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# Homogeneous Bianchi Type $VI_h$ String Dust Cosmological Model with Perfect Fluid in General Relativity

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**Abstract:** Homogeneous Bianchi type VIh string dust cosmological model in general relativity is investigated. To get deterministic solution of Einstein's field equation we assume the condition $A = \beta B^n$ and string tension density $\lambda$ is equal to rest density $\rho$ .Our model has a deceleration parameter value of $q = -0.77$ for $n = 0.16$ Also, some physical and geometrical parameters of the model are discussed. Our model represents expanding, shearing and non- rotating universe.
**Keywords**:Bianchi Type VIh, perfect fluid, cosmic string, dust and general relativity.

## INTRODUCTION

In the study of early universe, cosmic string plays an important role. These strings arise as topological defects during the phase transition of early universe[1, 2]. Also, many unified theories of fundamental forces are featured by the cosmic strings. There has been good amount of work on string cosmology [3-6]. David F.Chernoff and S.-H. Henry Tye,Bali and Pradhan,Wang have investigated different aspects of cosmic string [7-9].In general relativity, normally cosmological models are constructed under the presumption that the matter content is an idealized perfect fluid. Roy and Prasad have obtained solutions, which generalize the Bianchi type VIh cosmological models with perfect fluid [10]. Wainwright et al. have obtained solution for Bianchi type III, V, VIh models for stiff perfect fluid [11]. Das et al.[12] ,prescribed most general solution for perfect fluid cosmological universe. Tripathi et.al. Mishra et.al. Santhil et.al. [13-15] investigated Bianchi type VIh model in different aspects.

Apart from this, dust solution is a precise solution of the Einstein field equation in which pressure is zero. G. S. Rathore,Seema Tinker, Bali and Upadhyay,Swati Parikh, Hassan Amirhashchi and Hishamuddin Zainuddin, have studied cosmological models in different aspects with dust fluid [16-20].Raj Bali and Deo Karan Singh investigatedBianchi Type-V Bulk Viscous Fluid String Dust Cosmological Model in General Relativity [21]. Shohei Aoyama et al. investigated the evolution of dust in a cosmological volume[22].

In this paper we have investigated homogeneous Bianchi type VIh string dust cosmological model with perfect fluid in general relativity. To get deterministic solution of Einstein's field equations, we assumed that string tension density $\lambda$ is equal to rest density $\rho$ i.e. $\lambda = \rho.$ Also discussed some physical and geometrical parameters of the model.

## THE METRIC AND FIELD EQUATIONS

Consider Bianchi type $VI_h$ metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2hx} dy^2 + C^2 e^{2hx} dz^2 \qquad (1)$$

where metric potentials $A$, $B$ and $C$ are functions of't' only.

The energy momentum tensor for the cloud of string dust is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j \qquad (2)$$

where

$\rho$ is proper energy density,

$\lambda$ is string tension density,

$x_i$ is the unit space like vector specifying the direction of string satisfying the conditions

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$v_i v^i = -x_i x^i = -1,$ and $v^i x_i = 0$

The flow vector $v^i$ satisfying

$$g_{ij} v^i v^j = -1 \quad (3)$$

The co-ordinates are considered to be comoving so that $v^1 = 0 = v^2 = v^3$ & $v^4 = 1$ for the line element (1)

Choose $x^i$ parallel to x-axis so that $x^i$ =($A^{-1}$, 0, 0, 0)

The Einstein's field equation (in gravitational units $c = 1, 8\pi G = 1$)

$$R_i^j - \frac{R}{2} g_i^j = -T_i^j \quad (4)$$

For the line element (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{h^2}{A^2} = \lambda \quad (5)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{h^2}{A^2} = 0 \quad (6)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{h^2}{A^2} = 0 \quad (7)$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{h^2}{A^2} = \rho \quad (8)$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0 \quad (9)$$

**SOLUTION OF THE FIELD EQUATIONS**

The field equations (5) – (9) are a system of five equations with six unknown parameters $A, B, C, \lambda, p, \rho$.

To obtain explicit solution of the model one additional constraint is required.

