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Composite Labelling of Unary Operation of Comp Graph and 2-Tuple of Coconut Tree

[*1] T.R. Manikandan, [2*] V.E. Sasikala
[*1] M.Phil Scholar, [2*] Corresponding Author, Research Supervisor,
Department of Mathematics, Vels Institute of Science, Technology & Advanced Studies (VISTAS), Chennai, India.

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ABSTRACT: In this paper, we introduce the concept of composite labelling. All the graphs considered are undirected connected simple graphs with order n and size m. Let u, v, w ∈ V(G). A composite labelling is a bijective function f: (V(G) ∪ E(G)) → {1, 2, 3, ..., m + n} such that gcd (f(uv), f(vw)) ≠ 1. A graph that admits composite labelling is known as a composite graph. In this paper we investigate composite labelling for unary operation of comp graph and splitting graph.

INTRODUCTION: This project deals with graph labelling. All the graphs considered here are finite, simple and undirected. A graph labelling is an assignment of integers to the vertices or edges or both the subject to certain conditions. If the domain of the mappings is the set of vertices (or edges), then the labelling is called a vertex labelling (or an edge labelling). The concept of graph labelling was first introduced by Rosa in mid sixties. In the year 1967, Rosa introduced a new type of graph labelling which he named as β-labelling. Let G be any graph and m be the number of edges in G. Rosa introduced a function f from the set of vertices of G to the set of the integers {0,1,2,…,m}, so that each edge is assigned the label │f(u)-f(v)│, with all labels are distinct. Golomb independently studied the same type of labelling and named this labelling called graceful labelling. Since then, different properties of graceful labelling of graphs have been introduced and studied extensively by several graph theorists.

The concept of composite labelling was first introduced by Kureethara Joseph Varghese and P. Stephy Maria [2]. The composite labelling for tree, cycle and ladder graphs has been shown by Kureethara Joseph Varghese and P.Stephy Maria. In this study, we look into composite labelling of particular graph types and how it might be used.

Tri-diagonal matrices arise in many science and engineering areas, for example in parallel computing, telecommunication system analysis, and in solving differential equations using finite differences. In particular, the eigenvalues of tridiagonal symmetric matrices have been studied extensively starting with Golub in 1962. Moreover, a search on Math Sci Net reveals that over 100 papers with the words “tridiagonal symmetric matrices” in the title have been published since then. Tridiagonal real symmetric

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matrices are a subclass of the class of real symmetric matrices. A real symmetric matrix is a square matrix A with real-valued entries that are symmetric about the diagonal; that is, A equals its transpose AT. Symmetric matrices arise naturally in a variety of applications. Real symmetric matrices in particular enjoy the following two properties: (1) all of their eigen values are real and (2) the eigenvectors corresponding to distinct eigenvalues are orthogonal.

$$An = \begin{bmatrix} \mathbf{0} & \mathbf{1} \cdots & \mathbf{0} \\ \mathbf{1} & \ddots & \mathbf{1} \\ \mathbf{0} & \cdots \ \mathbf{1} & \mathbf{0} \end{bmatrix}$$

with ones on the superdiagonal and subdiagonal and zeroes in every other entry. These matrices, in particular, arise as the adjacency matrices of the path graphs and hence are fundamental objects in the field of spectral graph theory.

2 PRIMILINARY

2.1 Definition.

A graph G consists of a pair (V(G), E(G)) where V(G) is a non-empty finite set whose elements are called vertices and E(G) is a set of unordered pair of distinct elements of V(G). The elements of E(G) are called edges of the graph G.

2.2 Definition

H (V, E) is a directed less simple connected graph with order n of size m. Let uv and vw be two edges that intersect the shared vertex v. uvw is a two-length route in the graph H. f: (V(G) ∪ E(G)) → {1, 2, 3, ..., m + n} is a bijective function with the condition that gcd (f(uv),(vw))≠1. On composite graphs, the labels are also composite.

2.3 Definition

A vertex joined to itself by an edge is called a loop. Let G be a graph, if two or more edges of G have the same end vertices then these edges are called multiple edges.

