JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p.19-24
https://publishoa.com
ISSN: 1309-3452

Strong and Weak Perfect Dominating Set in Fuzzy Graph

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Received 2022 March 15; **Revised** 2022 April 20; **Accepted** 2022 May 10.

Abstract
The application of graph theory in simulating the key characteristics of systems with finite components has been well-documented. The telephone network, the train network, communication issues, the traffic network, and many other networks are shown graphically. Graph theoretic models can occasionally provide a helpful structure for the application of analytic methods. Modeling a relationship between a group of things using a graph is another option. An edge represents the relationship between two items if they have an unordered relationship, whereas a directed edge represents the relationship between two ordered objects.
Keywords: Graph Theory, Fuzzy graph, weak perfect Graph,

Introduction
An element of a subset of the universal set may be graded as belonging to the set if its value falls inside a closed interval of real numbers, as stated in Zadeh's work. Many of Zadeh's theories have been put to use in various branches of science and engineering. Fuzzy set theory has also had an impact on theoretical mathematics. Topology, abstract algebra, geometry, graph theory, and analysis have all benefited from the introduction of fuzzy set theory concepts into their work. Graph theoretic notions like routes, cycles, trees, and connectedness have fuzzy analogs, which Rosenfeld presented as fuzzy graphs. Fuzzy graphs were invented by Nagoor Gani and Chandrasekaran, who discovered the dominating set and dominance number.

Preliminaries
We present the imaginative idea of strong and weak perfect dominating sets in fuzzy graph, just as the meanings of a negligible strong and weak perfect dominating set, a base strong and weak perfect dominating set, and a strong and weak perfect dominating number. In a fuzzy graph, we show the documentation for a wide range of kinds of least strong and weak perfect dominating sets, just as various sorts of strong and weak perfect dominating numbers.

Definition: A collection of perfect fuzzy dominant sets is defined as follows: Ps is considered to be a powerful ideal dominating set. **(SPDS)** If $\mu(U,V) = \sigma(U) \wedge \sigma(V)$, and dN(u) ≥ dN(v).

Definition: A perfect dominating set Ps of a fuzzy graph is defined as follows: If Ps-v does not include a strong perfect dominating set of G for each vertex v in Ps, then G is said to be a minimum strong perfect dominating set for that vertex v.

Definition: The strong perfect dominance number G of a fuzzy graph is the smallest strong perfect dominating set that has the lowest fuzzy cardinality in the graph. It is signified by γspf(G).

Definition: The set with the maximum fuzzy cardinality of a minimum strong perfect dominating subset of a fuzzy graph G increases in size as the number of strong perfect dominations in the fuzzy graph G increases. It is denoted by (G) spf.

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Fuzzy Dominating Set

Definition: Consider the case when G = (σ,μ) is a fuzzy graph. If there is one such subset D for any v, V- D, then there exists a subset D of V that is dominant over G, and vice versa.

Definition: Set D in a fuzzy graph is dominant if and only if there is no minimum dominating set of G in existence. If there is no least dominating set of G in existence, then set D is dominant if and only there is no minimal dominating set of G in existence. The cardinality of all minimally dominant sets with the least value is represented by the dominance number G. The fuzzy independence number of G is the total number of maximum fuzzy independent sets, and it is represented by the symbol (G).

Strong (weak) Triple connected perfect domination number of a fuzzy graph

Other fundamental limitations on the strong(weak) triple linked perfect dominance number of a fuzzy graph, as well as some related findings, are discussed in this section.

Definition:

This is referred to as the strong (weak) triple connected perfect dominance set. The strongest (weakest) triple connected perfect dominating set with the least cardinality is referred to as the strong (weak) triple connected perfect dominance number. Furthermore, it is signified by the γstcp (γwtcp).

Definition:

A subset S of a nontrivial fuzzy graph G is said to be a strong (weak) triple connected perfect dominating set if <Ps(w)t> is a strong (weak) triple connected perfect dominating set and the induced sub graph Ps(w)t is a triple connected subgraph of Ps(w)t. If Ps(w)t is a strong (weak) triple connected perfect dominating set and the induced sub graph Ps(w)

Theorem

There is no such thing as a strong (or weak) triple linked perfect dominating set for any fuzzy graph.

