**JOURNAL OF ALGEBRAIC STATISTICS**
Volume 13, No. 3, 2022, p. 1149 - 1156
https://publishoa.com
ISSN: 1309-3452

# Graphical Partition of Triangular Number Graphs and Its Application.

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**ABSTRACT**
In this paper all graphs are finite natural numbers, simple and undirected. A graph labelling is assigned by an integer to the vertices. Here we have discussed some theoretical investigation and found the graphical partitions of triangular number graph, thereafter focused on an algorithm and forwarded an application relating to our theoretical investigation.

**Keyboard:** Triangular number. Triangular Number Graph, Planar Graph, Regular Graph, Graph Partitioning.

## 1. Introduction:

A triangular number or triangle number counts objects arranged in an equilateral triangle. Triangular numbers are a type of figurate number, other examples being square numbers and cube numbers. The nth triangular number is the number of dots in the triangular arrangement with n dots on each side, and is equal to the sum of the *n* natural numbers from 1 to *n*. The sequence of triangular numbers, starting with the $0^{th}$ triangular number, is 0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300, 325, 351, 378, 406, 435, 465, 496, 528, 561, 595, 630, 666. Graph Partitioning is a universally employed technique for parallelization of calculations on unstructured grids for finite element, finite difference and finite volume techniques. It is used in parallelization of matrix vector multiplication in iterative solvers such as PDE solvers using sparse matrix vector multiply. It is also used in parallelization of neural net simulations, particle calculation and VLSI circuit design. Graph Partitioning presents a way to exploit concurrency in a problem by decomposing it into separate units to be mapped onto parallel processors. For example, after finding concurrency in a problem using the Geometric Decomposition pattern, graph partitioning can be used to divide the problem into chunks to be mapped onto parallel processors. Graph Partitioning algorithms fall into the more general category of graph algorithms.

### 1.1 Definitions:

Polygonal numbers are the number of vertices only which will represents by a certain polygon. The first number in any group of polygonal numbers is one and a pendent vertex whose degree is always zero. The second number is equal to the number of vertices of the polygon. The third number is made by extension of sides of polygon from the second polygon number, thus extending the numbers of polygons by similar process expanding the numbers of vertices respectively.

### 1.2 Triangular Number:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55 ………………are known as triangular numbers. The $n^{th}$ triangular number is denoted by $T_n$.

$$\therefore\ T_n = \frac{n(n+1)}{2}, n \geq 1.$$

### 1.3 Triangular number graph:

The triangular numbers are assigned by vertices of the graph therefore numbers of vertices are respectively therefore 1, 3, 6, 10, 15, 21, 28, 36, 45, 55 ……………… and the nth term of vertex is assigned by $T_{vn}$ therefore $T_{vn} = \frac{n(n+1)}{2}, n \geq 1.$

**JOURNAL OF ALGEBRAIC STATISTICS**
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And the numbers of edges for triangular graph are respectively 0, 3, 9, 18, 30, 45, 63, 84, 108, 135…… accordingly and the nth term of edges are assigned as $T_{en}$ .

$$\therefore T_{en} = \frac{3n(n-1)}{2}, n \geq 1.$$

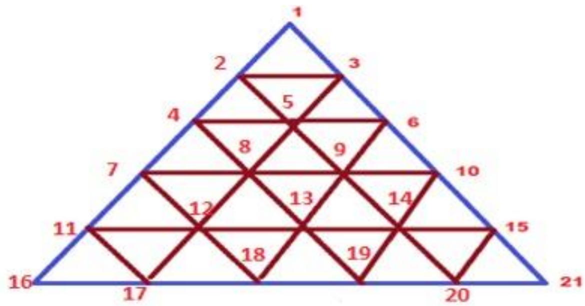


Figure1- Triangular Number graph

### 1.4 Construction of the triangular number graph

$$f(v_i) \rightarrow v_1\ v_{\frac{i^2+i+2}{2}} v_{\frac{(i+1)(i+2)}{2}}$$

Putting $i = 1\ \Rightarrow f(v_1) \rightarrow v_1\ v_2 v_3$

Putting $i = 2\ \Rightarrow f(v_2) \rightarrow v_1\ v_4 v_6$

Thus we can construct the triangular graph and we can expand the value of n and the graph will be enlarged more.

