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Separation Axioms on S-Topological BE-Algebras

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ABSTRACT:
An S-Topological BE-Algebras(STBE-Algebras) is a BE-Algebra equipped with a special type of topology that makes the operation (defined on it)as S-topological continuous. In this paper, we discuss the separation axioms on a STBE-Algebras.
Keywords: STBE-Algebra, Semi-open, Semi-closed, Semi-T_1, Semi-T_2

1. INTRODUCTION

In [1] H.A. Kim and Y.H. Kim introduced the notion of BE-algebras, which is a generalization of BCK-algebras. They also introduced the notion of commutative BE-algebras and studied their properties and characterization. In [6], Mehrshad S and Golzarpoor J studied the topological BE-algebras and discussed their properties. In [7], Jansi M and Thiruveni V introduced the notion of ideals in TSBF-algebras. Motivated by this, in our earlier paper, we introduced the notion of S-topological BE-algebras (STBE-Algebras). In this paper, we discuss the separation axioms of STBE-algebras.

2. PRELIMINARIES

Definition 2.1 [1] A BE-algebra is an algebra (X,*,1) of type (2,0) (that is, a non-empty set X with a binary operation * and a constant 1) satisfying the following conditions

1. $x * x = 1$
2. $x * 1 = 1$
3. $1 * x = x$
4. $x * (y * z) = y * (x * z),\ \forall x, y, z \in X.$

Definition 2.2 [2] A BE-algebra (X,*,1) is called a commutative BE-algebra if it satisfies the identity
$(x * y) * y = (y * x) * x, \forall x, y \in X.$

Definition 2.3 [2] If X is a commutative BE-algebra then $x * y = 1 \text{ or } y * x = 1,$ for all distinct $x, y \in X.$

Definition 2.4 [3] A subset A of a topological space is said to be semi-open if $A \subseteq \overline{Int\ A}$.

Definition 2.5 [3] The complement of a semi-open set is called semi-closed.

Definition 2.6 [3] The semi-closure of a subset A of a topological space is the intersection of all semi-closed set containing A. It is denoted by $\bar{A}^s$.

Definition 2.7 [3] A subset A of a topological space is said to be regular open if $A = \overline{Int\ A}$.

Definition 2.8 [4] A topological space (X, τ_s) is called semi-T_1 if for each two distinct points $x, y \in X,$ there exists

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two semi-open sets U and V such that U containing x but not y and V containing y but not x.

Definition 2.9 [4] A topological space (X, τ_S) is called semi-T_2 if for each two distinct points $x, y \in X$, there exists two disjoint semi-open sets U and V such that $x \in U$ and $y \in V$.

Definition 2.10 [5] A BE-algebra $(X, *, 1)$ equipped with a topology τ_S is called S-topological BE-algebra (STBE-algebra) is the function $f: X \times X \to X$ defined by, $f(x, y) = x * y$ has the property that for each open set O containing $x * y$, there exists a open set U containing x and a semi-open set V containing y such that, $U * V \subseteq O$, for all $x, y \in X$.

Definition 2.11 [6] Let $(X, *, 1)$ be a BE-algebra and $F \subseteq X$. Then F is a filter when it satisfies the conditions:

1) $1 \in F$,
2) $If\ 1 \neq x \in F$ and $x * y \in F$, then $y \in F$.

3. SEPARATION AXIOMS ON S-TOPOLOGICAL BE-ALGEBRAS

Definition 3.1 A S-topological BE-algebra $(X, *, \tau_S)$ is called semi-T_1 STBE-algebra if for each two distinct points $x, y \in X$ there exists two semi-open sets U and V such that U containing x but not y and V containing y but not x.

Definition 3.2 A S-topological BE-algebra $(X, *, \tau_S)$ is called semi-T_2 STBE-algebra if for each two distinct points $x, y \in X$ there exists two disjoint semi-open sets U and V such that $x \in U$ and $y \in V$.

Definition 3.3 Let $(X, *, \tau_S)$ be a STBE-algebra. A non-empty subset $A \subseteq X$ is called an ideal of X if

1) $1 \in A$,
2) $\forall y \in X$ and $\forall x \in A$, if $x * y \in A$, then $y \in A$.

Definition 3.4 Let $(X, *, \tau_S)$ be a STBE-algebra. Let $A \subseteq X$. $a \in A$ is said to be an interior point of A, if there exists an open set U such that $a \in U \subseteq A$.

Theorem 3.5 In a commutative STBE-algebra $(X, *, \tau_S)$ if $\{1\}$ is closed, then it is semi-T_2 STBE-algebra.

Proof: Suppose $\{1\}$ is closed and let x and y be any two distinct points in X. Then either $x * y \neq 1$ or $y * x \neq 1$.

Without loss of generality, suppose that $x * y \neq 1$.

Hence there exists an open set U containing x and a semi-open set V containing y such that $U * V \subseteq X - \{1\}$.

Hence, U is open (and hence semi-open) set containing x, V is a semi-open set containing y and $U \cap V = \phi$.

So, we obtain that X is semi-T_2 STBE-algebra.

Theorem 3.6 If the STBE-algebra $(X, *, \tau_S)$ is T_0, then it is semi-T_1 STBE-algebra.

Proof: Let $x, y \in X$ and $x \neq y$. Then either $x * y \neq 1$ or $y * x \neq 1$.

Suppose that $x * y \neq 1$.

Now, since X is T_0, there is an open set W containing one of $x * y$ and 1 but not the other.

Case (i) Suppose that $x * y \in W$ and $1 \notin W$.

