 5).
The ranking of these numbers with the revised method can be described as follows

$\bar{x}$ (A_1) = 0.633 $\bar{x}$ (A_2) = 0.7 $\bar{x}$ (A_1) = 0.767

There the ranking is $A_1 < A_2 < A_3$.

The ranking by RANFUW method results in
$U_T(A_1) = 0.3439$ $U_T(A_2) = 0.5$ $U_T(A_3) = 0.655$
which gives $A_1 < A_2 < A_3$.

FIGURE 5

Example 4

In this example the five numbers are taken two are triangular and other three are trapezoidal numbers and some of them are non-normalized fuzzy numbers. Q_1 = (3, 5, 7; 1), Q_2 = (3, 5, 7; 0.8), Q_3= (5, 7, 9, 10; 1), Q_4= (6, 7, 9, 10; 0.6), and Q_5 = (7, 8, 9, 10:0.4) are the fuzzy numbers which are to be ranked (see figure 6).

FIGURE 6

The revised method ranked these numbers as follows

$\bar{x}$ (Q_1) = 5 $\bar{x}$ (Q_2) = 5
$\bar{x}$ (Q_3) = 7.714 $\bar{x}$ (Q_4) = 8.0 $\bar{x}$ (Q_5) = 8.5
As the value of $\bar{x}$ is same for Q_1 and Q_2 therefore the comparison of the $\bar{y}$ is required.
Since $\bar{y}$ (Q_1) = 0.5 and $\bar{y}$ (Q_2) = 0.4 therefore the ranking becomes

$Q_2 < Q_1 < Q_3 < Q_4 < Q_5$

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With normalization we get $Q_1 = Q_2 < Q_3 < Q_4 < Q_5$

RANFUW gives the ranking with utility function as

$U_T(Q_1) = 0.1225$ $U_T(Q_2) = 0.125$ $U_T(Q_3) = 0.262$ $U_T(Q_4) = 0.2782$
$U_T(Q_5) = 0.3$

These utility values rank the numbers in the following order

$Q_1 < Q_2 < Q_3 < Q_4 < Q_5$

With the normalization of the fuzzy numbers we get $Q_1 = Q_2 < Q_3 < Q_4 < Q_5$ (same as the result in the previous method).

Here, we would like to present a note on the practical significance of normalization of fuzzy numbers. Comparison of non-normalised and normalized fuzzy numbers can be done purely with an academic interest by any methods proposed for ranking them. But when it comes to the applicability of these methods in a real-world situation (setting), ranking of non-normalised and normalized fuzzy numbers differ very much and this issue should be resolved in a best fitting manner. At this point it is to be noted that normalized fuzzy numbers could be considered as unbiased estimates, where as the non-normalised fuzzy numbers as biased towards the type of the person - expert or judge or decision maker (i.e., pessimistic or optimistic type). That is to say that optimist always tries to give maximum membership grade to a fuzzy number to be 1 (i.e., sup $\mu(x) = 1$), where as pessimist gives the maximum value less than 1 (i.e., sup $\mu(x) < 1$). To avoid this inconsistency and to compare fuzzy evaluations (numbers), all numbers could be normalised to certain level (preferably sup $\mu(x) = 1$). In this context RANFUW shows better performance than the other method.

Example 5

In this example fuzzy numbers of special type are taken in to consideration. In the same example three conditions i.e. on the scale of preference are tried. First is the extreme right fuzzy numbers, in second, middle value fuzzy numbers and finally extreme left fuzzy numbers are taken.

SPECIAL TYPE FUZZY NUMBERS

Case 1

For this condition the two fuzzy numbers are taken as shown in figure 7. These numbers can be called as extreme right numbers. Triangular fuzzy numbers $G_1 = (1, 1, 4)$, and $G_2 = (0, 3, 3)$ are taken as case.

FIGURE 7

By revised method

$\overline{x}(G_1) = \overline{x}(G_2) = 2.0$, therefore the comparison of the $\overline{y}$ is necessary
as $\overline{y}(G_1) = 0.4$ $\overline{y}(G_2) = 0.6675$ which leads to $G_1 < G_2$

RANFUW gives

$U_T(G_1) = 0.4107$ $U_T(G_2) = 0.589$
therefore $G_1 < G_2$

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Case 2

In this case two fuzzy numbers are middle value fuzzy numbers as shown in figure 8. Trial is taken on triangular fuzzy numbers H_1 = (4, 4, 7), and H_2 = (3, 6, 6).