2.4 Definition

A graph is called simple if it has no loops and no multiple edges.

2.5 Definition

A graph is called simple if it has no loops and no multiple edges.

2.6 Definition.

A comb graph is created by connecting each vertex of a route with a single pendant edge.

2.7 Definition.

The coconut tree is a graph established from the path P_m by appending n new pendant edges at an end vertex of P_m.

2.8 Definition.

Consider G be a simple graph and let G^1 is the another copy of G. Combine each vertex V of G to the corresponding vertex V^1 of G^1 by an edge. The new graph hence obtained is called 2-Tuple of G.

2.9 Definition.

The line graph H of a graph G is a graph the vertices of which correspond to the edges of G, any two vertices of H being adjacent if and only if the corresponding edges of G are incident with the same vertex of G.

2.10 Definition

Bistar graph $\boldsymbol{B_{n,n}}$ is a graph obtained by joining the apex vertices of two copies of star $\boldsymbol{k_{1,n}}$.

2.11 Definition

If P_n is a path graph with n vertices and K1 is a complete graph with one vertex then $P_n \cup K_1$ is a fan graph and it is denoted by Fn.

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2.12 Definition

A wheel graph W_n is a graph formed by connecting a single universal vertex to all vertices of a cycle.

2.13 Definition

The Helm graph H_n is a graph obtained from a wheel graph by attaching a pendant edge at each vertex of cycle Cn.

2.14 Definition (Eigenvalue and eigenvector).

1. An *eigenvector* of A is a nonzero vector v in R^n such that Av=λv, for some scalar λ.
2. An *eigenvalue* of A is a scalar λ such that the equation Av=λv has a nontrivial solution.

If Av=λv for v≠0, we say that λ is the *eigen value for* v, and that v is an *eigenvector for* λ.

3 Composite labelling of Comp Graph

3.1 Theorem. The comb graph $P_n oK_1$ admits composite labeling.

proof:

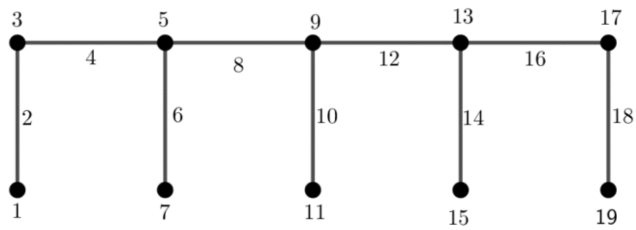
H = $P_n oK_1$ is a comb graph with 2n vertices and (2n − 1) edges. Let's call the vertices

v_1, v_2, ..., v_n and the edges u_1, u_2, ..., u_m. Note that, |V (H)| = 2n and |E(H)| = 2n − . A bijection f : (V (H) S E(H)) → 1, 2, 3, ..., as follows by f(vi) = 2i − 1 for all i and f(vivn) = 2i for all i.

where 1 ≤ i ≤ n(n + 1) .

Clearly gcd(f(uv),(vw))$\neq \mathbf{1}$. Thus the function defined above provides composite labeling for a graph H. That is, comb graph $P_n oK_1$ is a composite labelling.

3.2 Example:



Composite labelling of Comp Graph

4 MAIN RESULTS

Theorem 4.1 Line graph of Comp graph admit Composite labelling.

Proof: The comb graph $P_n \Theta K_1$ has 2n vertices and (2n-1) edges. Let $v_1, v_2, \ldots v_p$ be the path of length p. Let u_1, $u_2, \ldots u_q$ be the corresponding vertices of v_i. Join v_i and u_i for 1≤ i ≤ p, q.

Join v_i and u_{i+1} for 1 ≤ i ≤ p. G=L($P_n \Theta K_1$). Let V(G)={v_i u_i ; 1 ≤ i ≤ p, q} and E(G)= {V_i V_{i+1}; u_i v_i;$v_i u_{i+1}$; 1 ≤ i ≤ p; 1 ≤ i ≤ q}. Then G is of order 2n.