Since X is an STBE-algebra, there exists an open set U containing x and a semi-open set V containing y such that

$U * V \subseteq W$.

Then U and V are the required semi-open sets containing x and y respectively.

$\Rightarrow$ X is semi-T_1 STBE-algebra.

Case (ii) Suppose that $1 \in W$ and $x * y \notin W$. Then we have $x * x = 1 \in W$.

So, there exists an open set U_1 containing x and a semi-open set V_1 containing x such that $U_1 * V_1 \subseteq W$.

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Also, as $y * y = 1 \in W$, there exists an open set U_2 containing y and a semi-open set V_2 containing y such that $U_2 * V_2 \subseteq W$

Therefore, $G = U_1 \cap V_1$ and $H = U_2 \cap V_2$ are two semi-open sets containing x and y respectively.

It is clear that $y \notin G$ and $x \notin H$. $\Rightarrow$ X is semi-T_1 STBE-algebra.

Theorem 3.7 Let $(X,*,\tau_S)$ be a STBE-algebra and A be a filter on X. If 1 is an interior point of A, then A is open.

Proof: Let $x \in A$. Since $x * x = 1$ and $1 \in Int(A)$, there exists an open set U such that $1 \in U \subseteq A$.

Now, as X is an STBE-algebra, there exists an open set V containing x such that $V * x \subseteq U$.

Claim: $V \subseteq A$.

Suppose $y \in V \cap (X \setminus A)$. Then $y * x \in U \subseteq A$. (since $V * x \subseteq U$)

As A is an ideal and $x \in A$, we must have $y \in A$, which is a contradiction to $y \in V \cap (X \setminus A)$.

Hence $V \subseteq A$. That is V is an open set containing x and $V \subseteq A. \Rightarrow A$ is open.

Theorem 3.8 Let X be a STBE-algebra and A be an ideal in X, which is open. Then A is semi-closed and hence it is regular open.

Proof: Let $x \notin A$.

Since $x * x = 1$ and X is an STBE-algebra, there exists an open set U containing x and a semi-open set V containing x such that $U * V \subseteq A$.

Take $W = U \cap V$. Then W is a semi-open set containing x and $W * W \subseteq A$.

Claim: $W \subseteq X \setminus A$

Suppose not. Let $y \in W \cap A$. Then, as A is an ideal, we must have $W \subseteq A$, a contradiction.

Hence A is semi-closed.

Since A is open, we have $A \subseteq Int(\overline{A}) \subseteq A. \Rightarrow A = Int(\bar{A}) \Rightarrow A$ is regular open.

Theorem 3.9 Let $(X,*,\tau_S)$ be a STBE-algebra and F be a filter of X. If F is open, then it is closed.

Proof: Let F be an open filter of X. We show that X-F is open.

Let $x \in X - F$. Since F is open, 1 is an interior point of F.

Since $x * x = 1$, there exists an open set V containing x and a semi-open set W containing x such that $V * W \subseteq F$.

We claim that $V \subseteq X - F$.

If $V \nsubseteq X - F$, then there exists an element $y \in V \cap F$. For each $z \in W$, we have $y * z \in V * W \subseteq F$.

Since $y \in F$ and F is a filter, $z \in F$. Hence $W \subseteq F$ and so $x \in F$, which is a contradiction.

Therefore, $x \in V \subseteq X - F$, which implies that X-F is open and hence F is closed.

Definition 3.10 Let $(X,*,\tau_S)$ be a STBE-algebra, U be a non-empty subset of X and $a \in X$. The subsets Ua and aU are defined as follows:

$Ua = \{x \in X : x * a \in U\}$ and $aU = \{x \in X : a * x \in U\}$. Also if $K \subseteq X$, we define $KU = \bigcup_{a \in K} aU$ and $UK = \bigcup_{a \in K} Ua$.

Theorem 3.11 Let $(X,*,\tau_S)$ be a STBE-algebra, U and F be two non-empty subsets of X. Then the following are true.

(1) If U is open, the Ua is open and aU is semi-open.

(2) If F is closed, then Fa is closed and aF is semi-closed.`

Proof: (1) Let U be an open set, $a \in X$ and let $x \in Ua$. Then $x * a \in U$.

Since X is an STBE-algebra, then there exists an open set G containing x and a semi-open set A containing a such that

$G * A \subseteq U, x * a \in Ga \subseteq U,$ thus $G * a \subseteq U.$ Then $x \in G \subseteq Ua.$ So Ua is open.

To prove that aU is semi-open, let $x \in aU$. Then $a * x \in U.$

Since X is an STBE-algebra, there exists an open set A containing a and a semi-open set H containing x such that $A * H \subseteq U,$ so $a * x \in aH \subseteq U,$ thus $a * H \subseteq U.$ Hence $x \in H \subseteq aU.$ Therefore aU is semi-open.

(2) Let F be closed. Then F^C is open. Hence by (1), $(F^C)a$ is open and $a(F^C)$ is semi-open.

Clearly, $(Fa)^C = (F^C)a$and $(aF)^C = a(F^C).$ Hence, $(Fa)^C$ is open and $(aF)^C$ is semi-open.

Consequently, Fa is closed and aF is semi-closed.

4. CONCLUSION

Here we have discussed the separation axioms in STBE-Algebras using the concepts of open, semi-open, closed, semi-closed sets and also listed the properties of ideals and filters of an STBE-algebra in terms of these axioms. We can also study the properties of STBE-Algebras satisfying the separation axioms in further studies.

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