By revised method

$\bar{x}$ (H_1) = $\bar{x}$ (H_2) = 5.0, so the ranking is based on comparison of $\bar{y}$
as $\bar{y}$ (H_1) = 0.454 $\bar{y}$ (H_2) = .555 which leads $H_1 < H_2$

RANFUW gives

$U_T(H_1) = 0.4107$ $U_T(H_2) = 0.589$
therefore $H_1 < H_2$

FIGURE 8

Case 3

Triangular fuzzy numbers I_1 = (7, 7, 10), and I_2 = (6, 9, 9) are tried in this case, these are shown in figure 9.

FIGURE 9

The revised method gives

$\bar{x}$ (I_1) = $\bar{x}$ (I_2) = 8.0, so the ranking is based on comparison of $\bar{y}$
as $\bar{y}$ (I_1) = 0.4705 $\bar{y}$ (I_2) =0 .533 which leads $I_1 < I_2$

RANFUW gives

$U_T(I_1) = 0.4107$ $U_T(I_2) = 0.589$
therefore $I_1 < I_2$

Conclusion

From the above examples it can be seen that the RANFUW method ranks the fuzzy numbers in the same way as the revised method proposed by Wang and Lee without any check on the intermediate steps or requirements. In Wang and Lee method if the value of $\bar{x}$ is same for two fuzzy numbers, then the value of $\bar{y}$ need to be calculated for distinguishing the fuzzy numbers. This secondary check is not necessary in RANFUW method. Determination of inverse functions and the corresponding centroids for both the function and its inverse respectively and the areas bounded by the centroids is not required in RANFUW. So the computation time and complexities in the revised method of Wang and Lee could be avoided in RANFUW in ranking the numbers. Moreover, in the real-world setting, where the evaluations of the experts or judges or decision makers (i.e., the preference structure of fuzzy numbers given by them) are to be categorised or ranked, the bias of the type of personality (pessimistic or optimistic, similar to risk-taking or risk-aversing type of personalities) could be avoided by normalizing the evaluations (numbers). In this context RANFUW shows better performance than the other methods. Therefore, the trials presented in this comparative study justify that RANFUW is simple and straight forward compared to the other methods.

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References

[1] Wang, Y. J., H.-S. Lee, "The revised method of ranking fuzzy numbers with an area between the centroid and original points", Computers and Mathematics with Applications Volume 55 issue 9 (2008) pp 2033-2042.

[2] Chu, T. C., C.T. Tsao, "Ranking fuzzy numbers with an area between the centroid point and the original point", Computers and Mathematics with Applications, 43 (2002) 111–117.

[3] Anand Raj, P., D.N. Kumar, "Ranking multi-criterion river basin planning alternatives using fuzzy numbers", Fuzzy Sets and Systems, 100 (1998) 89-99.

[4] Anand Raj, P., D.N. Kumar, "Ranking alternatives with fuzzy weights using maximizing set and minimizing set", Fuzzy Sets and Systems, 105 (1999) 365–375.

[5] Jain, R., "Decision-making in the presence of fuzzy variables", IEEE Transactions on Systems, Man and Cybernetics, 6 (1976) 698–703.

[6] Bass, S. M., H. Kwakernaak, " Rating and ranking of multiple-aspect alternatives using fuzzy sets", Automatica 13 (1977) 47-57.

[7] J.F. Baldwin, N.C.F. Guild, "Comparison of fuzzy sets on the same decision space", Fuzzy Sets and Systems, 2 (1979) 213-231.

[8] Bortolan, J. J., R. Degani, "A review of some methods for ranking fuzzy numbers", Fuzzy Sets and Systems, 15 (1985) 1–19.

[9] Chen, S. J., C.L. Hwang, "Fuzzy Multiple Attribute Decision Making", Springer, New York, 1992.