## 2. Background:

A group theoretical approach to the partitioning of integers: Application to triangular numbers, squares, and cantered polygonal numbers has been discussed L.K. Mork et al [1]. The main result of their work is a group theoretic approach to partitioning integers into an arbitrary number of members of an arbitrary set of increasing non-negative integers. The work builds upon the existing literature on partitioning of numbers into triangular numbers and square numbers. Generating function formulae for the number of partitions and distinct partitions of the integers into arbitrarily many triangular numbers, squares, and cantered polygonal numbers are presented as applications.

Musa DEMİRCİ [3] has been discussed in their research paper that Graphs have wide applications in all areas of science and therefore the interest in Graph Theory is increasing every day. Applications of Graph Theory is in Chemistry, Pharmacology, Anthropology, Biology, Network Sciences etc, Graph theory is connected with algebra by means of a new graph invariant Ω and define triangular graphs as graphs with a degree sequence consisting of n successive triangular numbers and use Ω and its properties to give a characterization of them. Finally the conditions put for the realizability of a set D of n consecutive triangular numbers and also give all possible graphs for $1 \leq t \leq 4$.

N. D. Baruah, at el[10] in their research paper have been discussed that the Sums of Squares and Sums of Triangular Numbers Induced by partitions of 8.

Miklavic Stefko et al [2] have been examined the connections between equi-stable graphs, general partition graphs and triangle graphs. While every general partition graph is equistable and every equi-stable graph is a triangle graph, but not every triangle graph is equi-stable, and a conjecture due to Jim Orlin states that every equi-stable graph is a general partition graph. The conjecture holds within the class of chordal graphs; if true in general, it would provide a combinatorial characterization of equi-stable graphs. They have exploiting the combinatorial features of triangle graphs and general partition graphs.

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Tomescu A.M et al[4] have been proposed that a series of counting, sequence and layer matrices are considered precursors of classifiers capable of providing the partitions of the vertices of graphs. Classifiers are given to provide different degrees of distinctiveness for the vertices of the graphs. Any partition can be represented with colours, this fundamental idea, it was proposed to color the graphs according to the partitions of the graph vertices. Two alternative cases were identified: when the order of the sets in the partition is relevant (the sets are distinguished by their positions) and when the order of the sets in the partition is not relevant (the sets are not distinguished by their positions).

Gill Gurbinder et al[5] have been proposed that an experimental study of partitioning strategies for work-efficient graph analytics applications on large KNL and Skylake clusters with up to 256 machines using the Gluon communication runtime which implements partitioning-specific communication optimizations. Evaluation results show that although simple partitioning strategies like Edge-Cuts perform well on a small number of machines, an alternative partitioning strategy.

Kesarev Sergey etal [6] have been proposed that special types of graphs with represent multi-community landscape of Online Social Networks (OSNs). Compared to friendship networks, community-user graphs have super-spreaders and extremely small diameter hampering the achievement of balanced distributions of vertices between computational processes. They has speculative a model of a multi-community networks and test the quality of solutions provided by different graph partitioning algorithms for synthetic Multi-Community Graphs. Finally METIS provides balanced partitions than depth-first traversal heuristic in terms of the edge-cut while sizes of sub-networks are balanced in both cases.

Sha Yuanxia et al [7], have been focuses on the analysis of large-scale distribution network reconstruction fused with graph theory and graph partitioning algorithms. Graph theory and graph segmentation algorithms have been rushed by many researchers in the fields of medicine, drone, and neural network. It is a new comer in the field of computer vision. To improve the performance of distribution networks and reduce network losses, A multi-division model for distribution network construction and reconstruction is established, and a graph theory-based division algorithm method is proposed to effectively solve the problem of feeder-to-feeder reconstruction during large-scale distribution in distribution networks rough its superconductivity phenomenon and the characteristics of clustering algorithm division.

