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Method of Stability Analysis for Nonlinear Automatic Control Systems of Peltier Thermoelectric Modules

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ABSTRACT

Peltier thermoelectric modules (TEM) have several significant advantages over classic compressor-based cooling systems, such as better mass characteristics, non-dependence on special orientation, absence of moving parts, possibility of the precise temperature control over time, etc. In order to adjust different temperature regimes based on TEM, high quality automatic control systems are strongly needed. The development of such systems necessitates the use of complex models which take into account nonlinearities of the Peltier module itself, as well as different other blocks of the control system. The Functional diagram and model of the TEM-based climate control system have been represented. The transfer coefficients of nonlinear links and a scheme as a whole based on piecewise linear approximation have been obtained. A new methodology for modeling the nonlinear stability of climate control systems based on TEM under arbitrarily large impacting disturbances have been deleloped. The presented technique based on the Popov's frequency criterion and piecewise linear approximation allows an analytical study of absolute stability in general for any order of the model and with an arbitrary nonlinearities of components. Based on the proposed approach, Modeling of nonlinear stability of climate conltol systems based on TEM was done. As a result of the simulation, the stability areas of the system with various filters in the feedback loop were estimated. The possible application of the proposed approach in a future for optimization and synthesis of TEM control systems was briefly discussed.

Keywords: Peltier module, thermoelectric system, piecewise linear approximation, nonlinear stability, Popov's frequency criterion.

INTRODUCTION

Currently, cooling systems based on Peltier thermoelectric modules (TEM) are gaining great popularity due to a number of characteristics that favorably distinguish them from traditional compressor cooling systems [1]. In particular, such advantages are the possibility of using both for cooling and heating by simply changing the polarity of the supply voltage [2], high technological characteristics due to the absence of moving parts, ease of integration into trigeneration systems with additional use of renewable energy sources [3-5], etc. Studies aimed at improving thermoelectric systems consider, for example, the use of an additional cooling element [6], special operating modes to achieve maximum efficiency [7-10], thermoelectric energy converters with high operating current [8]. A separate direction is the development and research of control systems for thermoelectric elements to achieve high accuracy and speed of temperature adjustment in dynamic mode and the possibility of thermostating by changing the supply current [11, 12]. The improvement of control systems for thermoelectric elements requires the development and implementation of new methods for their modeling, allowing for the nonlinearity of the Peltier element in a wide range of control currents [13], a high order [14] and a complex system structure with a large number of control objects and, accordingly, control channels, which is typical for climate control systems in agriculture [15].

Despite a large number of studies and the widespread practical use of TEM in practice, the issues of ensuring their sustainability and quality indicators have not been sufficiently considered.

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The aim of the work is to develop and apply a methodology for modeling the nonlinear stability of a trigenerative climate control system based on Peltier thermoelectric modules.

FUNCTIONAL DIAGRAM AND MODEL OF THE TEM-BASED CLIMATE CONTROL SYSTEM

An example of a functional scheme of a trigenerative system based on TEM is shown in Fig. 1. The temperature controller (TC) compares the desired temperature T_{des} and the temperature at the output of the temperature sensor after the inertia compensator (InComp) T_s comp, and also generates a control current for the Peltier thermoelectric module (TEM) I_c. The output parameter of the ventilation system (VS) and the control system as a whole is the temperature of the point heat/cold source T_{ps}, which acts as a control parameter of the temperature sensor (TS) in the feedback circuit, having a temperature T_s at the output.