Proof

A linked graph is defined as a graph that is neither highly connected (triple connected) nor weakly connected by definition (weakly connected). Only linked fuzzy networks with a strong (weak) triple connected dominant set are addressed in this study; no other types of graphs are.

Theorem

In a fuzzy graph G, each perfect dominating set with a strong (weak) triple link corresponds to one of the perfect dominating sets of the fuzzy graph G.

Remark

The converse of the above theorem need not be true.

Perfect Dominating Set

Definition: When considering a fuzzy graph G, a perfect dominant set D is one in which every vertex v that is not included in D is adjacent to precisely one vertex included in D.

Definition: A set with the least number of perfect dominant components that is conceivable D is the dimension of a fuzzy graph. G is said to exist if D-v is not a dominating set for any vertex v in D and G is not a dominating set for any vertex v in D. A perfect dominating set with the shortest cardinality is referred to as a "minimum perfect dominating set." It is denoted by the Pf set of the letter G. When it comes to fuzzy graphs, the perfect dominance number is equal to the cardinality of the smallest perfect dominating set. It is represented by the letter pf (G).

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Theorem
ny strong, ideal dominating set that you can think of Ps (SPDS) is a perfect dominating set in the game of chess. The inverse is not true in this case.

Proof
Let Ps be a minimal strong perfect dominating set of G.
Suppose $v \in V$-Ps is not dominated by exactly one vertex u in Ps.
We know that by definition of strong perfect set is if every vertex $v \in V$-Ps is strongly perfect dominated by exactly one vertex u in Ps.
That is, u, $v \in V$, u dominated by exactly one vertex u in Ps if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ and $dN(u) \geq dN(v)$ then we have every vertex $v \in V$-Ps is dominated by exactly one vertex u in Ps which is a contradiction.
Therefore Ps is a perfect dominating set.

Theorem: A set that is absolutely dominant. As long as each vertex v in D is a perfect dominating vertex and Pnf [v, D] is not empty, D is a minimum perfect dominating set; otherwise, D is not a minimal perfect dominating set.

Proof:-
Assume that D is the lowest possible value and that v D is the highest possible value. There is a vertex w that is not in D-v but is next to at least two points in D-{v}, indicating that the vertex w is not in D-{v}.
If w=v, then v Pnf [v, D]. If w=v, w cannot be next to at least two vertices of D-{v}, since D is a perfect dominating set. As a consequence, no vertex of D-v is congruent with w.
Due to the fact that D is a perfect dominating set. Only v is close to W in D. N(w)TD = v. As a result, w Pnf [v, D].
Suppose, on the other hand, that v D and Pnf [v, D] contain some vertex w of G.
If w = v, then either w is adjacent to at least two D-{v} vertices or w is not adjacent to any D-{v} vertices. As a consequence, D-{v} does not constitute a perfect dominating set. If w=v, then N(w)TD=v shows that w is not a vertex of D-{v}. Thus, in all cases where v D is a perfect dominating set, D-{v} is not a perfect dominating set. As a consequence, the value of D is negligible.

Definition:
A collection of perfect fuzzy dominant components Pw is a phrase that refers to a weak perfect dominating set that has a low level of strength (WPDS). If $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ and $dN(v) \geq dN(u)$.

Definition:
The fuzzy graph's pw G is said to be a minimum weak perfect dominating set if Pw-v does not have a weak perfect dominating set of G for every vertex v in Pw.

Definition:
It is the smallest fuzzy cardinality of a minimum weak perfect dominating set of G that determines a graph's weak perfect dominance number. It is denoted by γwpf (G).

Definition:
If a fuzzy graph G has a larger weak perfect dominance number then the graph has a higher maximum fuzzy cardinality. It is denoted by Γ (G) wpf.

Proof:-
Each D1 vertex x has a single D2 vertex v(x) adjacent to it. Additionally, each y in D2 has a unique vertex u(y) in D1 that is close by. These functions have the unique property of being invertible. |D1| = |D2|.

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Corrolary:

As long as there are two perfect dominant sets D1, which equals D2, on a fuzzy graph G, D1 T D2 = 4.8. Corrolary Assume that G is an n-node fuzzy graph with a single edge per node. There is a perfect dominating set D if and only if |D| is equal to or larger than n/2.

Theorem

Any strong perfect dominating set Ps (SPDS) of a fuzzy graph G is a strong dominating set.