Graph partitioning using quantum annealing on the D-Wave 2X machine. Motivated by a recently proposed graph-based electronic structure theory applied to quantum molecular dynamics (QMD) simulations, graph partitioning is used for reducing the calculation of the density matrix into smaller subsystems rendering the calculation more computationally efficient have been proposed by Mwesigwa H.U et al [8]. Unconstrained graph partitioning as community clustering based on the modularity metric can be naturally mapped into the Hamiltonian of the quantum annealed. On the other hand, when constraints are imposed for partitioning into equal parts and minimizing the number of cut edges between parts, a quadratic unconstrained binary optimization (QUBO) reformulation is required. This reformulation may employ the graph complement to fit the problem in the Chimera graph of the quantum annealed.

## 3. New Approaches:

### 3.1 Theoretical investigation:

#### 3.1.1 Theorem:

The graphical partition of the number $a_n = 3n(n-1)$ for all $n \geq 1$, will represent a Triangular number.

**Proof:**

This theorem can be proving by the method of Mathematical induction only.

For $a_1 = 0$ hence in a pendent vertex always degree is zero which is true for $n = 1$.

For $n = 2 \implies a_2 = 6$, that we can split $6 = 3(2)$ hence there are three vertices of degree two. As shown in figure - 2, which will represents a triangle only, triangular number represents by 3,

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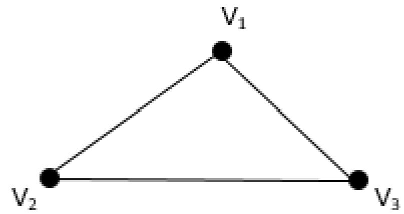


Figure-2 –Triangular Graph

For, $n = 3 \Longrightarrow a_3 = 18$, that we can split $18 = 3(2) + 3(4)$ hence there are six vertices, among them three vertices of degree 2 and remaining three vertices of degree 4. As shown in figure -3 and this will represent four triangles and triangular number will be represents by 6,

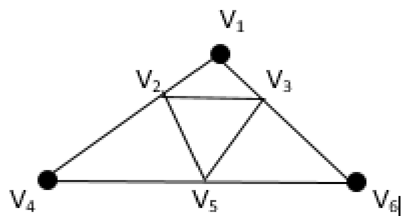


Figure-3- Four Triangular Graph

For $n = 4 \Longrightarrow a_4 = 36$, that we can split $36 = 3(2) + 6(4) + 6(1)$ hence there are ten vertices, among them three vertices of degree 2 and six vertices of degree 4 and one vertex of degree six. As shown in figure -4 and this will represent 9 triangles and triangular number represent by 10.

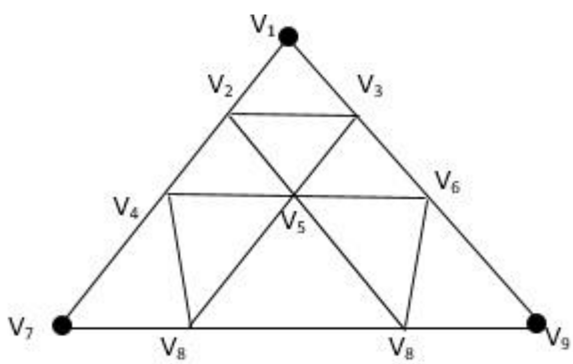


Figure-4: Triangular Graph present 9 Triangle

Let us consider that this is true for $n = k$ for the vertices $v_k = \frac{k(k+1)}{2}$ and the degrees of the vertices will be

$$a_k = 3k(k-1) = 3(2) + 4(3k-6) + 6\left\{\frac{(k-2)(k-3)}{2}\right\}$$

Here we have, there are three vertices of degree 2, $(3k-6)$ numbers of vertices of degree 4 and $\left\{\frac{(k-2)(k-3)}{2}\right\}$ numbers of vertices of degree 6.