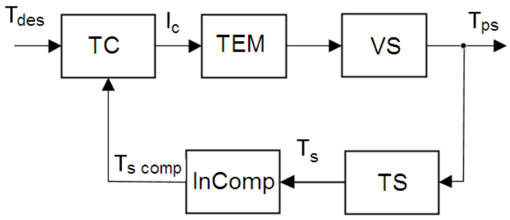

Figure 1. Functional diagram of the TEM-based climate control system with deviation control

The functional model of the TEM-based climate control system with combined regulation is shown in Fig. 2. The transfer characteristics of the model blocks in the forward and backward control circuits contain lower indices 1 and 2, respectively. The forward control circuit can be used to improve the dynamic properties of the system (increase the speed of the transient process, reduce the dumping factor) while maintaining stability and suppressing external interference determined by the parameters of the backward control circuit. The figure shows $x_{Г1,2}$ – destabilizing factors affecting the temperature sensors of the control circuits, ε – destabilizing effect on the temperature controller. The transfer characteristics of TEM and VS in the model are normalized to 1. The inertial properties of the model are given by blocks $M_{1,2}(p)$, where p is the differentiation operator. The nonlinear properties of the model are given by blocks $N_{1,2mn}$, $R_{1,2mn}$, E_m and N_m, where m и n are the numbers of nodes for approximating nonlinear characteristics.

The model of forming the deviation of the output parameter of the model Fig. 2 is determined by the expression

$$y = \frac{1 - n_1 M_1(p) N_{1mn}}{1 + n_2 M_2(p) N_{2mn}} x + \frac{n_1 M_1(p) N_{1mn}}{1 + n_2 M_2(p) N_{2mn}} x_{Г1} + \frac{n_2 M_2(p) N_{2mn}}{1 + n_2 M_2(p) N_{2mn}} x_{Г2} -$$
$$- \frac{N_m}{1 + n_2 M_2(p) N_{2mn}} \varepsilon - \frac{n_1 M_1(p) R_{1mn} + n_2 M_2(p) N_{2mn} + E_m}{1 + n_2 M_2(p) N_{2mn}}.$$

Here, the transfer coefficients of nonlinear links based on piecewise linear approximation are defined as

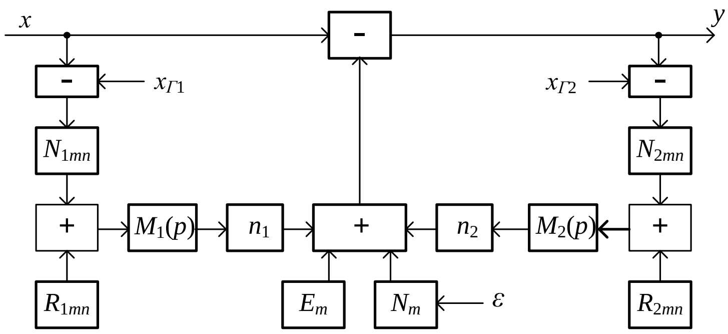

Figure 2. Functional model of the climate control system based on TEM with combined regulation

Here, the transfer coefficients of nonlinear links based on piecewise linear approximation are defined as

$$N_{1mn}=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}K_m K_{1n} Q_m Q_{1n},\ N_{2mn}=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}K_m K_{2n} Q_m Q_{2n},\ N_m=\sum_{m=0}^{M-1}K_m Q_m,$$
$$R_{1mn}=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}K_m B_{1n} Q_m Q_{1n},\ R_{2mn}=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}K_m B_{2n} Q_m Q_{2n},\ E_m=\sum_{m=0}^{M-1}B_m Q_m$$

where the symbols K and B are the coefficients of the approximating segments of the lines, $Q_m=Q_m(u+\varepsilon)$, $Q_{1n}=Q_{1n}(x-x_{\Gamma 1})$, $Q_{2n}=Q_{2n}(y-x_{\Gamma 2})$ are the inclusion functions of the segments of the approximating lines.

The inclusion functions Q_m and $Q_{1,2n}$ are either nonzero and equal to one only in the interval between nodes m and m+1, or between the nodes n and n+1, respectively

$$Q_m(u+\varepsilon)=\frac{1}{2\Delta}\sum_{\lambda=0}^{1}\sum_{\gamma=0}^{1}(-1)^{\lambda+\gamma}\left|u+\varepsilon-U_{p_m}-\gamma\Delta_u+\Delta(1-\lambda)\right|,$$