We assume the condition $A = \beta B^n$

Equation (9) leads to

$$C = \alpha B$$

where $\alpha$ is constant of integration

For simplicity we assume $\alpha = 1$

$$\therefore B = C \quad (10)$$

Now, equations (5), (6) and (7) becomes

$$\frac{2B_{44}}{B} + \frac{B_4^{\;2}}{B^2} + \frac{h^2}{A^2} = \lambda \quad (11)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{h^2}{A^2} = 0 \quad (12)$$

$$\frac{2A_4 B_4}{AB} + \frac{B_4^2}{B^2} - \frac{h^2}{A^2} = \rho \quad (13)$$

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Using the condition $\lambda = \rho$ in equation (13) and subtracting from equation (11) we get,

$$\frac{B_{44}}{B} - \frac{A_4 B_4}{AB} + \frac{h^2}{A^2} = 0 \qquad (14)$$

After using the condition $\frac{A_4}{A} = n\frac{B_4}{B}$ in equation (14)

Equation (14) leads to

$$\frac{B_{44}}{B} - 2n\frac{B_4^{\ 2}}{B^2} = \frac{-2h^2}{A^2}$$

$$2ff' - 4n\frac{f^2}{B} = \frac{-4h^2}{\beta^2 B^{2n-1}}$$

$$where\ B_4 = f(B),\ B_{44} = ff',\ f' = \frac{df}{dB}$$

$$B_4 = \sqrt{\frac{2}{1-3n}}\frac{h}{\beta}\frac{1}{B^{n-1}}$$

$$B = \left(\sqrt{\frac{2}{1-3n}}\ \frac{nh}{\beta}t + c_1\right)^{\frac{1}{n}}$$ where $c_1$ is constant of integration.

$$B = T^{1/n} \quad (16)$$

$$where,\quad T = \sqrt{\frac{2}{1-3n}}\ \frac{nh}{\beta}t + c_1$$

$$dt = \sqrt{\frac{1-3n}{2}}\ \frac{\beta}{nh}dT$$

(17)

The line element (1) becomes

$$ds^2 = \frac{-\beta^2(1-3n)}{2n^2h^2}dT^2 + T^{2/n}\left(\beta^2 T\ dx^2 + e^{-2hx}dy^2 + e^{2hx}dz^2\right) \quad (18)$$

## PHYSICAL AND GEOMETRICAL FEATURES

The model (18) starts with a big bang at $T = 0$ *and* $\omega = 0$. The deceleration parameter ( $q$ )for the model is given by

$$q = \frac{2(n-1)}{n+2},\quad n \neq -2 \quad (19)$$

The deceleration parameter $q < 0$ (accelerating phase) for $n < 1$ and $q > 0$ (decelerating phase)for $n > 1$ and the value of this Deceleration Parameter is in the range of observed value. The universe is accelerating at present as observed in recent observations of Type Ia supernova [23-28]. From this data the deceleration parameter of the universe is in the range $-1 \leq q \leq 0$

i.e. $q_0 \approx -0.77$ and for our model (18), the deceleration parameter is $q = -0.77$ for $n = 0.16$

The expansion ($\theta$) for the model (18)is given by

$$\theta = \sqrt{\frac{2}{1-3n}}\ \frac{(n+2)h}{\beta T} \quad (20)$$

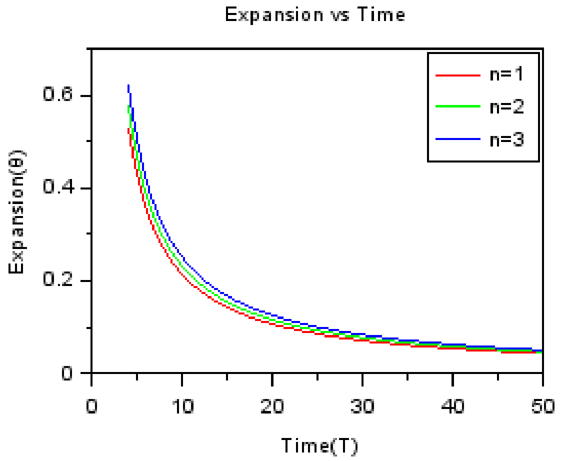


**Fig. 1 The plot of Expansion($\theta$) versus Time(T)**

Figure 1 shows that the expansion ($\theta$) is decreasing function of time (T) for n > 0, $h$ =1, $\beta = 1$ and approaches to zero as time(T) approaches to infinity. Also expansion in the model stop when
n = -2.

The string tension density ($\lambda$) for the model (18) is

$$\lambda = \frac{12nh^2}{(1-3n)\beta^2} \cdot \frac{1}{T^2} \quad (21)$$

Figure 2 depict that the string tension density ($\lambda$) is decreasing function of time (T) for n > 0,
$\beta = 1, h = 1$ and approaches to zero as time(T) approaches to infinity.