As follows, create a bijection labelling f : (V (H)∪E(H)) → 1, 2, 3, ..., m + n

Define the Vertex labelling

$$\boldsymbol{f(v_i) = n - W(even)\{0,2,4 \ldots.\} \quad if\ i = 1,2,\ldots q}$$

$$\boldsymbol{f(u_i) = n - i \quad if\ i = 1}$$

$$\boldsymbol{f(u_i) = n(2n+i) \quad if\ i = 2,\ldots p}$$

Define the Vertex labelling $\boldsymbol{f(e_i) = 2i \quad if\ i = \{1,2,\ldots r\} - \{3,6,12\ldots\}}$

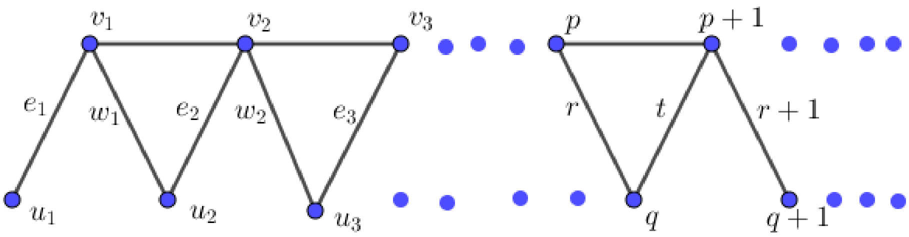
$$\boldsymbol{f(w_i) = i \quad if\ i = 1}$$

$$\boldsymbol{f(w_i) = 2n+1 \quad if\ i = \{2,3,\ldots.s\} - \{4,4+3,4+6\ldots..\}}$$

$$\boldsymbol{f(s_i) = 3n \quad if\ i = 1,2,\ldots\ldots.t}$$

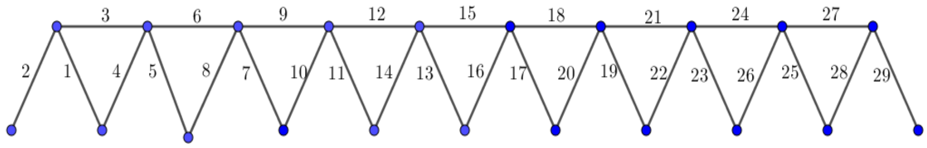
Clearly gcd(f(uv), (vw))$\neq \mathbf{1}$.

Thus the function defined above provides composite labeling for a graph H. That is, comb graph of line graph L(PnoK$_1$) is a composite labelling**.**

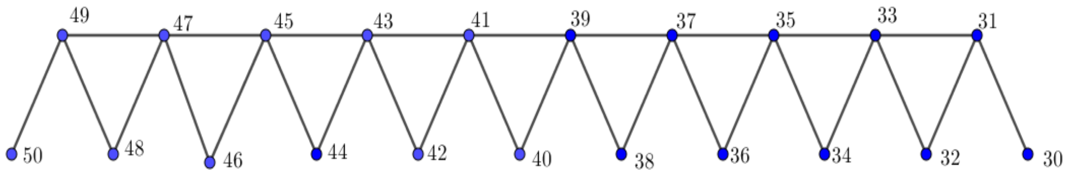


Comp graph of a line graph L(P$_n$oK$_1$)

Example 4.2 Consider a comb graph of line graph L(P$_9$oK$_1$)



Vertex Labelling of L(P$_9$oK$_1$)



Edge Labelling of L(P$_9$oK$_1$)

Clearly gcd(f(uv),(vw))$\neq$ **1**.

Thus the defined above graph provides composite labeling for a graph H. That is, comb graph of line graph L(PnoK$_1$) is a composite labelling**.**

5 Energy of the Graph:

We have to find the energy of the line graph for comp graph. The Corresponding Matrix of the Graph is M(L(P$_9$oK))$_1$=

$$\begin{bmatrix} 0 & n(n-1) & 0 & 0 & . & . & . & . & 0 \\ n(n-1) & 0 & n(n-1) & 0 & . & . & . & . & 0 \\ 0 & n(n-1) & 0 & n(n-1) & . & . & . & . & 0 \\ 0 & 0 & n(n-1) & 0 & . & . & . & . & 0 \\ 0 & 0 & 0 & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & n(n-1) \\ 0 & 0 & 0 & 0 & . & . & . & n(n-1) & 0 \end{bmatrix}$$

Definition 5.1 (Tridiagonal matrix). A tri-diagonal symmetric matrix is a Toeplitz matrix in which all entries not lying on the diagonal, super diagonal or sub diagonal are zero.