[10] Wang X., and E.E. Kerre, "Reasonalble properties for the ordering of fuzzy quantities (I)", Fuzzy Sets and Systmes, 118 (2001) 375-385.

[11] Zimmermann, H. J., "Fuzzy Sets Theory and its Applications", Kluwer Academic Publishers, Boston, 1985 p 1 - 399.

[12] Anand Raj, P., "Multi criterion decision making in fuzzy environment for river basin development and management", Thesis Report IIT Kharagpur 1998.

[13] A Nagoor Gani, V.N. Mohamed, "Solution of a Fuzzy Assignment Problem by Using a New Ranking Method", International Journal of Fuzzy Mathematical Archive, 2,(2013), 8-16.

[14] PhaniBushan, RaoPeddi , "Defuzzification method for ranking fuzzy numbers based on centroids and maximizing and minimizing set", Decision Science Letters 8 (2019) 411–428.

[15] Seyed Hadi Nasseri Nemataollah Taghi-Nezhad Ali Ebrahimnejad "A Note on Ranking Fuzzy Numbers with an Area Method using Circumcenter of Centroids" Fuzzy Information and Engineering 9(2017),259-268.

[16] Rao P.P.B., Shankar N.R. "Ranking fuzzy numbers with an area method using circumcenter of centroids", Fuzzy Information and Engineering, 1 (2013), 3-18.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 74 - 88
https://publishoa.com
ISSN: 1309-3452

[17] Namita Saini, Rakesh Kumar Bajaj, Neeraj Gandotra, Ram Prakash Dwivedi, “Multi-criteria Decision Making with Triangular Intuitionistic Fuzzy Number based on Distance Measure & Parametric Entropy Approach”, Procedia Computer Science 125 (2018), 34–41.

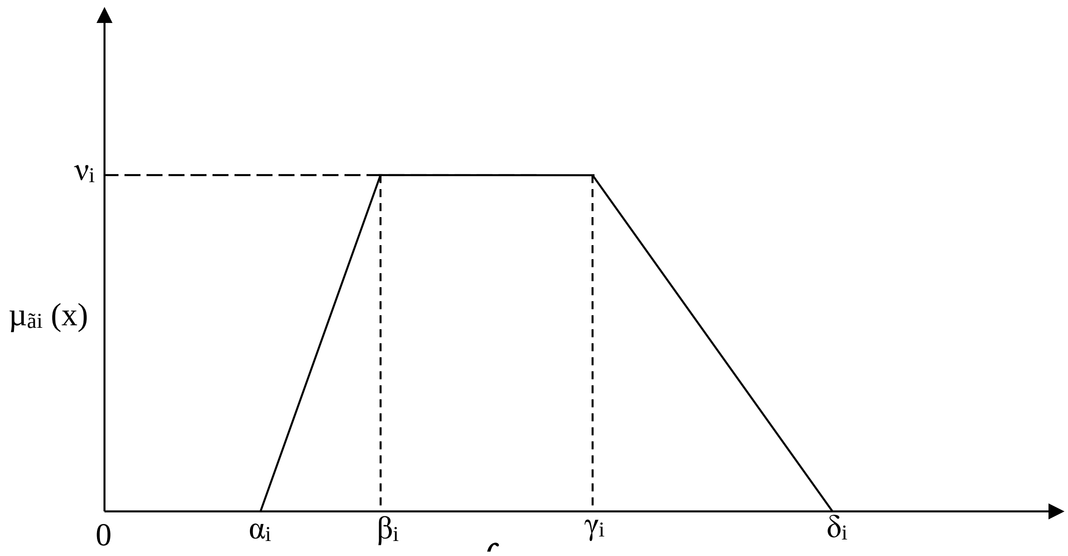

Figure 1: Graphical representation of the membership function of a fuzzy number $\tilde{a}_i$