$$Q_{1n}(x-x_{\Gamma 1})=\frac{1}{2\Delta}\sum_{\lambda=0}^{1}\sum_{\gamma=0}^{1}(-1)^{\lambda+\gamma}\left|x-x_{\Gamma 1}-D_{1n}-\gamma\Delta_{d1}+\Delta(1-\lambda)\right|,$$

$$Q_{2n}(y-x_{\Gamma 2})=\frac{1}{2\Delta}\sum_{\lambda=0}^{1}\sum_{\gamma=0}^{1}(-1)^{\lambda+\gamma}\left|y-x_{\Gamma 2}-D_{2n}-\gamma\Delta_{d2}+\Delta(1-\lambda)\right|,$$

where Δ is an arbitrarily small quantity (($\Delta\to 0$), λ and γ are integers,

METHODOLOGY FOR MODELING THE NONLINEAR STABILITY OF CLIMATE CONTROL SYSTEMS BASED ON TEM UNDER ARBITRARILY LARGE IMPACTING DISTURBANCES

According to V.M. Popov's criterion, for the absolute stability of the equilibrium position of a nonlinear system with a stable linear part, the existence of a real g is sufficient for which the condition is satisfied

$$\forall\omega\geq 0:\ \mathrm{Re}\left[(1+j\omega g)W(j\omega)\right]>-1/k, \qquad (1)$$

where k the angle of absolute stability, $W(j\omega)=W_R(\omega)+jW_I(\omega)$ is the complex transfer function of the filter in the feedback circuit in the form of the sum of the real and imaginary parts.

We introduce a modified complex transfer function

$$W^*(j\omega)=W_R(\omega)+jW_I^*(\omega), \qquad (2)$$

where $W_I^*(\omega)=\omega W_I(\omega)$.

Then the sufficient condition of absolute stability (1) will take the form $W_R(\omega)-gW_I^*(\omega)>-1/k$. At the stability boundary, the condition takes the form of equality (the equations of the Popov line)

$$W_R(\omega)-gW_I^*(\omega)=-1/k. \qquad (3)$$

The expressions of the lines approximating the left part (4) at the current nodes will take the form

$$W^*_{I\ m,n}(W_R) = g_{m,n}(W_R - b_{m,n}), \tag{4}$$

where n, m are current numbers of approximation nodes,, $g_{m,n} = (W^*_{I\ m} - W^*_{I\ n})/(W_{R\ m} - W_{R\ n})$ are angular coefficients, $b_{m,n} = W_{R\ m} - W^*_{I\ m,n} / g_{m,n}$ are abscissas of approximating lines.

We introduce a piecewise linear function

$$Q_{m,n}(\vartheta) = K_\sigma \sum_{\lambda=0}^{1}\sum_{\gamma=0}^{1}(-1)^{\lambda+\gamma}\left|\vartheta + \vartheta_n - \vartheta_m(1-\gamma) - \frac{\lambda}{2K_\sigma}\right|$$

where K_σ is the steepness of the lateral components of the inclusion function. The function is equal to 1 if its argument belongs to the section [ω_n; ω_m], and 0 otherwise. Thus, the function excludes the "false" values of the abscissa $b_{m,n}$ outside the section [ω_n; ω_m]:

$$b^*_{m,n} = b_{m,n} Q_{m,n}(b_{m,n}). \tag{5}$$

The boundary values of k for each true abscissa $b^*_{m,n}$ are obtained by substituting (5) into the right part of (3)

$$\mathrm{k}_{m,n} = -1/b^*_{m,n}. \tag{6}$$

The lower bound of stability N_2 corresponds to the maximum of all negative values $k_{m,n}$:

$$\widetilde{N}_2^{lower} = \max\{k_{m,n}[1 - \widetilde{q}(k_{m,n})]\}, \tag{7}$$

where $\widetilde{q}(\vartheta) = \frac{1}{2\Delta}[|\vartheta + \Delta| - |\vartheta| + \Delta]$ is the inclusion function taking the value 1 at $\vartheta \geq 0$ and 0 at $\vartheta < 0$. The upper limit of stability is determined by a minimum of all positive values $k_{m,n}$:

$$\widetilde{N}_2^{upper} = \min\{k_{m,n}\widetilde{q}(k_{m,n})\}. \tag{8}$$

The presented technique based on piecewise linear approximation allows an analytical study of absolute stability in general for any order of the model and with an arbitrary nonlinearities of components.