**JOURNAL OF ALGEBRAIC STATISTICS**
Volume 13, No. 3, 2022, p. 1149 - 1156
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Here we have observed that when number of vertices increases by $(k+1)$ then number of degrees of the total vertices will increases by $6k$ then it can be expressed as

$v_k = \frac{k(k+1)}{2} \Rightarrow \frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2} = v_{k+1}$ .Which is true for $(k+1)$ increases of vertices.

Again for the degrees of vertices $a_k = 3k(k-1)$ increase by $6k$ then it can be expressed as

$$a_k = 3k(k-1) \Longrightarrow 3k(k-1) + 6k = 3k(k+1)$$

$$= 3(2) + 4[3\{k+1\} - 6] + 6\left\{\frac{\{(k+1)-2\}\{(k+1)-3}{2}\right\} = a_{k+1}$$

This is also true for $k+1$ for increases of degrees and its graphical partitions.

Hence theorem is true for all real number of $n$.

**Table -1**

| SN | No of vertices Implies Triangular Number ($T_{vn}$) | No of edges ($T_{en}$) | Recurrence relation of degrees | Partition of degrees |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 2 | 3 | 3 | 6 | 3(2) |
| 3 | 6 | 9 | 18 | 3 (2)+4 (3) |
| 4 | 10 | 18 | 36 | 3 (2)+4 (6)+6 (1) |
| 5 | 15 | 30 | 60 | 3 (2)+4(9)+6 (3) |
| 6 | 21 | 45 | 90 | 3 (2)+4 (12)+6 (6) |
| 7 | 28 | 63 | 126 | 3 (2)+4 (15)+6 (10) |
| 8 | 36 | 84 | 168 | 3 (2)+4 (18)+6 (15) |
| 9 | 45 | 108 | 216 | 3 (2)+4 (21)+6(21) |
| 10 | 55 | 135 | 270 | 3 (2)+4 (24)+6 (28) |
| nth term | $v_n = \frac{n(n+1)}{2}$ | $e_n = \frac{3n(n-1)}{2}$ | $a_n = 3n(n-1)$ | $3(2) + 4(3n-6) + 6\left\{\frac{(n-2)(n-3)}{2}\right\}$ |

**3.1.2 Proposition: Triangular graph represents the triangular numbers is a planner graph.**

**3.1.3 Proposition: Triangular graph represents the triangular numbers having the degrees of the vertices $2 < d(v_i) < 6$.**

**3.2 Algorithm:**

***Input:*** Let $3n(n-1)$, for $n \geq 1$ be any real numbers which obviously represents the triangular numbers graphs.

***Output:*** To find out different form of graphical partitions is in the form of

$$a_n = 3n(n-1) = 3(2) + 4(3n-6) + 6\left\{\frac{(n-2)(n-3)}{2}\right\},$$

Step-1 - Study the triangular number

Step-2- Consider the triangular number as the number of vertices

Step-3- Found the number of vertices $v_n = \frac{n(n+1)}{2}$

Step-4- Find the number of edges of the triangular number of vertices

Step-5- Found the number of edges $e_n = \frac{3n(n-1)}{2}$

Step-6- Sum of degrees of the vertices is the twice of numbers of edges $\sum d(v_i) = 2e$.

Step-7- Found the sum of degrees of $\sum d(v_i) = 3n(n-1)$

Step-8- Then go the step for partitioning function of the graph by properties

Step-9- Get partition as $3n(n-1) = 3(2) + 4(3n-6) + 6\left\{\frac{(n-2)(n-3)}{2}\right\}$

Step-10- Stop.