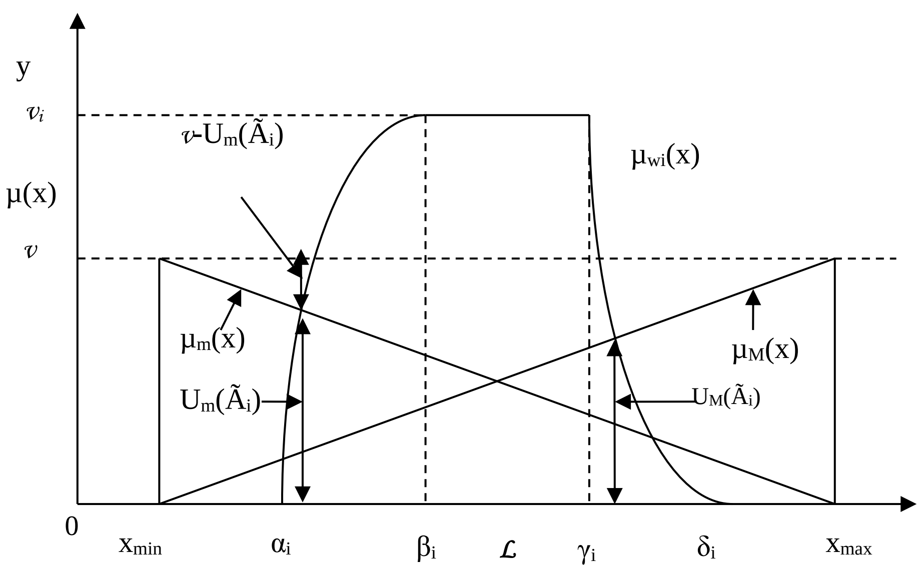

Figure 2: Graphical representation of $\mu_m(x)$, $\mu_M(x)$ and $\mu_{wi}(x)$

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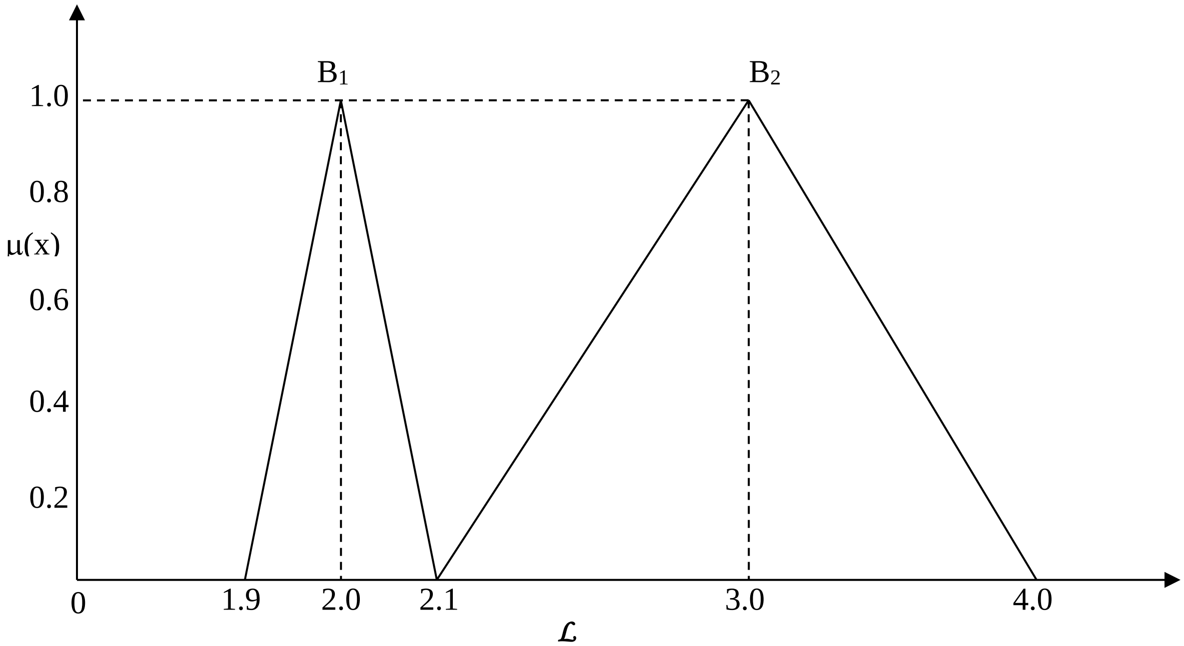

