

Multi – Attribute Decision Making for Selected Students Academic Performance Using Intuitionistic Fuzzy Matrices

J. Boobalan

Department of Mathematic,
Annamalai University,
Annamalainagar - 608 002, India

Deputed To:

Manbumigu Dr. Puratchithalaivar M.G.R. Government Arts and Science College,
Kattumannarkoil – 608 301, India
E-mail:jboobalan@hotmail.com

ABSTRACT

In this paper, we study the multi-attribute decision making problem on a real life situation using intuitionistic fuzzy matrices. We developed a new algorithm to achieve significant result for the above problem.

Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy matrix, Multiple Criteria Decision Making(MCDM), Multi-Attribute Decision Making(MADM)

1. Introduction

Fuzzy set theory was initiated by Zadeh [9] as a means of representing mathematically any imprecise or vague system of information in the real world and for the purpose of developing expert systems and soft computing. The mathematical formulation and the membership function of fuzzy set was introduced by Zadeh in 1965 as a generalization of a crisp set. Matrices play an important role in various areas in Science and Engineering to represent any binary relation. However, we cannot successfully use classical matrices because of various types of uncertainties present in real world situations such as problems in Economics, Engineering, Environment, Medical Science etc. do not always involve crisp data.

In system models based on fuzzy sets, one often uses fuzzy matrices(matrices over fuzzy algebra) to define fuzzy relations. Thomason [7] established some sufficient conditions for convergence under max-min products of the powers of a square fuzzy matrix. The concept of intuitionistic fuzzy sets proposed by Atanassov [2] is an important tool dealing with imperfect and imprecise information. An intuitionistic fuzzy set gives membership degree and non membership degree to describe the level of an element belonging to a set. Hence handling imperfect and imprecise information is more flexible and effective by intuitionistic fuzzy sets. Recently, intuitionistic fuzzy matrices has attracted the attention of several authors and they have developed the theory to some extent.

The concept of intuitionistic fuzzy matrix, a generalization of fuzzy matrix was introduced and developed by Shyamal and Pal [5]. The concept of intuitionistic fuzzy matrix is one of the recent topics developed for dealing with uncertainties present in most of our real life situations. Pal [4] introduced the intuitionistic fuzzy Determinant and studied some properties on it. Xu [8] introduced intuitionistic fuzzy ideal solutions. Adak et.al [1] applied generalized intuitionistic fuzzy matrix in Multi-Criteria Decision Making (MCDM) Problem. In [3] Hao et.al developed some novel intuitionistic fuzzy decision making models by using decision field theory. Motivated by this, we apply Multi-Attribute Decision Making (MADM) method to a real life situation by using intuitionistic fuzzy matrices.

II PRELIMINARIES

We briefly introduce the basic definitions relevant to this paper:

Definition 2.1[1]:1 An intuitionistic fuzzy matrix is a matrix of pairs $A = (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle)$ of non negative real numbers satisfying $\mu_{a_{ij}} + \nu_{a_{ij}} \leq 1$ for all i and j .

Definition 2.2[5]: Let $A = (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle)$ and $B = (\langle \mu_{b_{ij}}, \nu_{b_{ij}} \rangle)$ be two intuitionistic fuzzy matrices of order $m \times n$. Then

(i) $A \oplus B = (\langle \mu_{a_{ij}} + \mu_{b_{ij}} - \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}}\nu_{b_{ij}} \rangle)$ is called the algebraic sum of A and B .

(ii) $A \otimes B = (\langle \mu_{a_{ij}}\mu_{b_{ij}}, \nu_{a_{ij}} + \nu_{b_{ij}} - \nu_{a_{ij}}\nu_{b_{ij}} \rangle)$ is called the algebraic product of A and B .

Definition 32.2[6]: Let A and B be two intuitionistic fuzzy matrices, such that $A = (\langle \mu_{a_{ij}}, \nu_{a_{ij}} \rangle)$, $B = (\langle \mu_{b_{ij}}, \nu_{b_{ij}} \rangle)$ then

$$A \vee B = (\langle \max(\mu_{a_{ij}}, \mu_{b_{ij}}), \min(\nu_{a_{ij}}, \nu_{b_{ij}}) \rangle)$$

$$A \wedge B = (\langle \min(\mu_{a_{ij}}, \mu_{b_{ij}}), \max(\nu_{a_{ij}}, \nu_{b_{ij}}) \rangle).$$

Definition 42.3[5]: The complement of an intuitionistic fuzzy matrix A which is denoted by A^c and is defined by $A^c = (\langle \nu_{a_{ij}}, \mu_{a_{ij}} \rangle)$ for all i, j .