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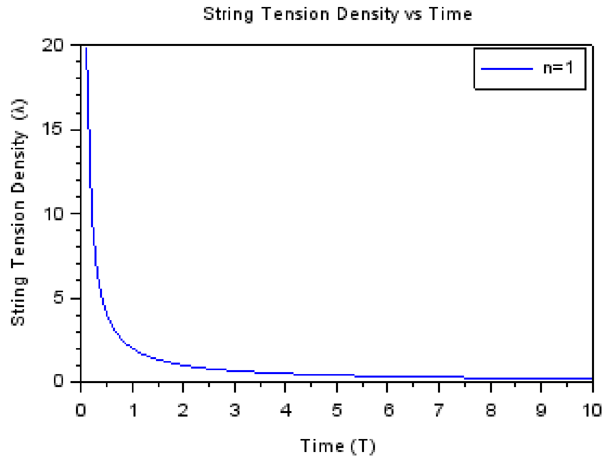


**Fig.2 The plot of String Tension Density(λ) versus Time(T)**

The volume ($V$) of the model (18) is given by

$$V = \beta T^{(n+2)/n} \quad (22)$$

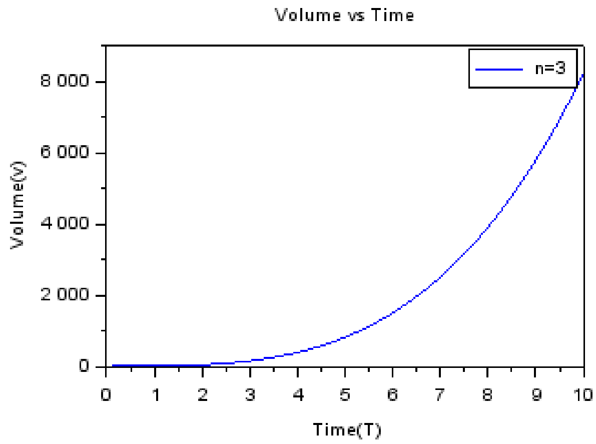


**Fig.3 The plot of Volume($V$) versus Time(T)**

Figure 3 represent the volume $V$ of the universe is zero at the initial epoch $T = 0 \ (if \ \ c = 0)$. As time increases, volume of the universe increases and $T \to \infty, \ V \to \infty$.

The Hubble Parameter (H) for the model (18) is

$$H = \sqrt{\frac{2}{1-3n}} \ \frac{h}{3\beta T} \quad (23)$$

Figure 4 shows that the Hubble parameter value decreases as time increases i.e. $\mathrm{H} \to 0$ as $T \to \infty$

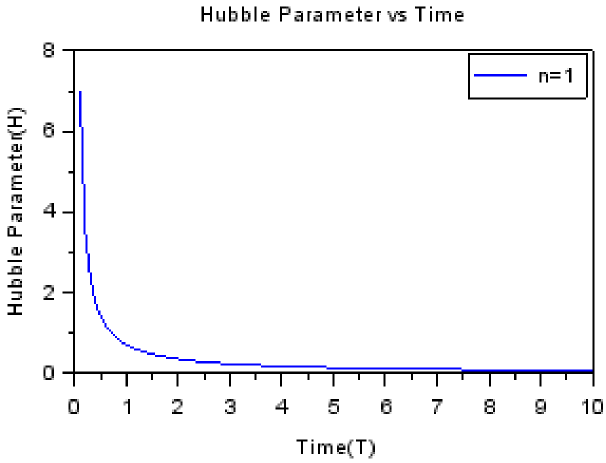


**Fig.4 The plot of Hubble Parameter(H) versus Time(T)**

The Shear of the model(18) is given by

$$\sigma = \sqrt{\frac{2}{3(1-3n)}}\,\frac{h(n-1)}{\beta T}$$

*and*

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}\,(n+2)}\ , n \neq -2 \qquad (24)$$

The ratio of shear '$\sigma$' and the expansion '$\theta$' tends to a finite value $\frac{\sigma}{\theta} \to \frac{1}{\sqrt{3}}\left(\frac{n-1}{n+2}\right)$, $n \neq -2$ the model isotropies for $n = 1$.

**CONCLUSION**

The model (18) starts with a big bang at $T = 0$. The expansion ($\theta$) is decreasing function of time (T) and approaches to zero as time(T) approaches to infinity.Also expansion in the model stop when n = -2. The universe is accelerating at present as observed in recent observations of Type Ia supernova. From this data the deceleration parameter of the universe is in the range $-1 \leq q \leq 0$ i.e. $q_0 \approx -0.77$ and for our model (18), the deceleration parameter is $q = -0.77$ for $n = 0.16$ and the strings disappear from the universe at a later time(i.e. present epoch).
Thus, the model represents expanding, shearing and non-rotating universe.

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