Figure 3: Triangular Fuzzy Numbers $B_1(1.9,2,2.1)$ and $B_2(2.1,3,4)$

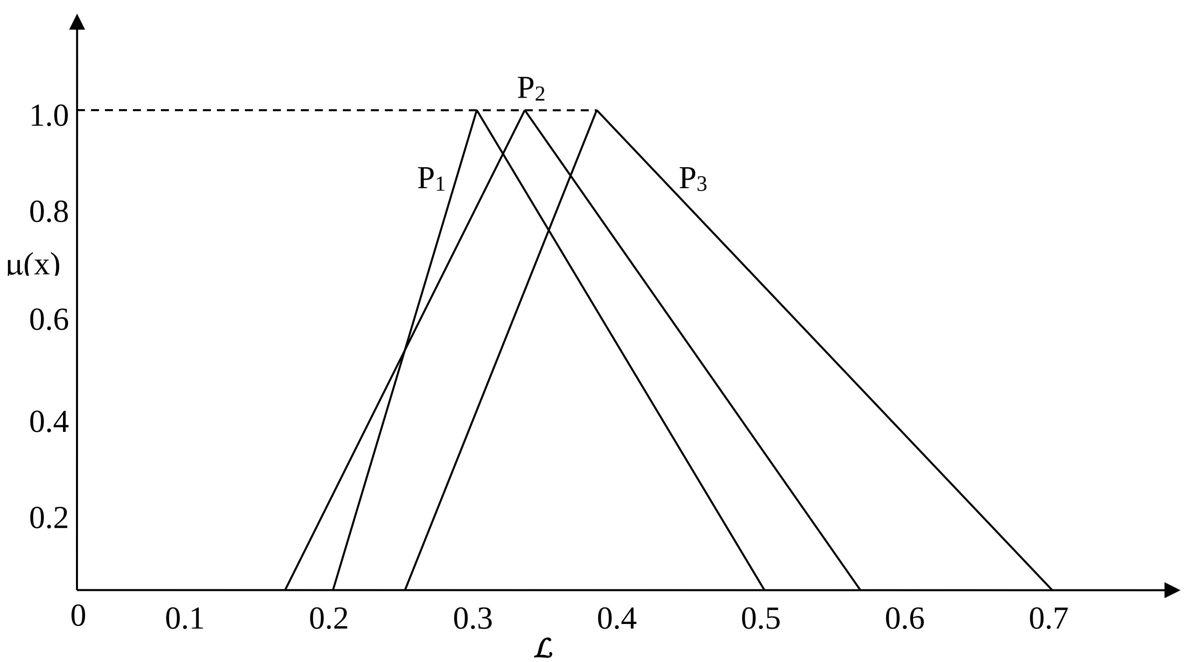

Figure 4: Triangular Fuzzy Numbers $P_1(0.2,0.3,0.5)$, $P_2(0.17,0.32,0.58)$ and $P_3(0.25,0.4,0.7)$

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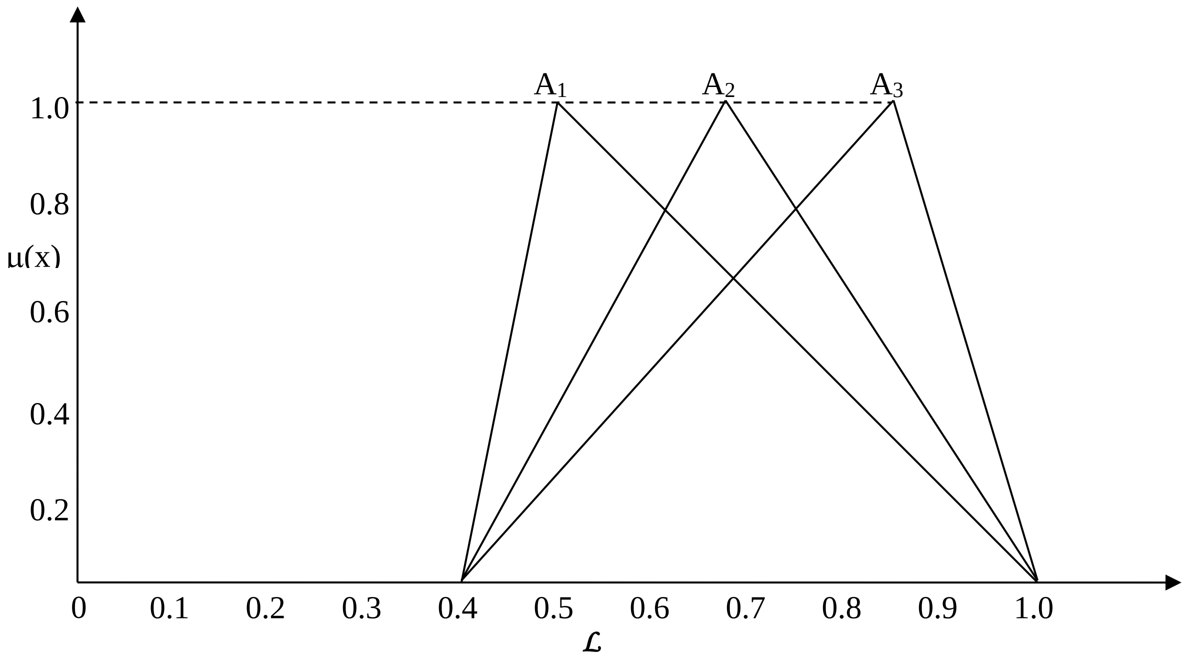