3. Application

In this section, we are working on the intuitionistic fuzzy matrices in connection with the Multi - Attribute Decision Making (MADM) as an application. We consider a real life problem using different attributes for different scales using MADM.

Decision making is a process of selecting the best option (or options) from a finite number of feasible options. It is a very common process present in almost every human functioning and plays an important role in business, finance, management, economics, social and political science, engineering and computer science, biology and medicine etc. Multiple criteria decision making (MCDM) refers to select or ranking options from available options with respect to multiple, usually conflicting criteria in the presence of single decision maker or multiple decision makers. In general, multicriteria decision making can be classified as (a) multiple attribute decision making (MADM) and (b) multiple objective decision making (MODM). MODM problems involve designing the best option given a set of conflicting objectives.

A typical example is mathematical programming problems with multiple objective functions. In contrast to MODM problems, MADM involves in the evaluation, selection and ranking of options from available alternatives with respect to various criteria. In last few decades, the multiple criteria decision making has received a great deal of interest from researchers and practitioners in many disciplines. In many real life decision making situations, the available information is vague or imprecise. To adequately solve decision problems with vague or imprecise information, intuitionistic fuzzy matrix theory have become powerful tool. In last two decades, several multiple criteria decision making theories and methods under intuitionistic fuzzy environment have been proposed for effectively solving the multiple criteria decision making problems and numerous applications have been reported in the literature.

In a given situation, let $S = \{S_1, S_2, \dots, S_n\}$ be the set of n students,

$M = \{M_1, M_2, \dots, M_k\}$ be the set of marks and $P = \{P_1, P_2, \dots, P_m\}$ be the set of problems.

Define Intuitionistic Fuzzy relation R from the set of marks M to the set of problems P (i.e., on $M \times P$) which reveals the degree of association and the degree of non-association between marks and problems. Construct Intuitionistic fuzzy Matrix (F, S) over S where F is a mapping $F : S \rightarrow IFM$ is the collection of all intuitionistic fuzzy submatrices of S . This Intuitionistic fuzzy matrix gives a relation matrix A called student-mark matrix, then construct another intuitionistic fuzzy matrix (G, P) over D where G is a mapping $G : P \rightarrow IFM$ is the collection of all intuitionistic fuzzy submatrices of D .

This intuitionistic fuzzy matrix gives a relation matrix B called Mark- Problem matrix, where each element denotes the weight of the marks for a certain Problems. Compute the complement and score matrix.

Finally, find the maximum score for each student S_i in the score matrix and then conclude that the student S_i is suffering from Problem P_j .

Algorithm:

Step 1: Choose the set of parameters

Step 2: Construct the Intuitionistic fuzzy matrices for each of the parameters Step 3: Calculate the Intuitionistic fuzzy complement matrices

Step 4: Compute the Intuitionistic fuzzy max-min average composition

Step 5: Compute the matrices V, W and obtain the score matrix S using above results

Step 6: Identify the maximum score S_i , for each student.

Then we conclude that the student S_i is affected from Problem P_j

Case Study:

Let there are five Students Arul, John, Michel, Ram and Rahim in a college. (i.e), $S = \{Arul, John, Michel, Ram, Rahim\}$
 The set of marks considered is, $M = \{M1, M2, M3, M4, M5\}$. Let the set of Problems be $P = \{Lethargy, Adolescent Problem, Family Problem, Economic Problem, Memory Loss\}$.

Table 1: represents the Determination of Marks

A	M1	M2	M3	M4	M5
Arul	(.4,.5)	(.5,.2)	(.4,.7)	(.6,.3)	(.3,.7)
John	(0,.6)	(.5,.4)	(.6,.7)	(.3,.7)	(.2,.4)
Michel	(.7,.4)	(.5,.6)	(.4,.5)	(.7,.9)	(.3,.4)
Ram	(.3,.4)	(.5,.2)	(.5,.8)	(.6,.5)	(.4,.2)
Rahim	(.4,.1)	(.2,.1)	(.1,.4)	(.6,.9)	(.7,.5)

Table 2: represents the relationship between marks and problems of students

B	Lethargy	Adolescent Problem	Family Problem	Economic Problem	Memory loss
M1	(.5,.4)	(.3,.9)	(.6,.3)	(.5,.2)	(.1,.4)
M2	(.7,.2)	(.4,.6)	(.2,.2)	(.1,.6)	(.2,.7)
M3	(.3,.7)	(.2,.3)	(.4,.6)	(.6,.4)	(.7,.2)
M4	(.1,.6)	(.5,.2)	(.4,.5)	(.3,.7)	(.7,.4)
M5	(.4,.3)	(.7,.9)	(.2,.2)	(.1,.6)	(.2,.7)