Definition 5.2 (The S_n matrices). Let S_n be an n×n tri-diagonal symmetric matrix in which the diagonal entries are zero, and the super diagonal and sub diagonal entries are all one.

Example 5.3. Here are the S_n matrices for n = 1, . . . , 4.

$S_1 = (0)$

$$S_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$S_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

A typical method of finding the eigenvalues of a square matrix is by calculating the roots of the characteristic polynomial of the matrix.

Definition 5.4 (The characteristic polynomial $f_n(\lambda)$). Given the matrix S_n. The characteristic polynomial $f_n(\lambda)$ is the determinant of the matrix $S_n - \lambda I_n$; that is, $f_n(\lambda) = |S_n - \lambda I_n|$.

Remark 5.5 If we set $f_n(\lambda) = 0$, then clearly the n roots (i.e., eigenvalues) of this characteristic equation give the spectrum of S_n. Moreover, these eigenvalues are real since S_n is a real symmetric matrix, and these eigenvalues are distinct since the sub diagonal and super diagonal entries of S_n are nonzero. In particular, S_n has n distinct real eigenvalues.

Example 5.6 Consider the matrix S_4 and the determinant $|S_4 - \lambda I_4|$, which gives the characteristic polynomial $f_4(\lambda)$.

$$S_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|S_4 - \lambda I_4| = \begin{bmatrix} -\lambda & .1 & 0 & 0 & 1 \\ -\lambda & 1 & 0 & 0 & 1 \\ -\lambda & 1 & 0 & 0 & 1-\lambda \end{bmatrix} = \lambda^4 - 3\lambda^2 + 1.$$

Thus the characteristic polynomial is $f_4(\lambda) = \lambda^4 - 3\lambda^2 + 1$. The roots of the corresponding characteristic equation $f_4(\lambda) = 0$ yield the four distinct eigenvalues

$$\lambda_1 = \frac{1+\sqrt{3}}{2} = \varphi \approx 1.61803$$

$$\lambda_2 = \frac{-1+\sqrt{5}}{2} = \frac{1}{\varphi} \approx .61803$$

$$\lambda_3 = \frac{1-\sqrt{5}}{2} = -\frac{1}{\varphi} \approx -.61803$$

$$\lambda_4 = \frac{-1-\sqrt{5}}{2} = -\varphi \approx -1.61803$$

where φ is the golden ratio.

The n distinct eigenvalues of each S_n matrixis

$$\lambda_s = 2\cos\frac{s\pi}{m+1} \; for\; s = 1,\dots,n$$

for the n distinct eigenvalues of each S_n matrix to give a sufficiency criterion for when the eigenvalues of the matrix S_m are also eigen values of the matrix S_n.

Therefore the Energy of the $M(L(P_9 o K_1))$ Matrix is $|2\cos\frac{s\pi}{m+1}|$

4 MAIN RESULTS

Theorem 4.3 2-Tuple of coconut tree admits Composite labelling.

Proof: Consider the graph 2-Tuple of CT(m, n) with the vertices v_i , u_j ,v_i' , u_j' , i = 1,2…m and j=1,2,…n and the edges$E = E_1 \cup E_2$, where E_1 = v_i , $v_i'$and E_2= u_j, u_j'. Now, |V(G)| =2(m+n) and |E(G)| =3(m+n)-2. Let the vertex function f defined as:

$1 \le i \le m$, and $1 \le j \le n$

$f(v_i)$=2(12+i) +1 if i=1,2,….m

$f(u_j)=2(12+i)$ $if \quad i = 1,2, \dots n$

$f(v_i')=4(3i+1)$ $if \quad i = 1,2, \dots m$

$f(u_j')=2(9+w(2n))$ $if \quad i = 1,2, \dots n$

$f(u_j')=25$ $if \quad i = 3$

Let the edge function f defined as: $1 \le i \le m$, and $1 \le j \le n$

$f^*(v_i v_{i+1})$ = 4(i-1)+1 ; $1 \le i \le m$

$f^*(u_j u_{j+1})$ = 2(i+W)+1 ; $1 \le i \le n$

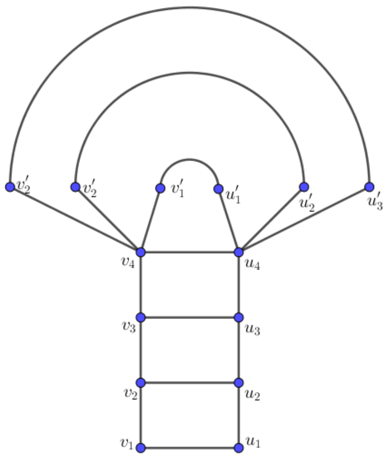
$f^*(v_i u_j)$ = 2i ;$1 \le i \le m$

$f^*(v_i {v'}_i)$ = [4(i+2) +1] ;$1 \le i \le m$

$f^*(u_j {u'}_j)$ = [1+2(i+2)] ; $1 \le i \le n$

$f^*({v'}_i {u'}_j)$ = i+4 ;$1 \le i \le m$

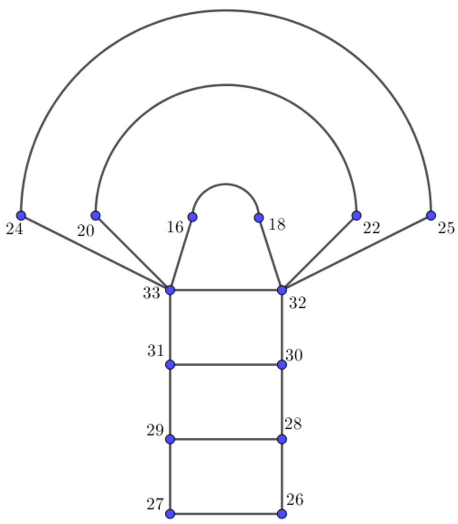
Clearly gcd(f(uv),(vw))≠ 1. Thus the function defined above provides composite labeling fora graph H. That is, 2-Tuple of coconut tree is composite labelling.



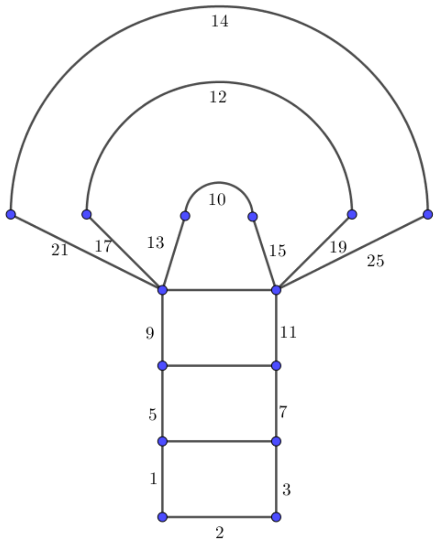
Composite Labelling of 2-Tuple of coconut tree CT(m, n)

Example 4.4 Consider a CT(4, 3) Graph

Consider a 2-Tuple of coconut tree CT(4,3)



Vertex Labelling of the CT(4,3)



Edge Labelling of the CT(4, 3)

Clearly gcd(f(uv),(vw))$\neq$ **1**. Thus the defined above graph provides composite labeling for a graph H. That is, comb graph of line graph L(P_noK_1) is a composite labelling**.**

6. CONCLUSION:

It's very Interesting to study graphs which admit of Composite labeling of various classes of graphs such as The comb graph ($P_n \times K_1$), The Line graph for Comb Graph, 2-Tuple of coconut tree are established. Also we find the Energy of the the Line graph for Comb Graph. Composite labelling of other sorts of graphs is still a work in progress and it will be completed later.

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