## 4 Application

### 4.1 Application of Graph Partitioning:

Graph partitioning algorithms have been used in several applications ranging from structural mechanics to VLSI design. A good graph partitioning algorithm always aims to reduce the communication between machines in their distributed environment and distribute vertices roughly equal to all the machines. Graph partitioning is essential and applicable to following real-world applications:

**4.1.1 Complex Network**: The major complex networks are included: biological networks, social networks, transportation, networks etc.

i) Social Networks: Social networks are very important and widely used network in this era of technology. The most useful social networks are Facebook, Twitter, and LinkedIn. Graph theory is used to make the application of all the social networks They use graph partitioning technology in order to process user query efficiently, as replying a query in a distributed manner is very handy and effective.

ii) Biological Networks: Most of the biological network can be represented by using the graph abstraction. They are protein-protein interaction, metabolic networks, and gene co-expression network. This kind of network is structured by using biological entities (for example protein and genes) and their interactions among them.in this structure biological entities are vertices and their interaction with each other is the edge. This graph-structured network is playing a pivotal role in solving biological interaction problem in a huge a biological network.

iii) Transportation Networks: Planning a trip and route is very common from transportation network by using a GPS (Global Positioning System) tool in the digital era. Graph partitioning can speed up and could be effective in planning a route by leveraging a good graph partitioning algorithm.

**4.1.2 PageRank**: PageRank is an application compute the rank of web rank from web network. PageRank denotes the rank or how important the web page is than others. Finding a rank and importance of a web page from web-graph would be effective by partitioning the whole graph into several distributed machine.

VLSI Design: Very large-scale integration (VLSI) system is one of the graph partitioning problems in order reduce the connection between circuits in designing VLSI. The main objective of this partitioning is to reduce the VLSI design complexity by splitting them into a smaller component. Another goal of a good partitioning is to reduce the number of connection among those circuit components. Here, vertices are the cells of and edges are the wires between them.

Image Processing: Partitioning of Graph has tremendous applications in image segmentation and it is a fundamental rule. Partitioning of Graph is one of the most useful and attractive tools to split into several components of a picture. Pixels are denoting as a vertex and if there are similarities between pixels are represented as an edge.

### 4.2 Application of Triangular number Graph:

**Triangle shape of metallic Bonding structure**:

Let us introduce the different types of chemical bonding using the van Arkel-Ketelaar triangle of bonding. This triangle is a method that utilizes electro-negativity values to predict the type of bonding between two different elements. The more about the triangle is that it allows bonding to be interpreted as a continuum rather than discrete. The metals and metal alloys to be included in the discussion of electro-negativity and its bonding, along with being a good visual representation of polar

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covalent bonds being intermediate between pure covalent and ionic. Tetrahedral of Structure, Bonding & Material Types which is shown in figure-5.