MODELING OF NONLINEAR STABILITY OF CLIMATE CONTROL SYSTEMS BASED ON TEM

Tables 1-6 show the boundary values of the stable coefficients of the climate control system with band-pass filter (BPF) of 4, 6 and 8th orders, as well as the rejection filter (RF) of 6, 8, and 10th orders in the feedback circuit.

Table 1. Lower limit of stability with a 4th-order rejection filter in a feedback circuit operating in linear and nonlinear mode

γ	1	2	3	4	5

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$\widetilde{N}_2^{lower}$ (linear mode)	-0.98	-0.96	-0.95	-0.94	-0.93
$\widetilde{N}_2^{lower}$ (nonlinear mode)	-0.97	-0.93	-0.905	-0.88	-0.87

Table 2. Lower limit of stability with a 6th-order rejection filter in a feedback circuit

γ	1	2	3	4	5
$\widetilde{N}_2^{lower}$ (linear mode)	-0.98	-0.95	-0.93	-0.9	-0.89
$\widetilde{N}_2^{lower}$ (nonlinear mode)	-0.98	-0.94	-0.9	-0.88	-0.86

Table 3. Lower limit of stability with a 8th-order rejection filter in a feedback circuit

γ	1	2	3	4	5
$\widetilde{N}_2^{lower}$ (linear mode)	-0.99	-0.96	-0.92	-0.9	-0.88
$\widetilde{N}_2^{lower}$ (nonlinear mode)	-0.99	-0.95	-0.91	-0.88	-0.86

Table 4. Upper limit of stability with a 6th-order band-pass filter in a feedback circuit

γ	1	2	3	4	5
$\widetilde{N}_2^{upper}$ (linear mode)	64	27	18.9	15.63	13.82
$\widetilde{N}_2^{upper}$ (nonlinear mode)	32	13.5	9.5	7.8	6.9

Table 5. Upper limit of stability with a 8th-order band-pass filter in a feedback circuit

γ	1	2	3	4	5
$\widetilde{N}_2^{upper}$ (linear mode)	64	20.3	12.6	9.8	8.3
$\widetilde{N}_2^{upper}$ (nonlinear mode)	46.2	14.6	9.1	7	6

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Table 6. Upper limit of stability with a 10th-order band-pass filter in a feedback circuit

γ	1	2	3	4	5
$\tilde{N}_2^{upper}$ (linear mode)	92.3	21.9	12.2	8.8	7.2
$\tilde{N}_2^{upper}$ (nonlinear mode)	75.9	18	9.9	7.2	5.9

The transfer functions of the filters have the following form:

$$M_2^{LPF}(p) = 1/(1+Tp)^I, \quad M_2^{HPF}(p) = (Tp)^I/(1+Tp)^I,$$

$$M_2^{BPF}(p) = \mathrm{H}_{\mathrm{LPF}}(p)\mathrm{H}_{\mathrm{HPF}}(p) = (\gamma Tp)^{0.5I}/[(1+Tp)(1+\gamma Tp)]^{0.5I},$$

$$M_2^{RF}(p) = 1 - M_2^{ЇÔ}(p).$$

Here T is the time constant of the link in the LPF and the LPF, γ is the ratio of LPF and HPF time constants in the composition of the BPF and the RF. Modeling has shown that in the nonlinear mode of operation of the climate control system, its stability area significantly narrows compared to the linear mode. Thus, the upper boundary value of stability for the 6th order BPF decreases by a factor of 2 (Table 4). The difference in the stability regions decreases with increasing order of the filter of both types (both bandpass and rejection). The results of the calculation of the lower bound of absolute stability for PF coincide with the results for the linear regime.