Table 3: represents the max-min average composition matrices

$A \Phi B$	Lethargy	Adolescent Problem	Family Problem	Economic Problem	Memory loss
Arul	(.54,.36)	(.62,.42)	(.45,.72)	(.74,.23)	(.25,.47)
John	(.24,.45)	(.47,.34)	(.56,.57)	(.23,.27)	(.52,.45)
Michel	(.47,.24)	(.75,.18)	(.64,.55)	(.17,.29)	(.23,.54)
Ram	(.36,.18)	(.75,.32)	(.65,.78)	(.45,.25)	(.51,.42)
Rahim	(.74,.21)	(.32,.11)	(.63,.41)	(.46,.29)	(.57,.36)

Table 4: Complementation Matrix Score

$A^c \Phi B^c$	Lethargy	Adolescent Problem	Family Problem	Economic Problem	Memory loss
----------------	----------	--------------------	----------------	------------------	-------------

Arul	(.36,.54)	(.42,.62)	(.72,.45)	(.23,.74)	(.47,.25)
John	(.45,.24)	(.34,.47)	(.57,.56)	(.27,.23)	(.45,.52)
Michel	(.24,.47)	(.18,.75)	(.55,.64)	(.29,.17)	(.54,.23)
Ram	(.18,.36)	(.32,.75)	(.78,.65)	(.25,.45)	(.42,.51)
Rahim	(.21,.74)	(.11,.32)	(.41,.63)	(.29,.46)	(.36,.57)

Table 5: V Matrix

V	Lethargy	Adolescent Problem	Family Problem	Economic Problem	Memory loss
Arul	0.34	0.42	0.36	0.25	0.40
John	0.25	0.41	0.21	0.38	0.28
Michel	0.27	0.31	0.28	0.18	0.42
Ram	0.21	0.41	0.23	0.32	0.29
Rahim	0.16	0.29	0.24	0.41	0.39

Table 6: W Matrix

W	Lethargy	Adolescent Problem	Family Problem	Economic Problem	Memory loss
Arul	0.47	0.52	0.49	0.54	0.68
John	0.34	0.61	0.58	0.52	0.45
Michel	0.46	0.52	0.47	0.27	0.56
Ram	0.48	0.51	0.61	0.68	0.59
Rahim	0.16	0.42	0.25	0.33	0.21

Table 7: Score Matrix

$S = V - W$	Lethargy	Adolescent Problem	Family Problem	Economic Problem	Memory loss
Arul	-0.13	-0.1	-0.13	-0.29	-0.28
John	-0.09	-0.2	-0.37	-0.14	-0.17
Michel	-0.19	-0.21	-0.19	-0.09	-0.14
Ram	-0.27	-0.1	-0.31	-0.36	-0.3
Rahim	0.0	-0.13	-0.01	0.08	0.18

Conclusions:

From the above Table 7, we conclude that Arul suffer from Adolescent Problem, John's study impacted by lethargy, Michel was disturbed by Economic Problem, Ram had Adolescent Problem and Rahim was disturbed by Memory Loss.

Acknowledgement:

The author would like to thank the Tamil Nadu State Council(TANSICHE), Chennai, India for granting financial support in the form of Minor Research Project to complete this work.

References

- [1]. Adak. A.K, Bhowmik. M and Pal. M, Application of Generalized Intuitionistic Fuzzy Matrix in Multi-Criteria Decision Making Problem, *J. Math. Comput. Sci*, 1(1) (2011) 19-31.
- [2]. Atanassov. K, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20(1) (1986) 87-96.
- [3]. Hao. Z.N, Xu. Z.S, Zhao. H and Zhang. R, Novel intuitionistic fuzzy decision making models in the framework of decision field theory, *Inf. Fusion* 33(2017):57–70.
- [4]. Pal. M, Intuitionistic fuzzy determinant, *Vidyasagar University Journal of Physical Sciences*, 7 (2001) 87-93.
- [5]. Shyamal. A.K and Pal M, Distance between intuitionistic fuzzy matrices, *Vidyasagar University Journal of Physical Sciences*, 8 (2002) 81-91.
- [6]. Shyamal. A.K and Pal. M, Two new operators on fuzzy matrices, *Journal of Applied Mathematics and Computing* 15(1-2) (2004) 91-107.
- [7]. Thomason. M. G, Convergence of powers of a fuzzy matrix, *Journal of Mathematical Analysis and Applications*, 57 (1977) 476-480.
- [8]. Xu. Z.S, (2007b) Models for multiple attribute decision making with intuitionistic fuzzy information, *Int J Uncertain Fuzziness Knowl-Based Syst.* 15(3) (2007b):285–297.
- [9]. Zadeh L.A., *Fuzzy Sets, Information and Control*, 8(3)(1965) 338- 353.