Figure 5: Triangular Fuzzy Numbers $A_1(0.4,0.5,1.0)$, $A_2(0.4,0.7,1.0)$ and $A_3(0.4,0.9,1.0)$

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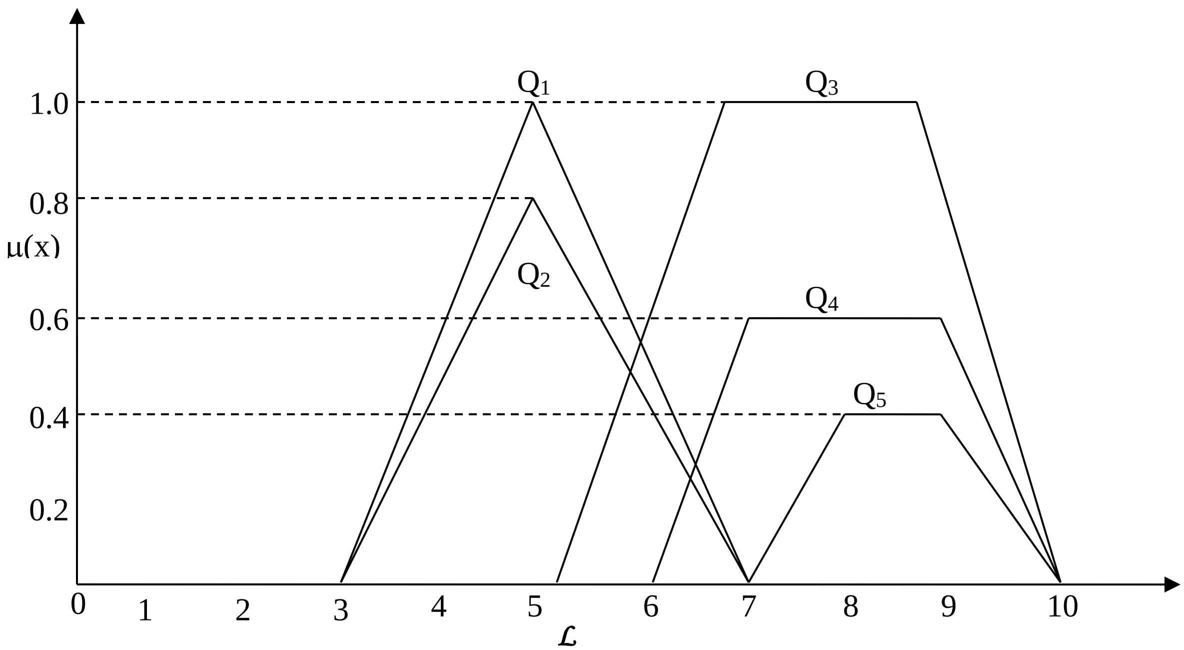