CONCLUSION

The relevance of the study of the absolute stability of climate control systems based on Peltier thermoelectric modules at an arbitrary magnitude of the impacting disturbances (stability "as a whole") is noted. A method for analyzing the absolute stability of high-order thermoelectric climate control systems with various types of control path filters has been developed. The proposed approach is based on the Popov's frequency criterion and piecewise linear approximation of the frequency characteristics of the system. Modeling of the stability regions of a thermoelectric system with filters of various types and orders is performed. The conducted studies revealed a significant difference in the calculated boundary coefficients of the device regulation in linear and nonlinear mode. The developed technique makes it possible to analyze the stability of a system of arbitrary order with variable coefficients, which is important for diagnosing a system with parameters that degrade during operation.

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REFERENCES

[1] Yu-Wei Chang, Chih-Chung Chang, Ming-Tsun Ke, Sih-Li Chen, Thermo-electric air-cooling module for electronic devices, Appl. Therm. Eng. 29 (2009) 2731e2737.

[2] Mark Gillott, Liben Jiang, Saffa Riffat, An investigation of thermoelectric cooling devices for small-scale space conditioning applications in buildings, Int. J. Energy Res. (2009).

JOURNAL OF ALGEBRAIC STATISTICS
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https://publishoa.com
ISSN: 1309-3452

[3] Huang, Y.; Wang, Y.; Rezvani, S.; McIlveen-Wright, D.; Anderson, M.; Hewitt, N. (2011): Biomass Fueled Trigeneration System in Selected Buildings. Energy Conversion and Management, 52, pp. 2448–2454

[4] Wilfried V.S., Feasibility of photovoltaic - thermoelectric hybrid modules. Appl Energy 2011; 88:2785–90.

[5] Hara T.O., Azum H.J., Cooling performance of solar cell driven, thermoelectric cooling prototype headgear. Appl Therm Eng 1998; 18:1159–69.

[6] Gorobets N.V., Ohrem V.G. Peltier thermoelectric cooler with additional conductive element / Applied physics. - № 4. - 2007. - Pp. 124-127.

[7] Zone A.P. Determining the conditions for obtaining the maximum energy efficiency of the Peltier element / News of Southwestern State University. - 2016, №3(20). -Pp. 153-158.

[8] Ohrem V.G. Thermoelectric cooling using the Peltier effect / Applied physics. - № 5. - 2011. - Pp. 123-126.

[9] Evdulov O.V. Development of devices and systems for cooling based on high-current thermoelectric energy converters, Dissertation for the degree of Doctor of Technical Sciences, Makhachkala. – 2019. - 330 p.

[10] Gnusin P.I. Study of the Peltier element efficiency in various operating modes / Video science. - №1(1). - 2016. – Pp. 20-27.

[11] Grinkevich V.A. Synthesis of a temperature controller for Peltier element // Collection of scientific works NSTU. – 2019. – No 1 (94). – P. 7-31. – DOI: 10.17212/2307-6879-2019-1-7-31.

[12] Grinkevich V.A. Investigation of a mathematical model of a thermostat based on the Peltier element / Collection of scientific works of NSTU. – 2017. – No 3 (89). – Pp. 62-77.

[13] Surzhik, D.I., Vasilyev, G.S., Kuzichkin, O.R., Konstantinov, I.S. Construction of energy-saving cooling and thermoelectric regenerative systems based on peltier modules (2020) International Multidisciplinary Scientific GeoConference Surveying Geology and Mining Ecology Management, SGEM, 2020-August (4.1), pp. 37-44.

[14] Kuzichkin Oleg R., Vasilyev Gleb S., Surzhik Dmitry I. Modeling of nonlinear control systems for thermoelectric cooling and regenerative systems based on Peltier modules. // International Journal of Engineering Research and Technology. ISSN 0974-3154, Volume 13, Number 12 (2020), pp. 5268-5273.

[15] Surzhik, D.I., Kuzichkin, O.R., Vasilyev, G.S. An integrated approach to the construction of energy-saving trigeneration systems for objects of the agro-industrial complex (2021) International Journal of Engineering Research and Technology, 13 (12), pp. 4622-4626.