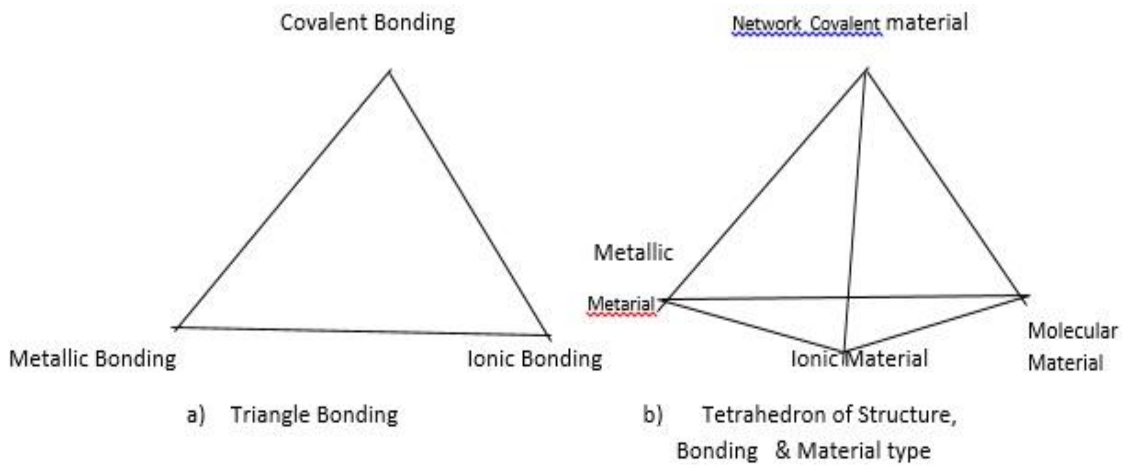


Figure5- Different Metallic Bonding

In 1941 van Arkel recognised three extreme materials and associated bonding types. Using 36 main group elements, such as metals, metalloids and non-metals, he placed ionic, metallic and covalent bonds on the corners of an equilateral triangle, as well as suggested intermediate species. The bond triangle shows that chemical bonds are not just particular bonds of a specific type. Rather, bond types are interconnected and different compounds have varying degrees of different bonding character (for example, covalent bonds with significant ionic character are called polar covalent bonds).

A covalent bond is a chemical bond that involves the sharing of electron pairs between atoms. These electron pairs are known as shared pairs or bonding pairs, and the stable balance of attractive and repulsive forces between atoms, when they share electrons, is known as covalent bonding

Ionic bonding is a type of chemical bonding that involves the electrostatic attraction between oppositely charged ions, or between two atoms with sharply different electro negativities and is the primary interaction occurring in ionic compounds. It is one of the main types of bonding along with covalent bonding and metallic bonding.

Metallic bonding is a type of chemical bonding that arises from the electrostatic attractive force between conduction electrons and positively charged metal ions.

The triangular metallic bonding of Van Arkel is shown in Figure-6, which is resembles with triangular number graph.

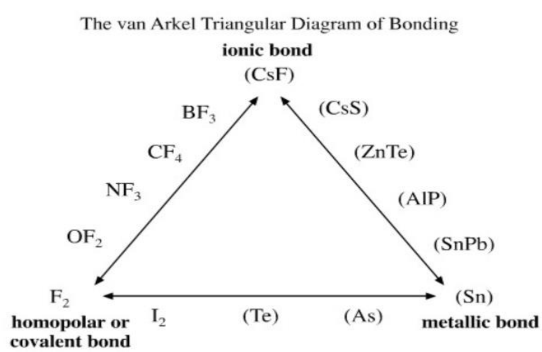


Figure-6: Van Arkel Triangular Diagram of Bonding

The Triangular shape of Zintl ions groups is shown in Figure-7 which is resembles in our triangular number graph structure.

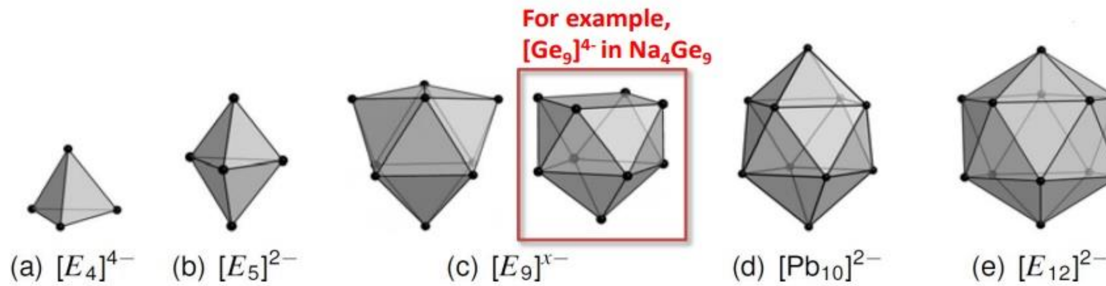


Figure-7: Group up Structure of Zintl ions groups

## 5 Conclusion:

Graphical partitions of Triangular number graph with respect to the construction of Triangular number of graphs are studied here, relating to this work we have developed an algorithm of the theoretical investigation. Finally, some applications of Triangular number graph which structures are resembles with different Van Arkel Triangular Bonding and Zintl ion groups are focused.

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