Figure 6: Fuzzy Numbers $Q_1(3,5,7;1)$, $Q_2(3,5,7;0.8)$, $Q_3(5,7,9,10;1.0)$, $Q_4(6,7,9,10;0.6)$ and $Q_5(7,8,9,10;0.4)$

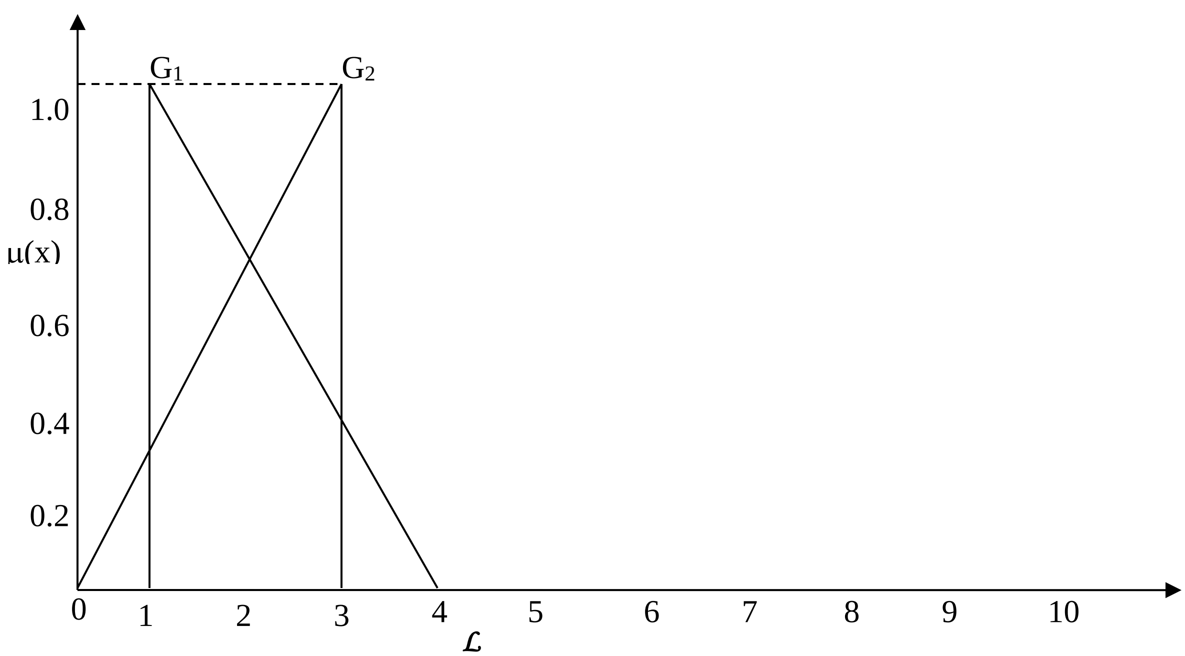

Figure 7: Triangular Fuzzy Numbers $G_1(1,1,4;1)$ and $G_2(0,3,3)$