Proof

Suppose that the perfect dominating set Ps is the smallest strong set of G. vPs, Assume that no vertex v in Ps has a significant effect on u. There is just one vertex in V-Ps that is significantly dominated by one vertex in Ps. There are uPs such that u dominates v for every vertex v, V-Ps, which violates our theory. As a consequence, u has a huge advantage over v.

Theorem

Any weak perfect dominating set (WPDS) of a fuzzy graph G is a weak dominating set.

Proof

Let Pw be a minimal weak perfect dominating set of G. Let u, v∈Pw. Suppose u is not weakly dominated by some vertex v in Pw. We know that if every vertex v∈V-Pw is weakly dominated by exactly one vertex in Pw. Then we get every vertex v∈V-Pw , there exists u∈Pw such that u weakly dominates v which is contradiction to our assumption. Therefore u is weakly dominates v. That is Pw is a weak dominating set.

Strong and weak perfect dominating set in strong fuzzy graph

A fuzzy graph with a strong graph and its strong and weak perfect dominating sets are presented and examined in this section.

Theorem

Let G be a fuzzy wheel Wn+1 strong fuzzy graph, then γspf(G) = {σ(v): v is the centre vertex of G}.

Proof

Let G be a fuzzy wheel Wn+1 with strong fuzzy graph. The vertex set of G are {v, v1, v2,…, vn} , v dominates vi for i= 1, 2, ⋯, n where v is the centre vertex of G. Let Ps be the strong perfect dominating set of a strong fuzzy graph which contains {σ(v): v is the centre vertex of the fuzzy wheel} such that v is the only dominating set. Therefore, strong perfect domination number is σ(v).

THEOREM:

For any diamond strong fuzzy graph G, then γspf (G) = {minσ(u) : u is a perfect dominating vertex} and γwpf (G) ={ maxσ(u) : u is a perfect dominating vertex} .

Proof

Assume that G is a fuzzy graph of diamond strength. Assume that every vertex has a unique membership value. In this instance, each vertex is totally flawless in relation to all other vertices. The perfect dominating set is referred to as a strong perfect dominating set if it includes the vertices with the fewest membership values. The phrase "strong perfect domination number" refers to a strong perfect dominating set's fuzzy cardinality. This means that spf (G) = min (u), where u is a perfectly dominant vertex. The vertices with the greatest membership values form the weak perfect dominating set. The fuzzy cardinality of a weak perfect dominating set is represented by a weak perfect dominance number. This means that wpf (G) = max (u), where u is a perfectly dominant vertex.

THEOREM:
If D is the weakest (strongest) dominating set of a linked IFG G, then V-D is the weakest (strongest) dominating set of G.

Proof:
Assume D is an IFG, G's minimum dominating set. Allow for the possibility that there exists a vertex such that I uv is a strong arc.
Therefore we note u weakly dominates v in G.
Clearly is a weak dominating set of G. DV u−∈Dv∈) () (ud vd N N ≥ D V −
Similarly we prove for minimal weak domination set

Theorem
If G be a fuzzy graph, then $\gamma s(G) \leq \gamma spf(G)$.

Proof
Assume that Ps is the smallest strong perfect dominating set of G. Each strong perfect dominating set of a fuzzy graph is equivalent to a strong dominant set of a fuzzy graph G. There exists a set in which vV-Ps dominates at least one vertex and dN(u) dN(v), implying that Ps is a strong dominating set of G. Therefore we have $\gamma s(G) \leq \gamma spf(G)$.

Theorem
Any weak perfect dominating set (WPDS) of a fuzzy graph G is a weak dominating set.

Proof
Let Pw be a minimal weak perfect dominating set of G. Let u, v∈Pw. Suppose u is not weakly dominated by some vertex v in Pw. We know that if every vertex v∈V-Pw is weakly dominated by exactly one vertex in Pw. Then we get every vertex v∈V-Pw , there exists u∈Pw such that u weakly dominates v which is contradiction to our assumption. Therefore u is weakly dominates v. That is Pw is a weak dominating set

CONCLUSION:
In this paper, we have discussed strong and weak domination perfect set in fuzzy graphs and strong fuzzy graphs. Some definitions, theorems of strong and weak perfect dominating set in fuzzy graph, properties of these parameters have been proved.

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