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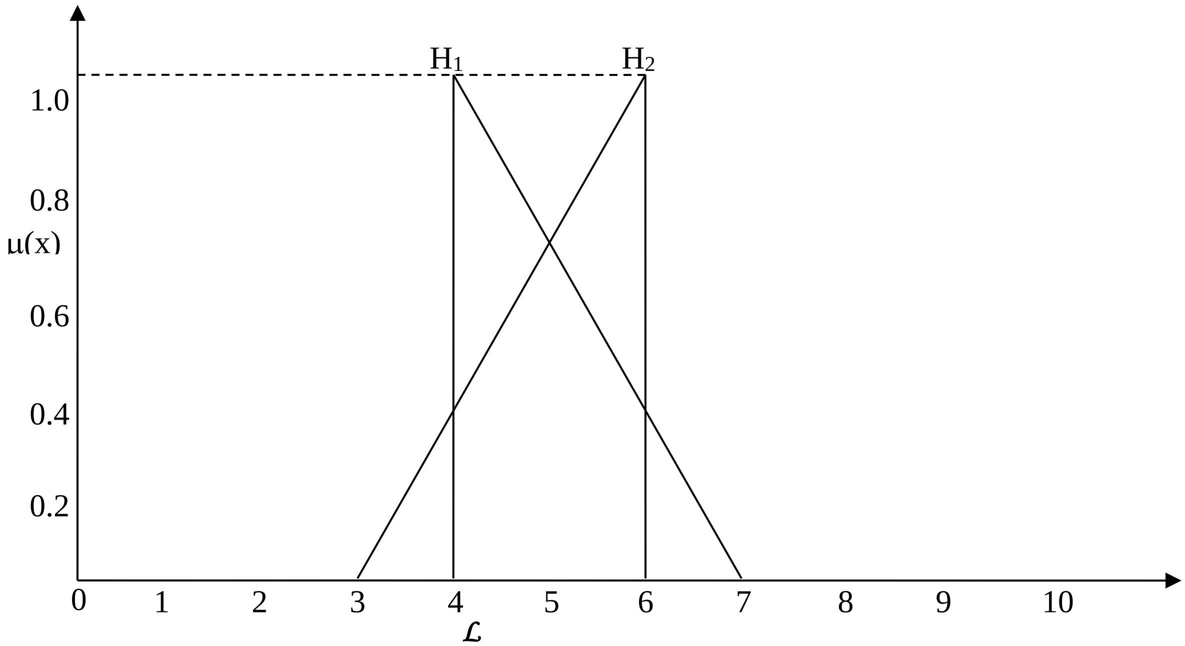

Figure 8: Triangular Fuzzy Numbers $H_1(4,4,7)$ and $H_2(3,6,6)$

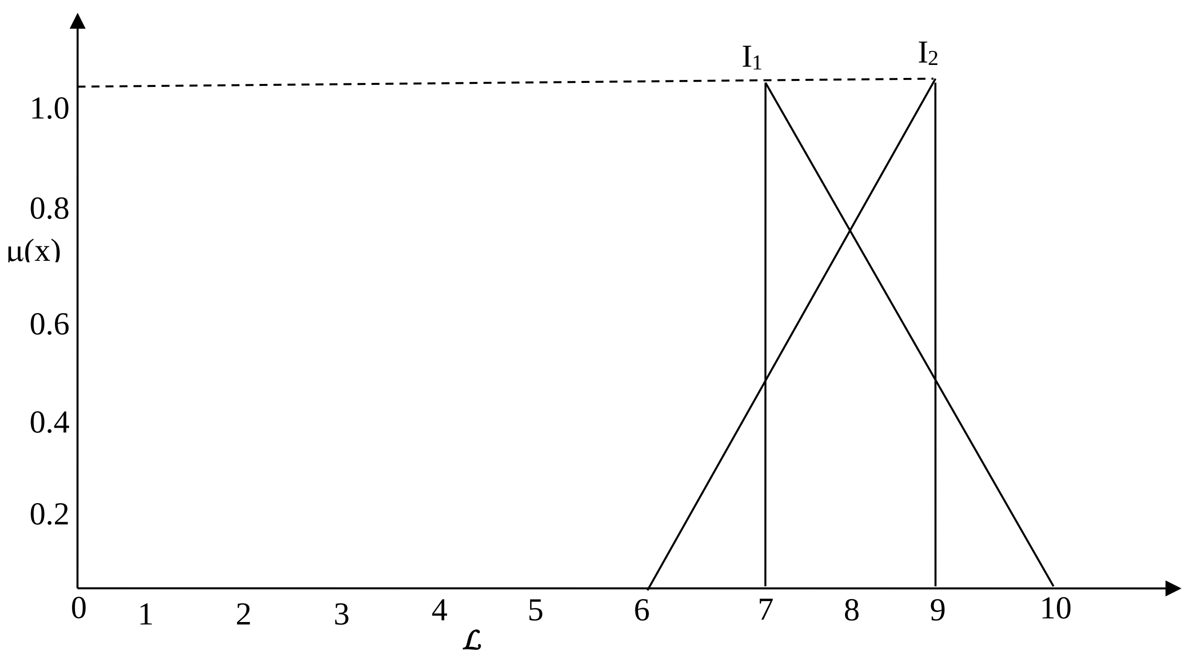

Figure 9: Triangular Fuzzy Numbers $I_1(1,1,4;1)$ and $I_2(0,3,3)$