

GMI Method-Fuzzy EOQ model without shortages

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ABSTRACT

For an EOQ model without shortages, the set up cost and the demand are fuzzified and an optimal total cost value is calculated. In this research, we have applied the method of graded mean integration of defuzzy the fuzzy objective function and then solved to estimate optimal values of order quantity and the respective annual total cost.

Keywords. Fuzzy optimal function, fuzzy numbers, GMI, EOQ Model

Introduction

Zadeh first introduced the concept of fuzziness, later it was developed and used in most of the field among which the decision making plays as important role. Most of the researchers fuzzified the input parameters, variables and also the objective functions to obtain an optimal solution which seems to more accurate when compared to the crisp values. After this arised the defuzzification process by many methods like signed distance, graded mean integration, centroid method etc.

Chang,Yao and Lee studied an EOQ model for fuzzy backorder quantities. Later Chang & Lee improved the model involving backorders using the method of signed distance for defuzzification. Ishii & Konno determined a stochastic inventory problem with fuzzy storage cost. Lee and Chang again determined fuzzy production inventory model using signed distance method approach.

Lee & Yao calculated an optimal EOQ in fuzzy seems for an inventory without backorders. Lin & Sai solved a transportation problem using genetic algorithm where the demand and supply are imprecise in nature. Lin & Lee also used the signed distance method for defuzzification process in thesis assessment for sampling survey. Vujoservic determined an EOQ formula for a fuzzy inventory cost.

In this paper the demand rate (r) and the cost of holding (a) are fuzzified by considering them as triangular fuzzy numbers (TFN) of an EOQ model with no shortages. Hence estimated a fuzzy inventory cost which is further defuzzified by GMI technique to calculate the total inventory cost.

Preliminaries

For the proposed model, the relevant definition of fuzzy sets are mentioned below:

2.1. Definition

Let $\tilde{u}$ be a fuzzy set on R . The membership function of the fuzzy point is

$$\mu_{\tilde{u}}(x) = \begin{cases} 1, & x = \mu \\ 0, & x \neq \mu \end{cases}$$

2.2. Definition

Let $[u, v; \alpha]$ be a fuzzy set on R . The membership function of the α -level fuzzy point is

$$\mu[u, v; \alpha] = \begin{cases} \alpha, & u \leq x \leq v \\ 0, & \text{otherwise} \end{cases}$$

where $0 \leq \alpha \leq 1$ and $u < v$.

2.3. Definition

Let $\tilde{B} = (x, y, z)$ represent a triangular fuzzy number. The GMI method of $\tilde{B}$ is determined as

$$\theta(\tilde{B}) = \frac{1}{6}(x + 4y + z)$$

1. EOQ Model without shortages based on GMI method.

T- Time cycle

a- Holding price per unit of item

q- Order quantity

c- Amount for ordering

r- Demand

TC- Total annual cost

The crisp total cost function without shortages is

$$G(q) = \frac{1}{2}aTq + \frac{cr}{q}, q > 0 \quad (1)$$

The crisp optimal solution of equation (1) are as follows:

$$\text{Optimal order quantity value } q_c = \sqrt{\frac{2cr}{aT}} \quad (2)$$

Equation (2) is calculated by assuming fixed lead time. But it is not true in real life as it will vary atleast in a small quantity and hence we set the input parameters 'r' and 'a' as follows:

$$\tilde{a} = (a - \phi_1, a, a + \phi_2)$$

$$\tilde{r} = (r - \phi_3, r, r + \phi_4)$$

The above are called the triangular fuzzy number which we use in equation (1) to fuzzify the total cost.

The fuzzy total cost is

$$G(\tilde{a}, \tilde{r}) = \frac{(\tilde{a}Tq)}{2} \oplus \frac{c\tilde{r}}{q}$$

$$G(\tilde{a}, \tilde{r}) = \frac{1}{2}(a - \phi_1, a, a + \phi_2)Tq + \frac{c[r - \phi_3, r, r + \phi_4]}{q}$$

$$= (G_1, G_2, G_3)$$

By GMI method

$$\begin{aligned}
 G_1 &= \frac{(a - \phi_1)Tq}{2} + \frac{c(r - \phi_3)}{q} \\
 &= G(q) - \left[\frac{Tq\phi_1}{2} + \frac{c\phi_3}{q} \right] \\
 G_2 &= G(q) \\
 G_3 &= \frac{(a + \phi_2)Tq}{2} + \frac{c(r + \phi_4)}{q} \\
 &= G(q) + \left[\frac{Tq\phi_2}{2} + \frac{c\phi_4}{q} \right]
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 \phi[G(q)] &= \frac{1}{6}[G_1 + G_2 + G_3] \\
 &= \frac{1}{6} \left\{ G(q) - \left[\frac{Tq\phi_1}{2} + \frac{c\phi_3}{q} \right] + 4G(q) + G(q) + \left[\frac{Tq\phi_2}{2} + \frac{c\phi_4}{q} \right] \right\} \\
 &= G(q) + \frac{Tq}{12}(\phi_2 - \phi_1) + \frac{c}{6q}(\phi_4 - \phi_3) \\
 &= \frac{aTq}{2} + \frac{cr}{q} + \frac{Tq}{12}(\phi_2 - \phi_1) + \frac{c}{6q}(\phi_4 - \phi_3) \\
 \frac{d}{dq}[G(q)] &= 0, \text{ gives} \\
 \frac{aT}{2} - \frac{cr}{q^2} + \frac{T}{12}(\phi_2 - \phi_1) - \frac{c}{6q^2}(\phi_4 - \phi_3) &= 0 \\
 q^* &= \sqrt{\frac{2c[6r + \phi_4 - \phi_3]}{T[6a + \phi_2 - \phi_1]}}
 \end{aligned} \tag{4}$$

$$\text{Also } \frac{d^2}{dq^2}[G(q)] = \frac{2cr}{q^3} + \frac{c}{3q^3}(\phi_4 - \phi_3) > 0$$

And the estimated value of TC will be minimum.

2. Analysis of the model with example

If we choose the order cost $C = 150$, storage cost $a = 0.25$, demand $r = 60,000$ and the time plan $T = 6$, the crisp values of order quantity and total annual cost are $q_c = 3464.1$ and $TC = 5196.5$.

On the other hand if the storage cost a and demand r are taken as triangular fuzzy numbers $\tilde{a} = (0.11, 0.25, 0.38)$, $\tilde{r} = (30,000, 60,000, 72,000)$ then the estimated values of fuzzy order quantity and total cost are $q_f = 3387.7$ and $TC = 5044.29$.

This indicates that the fuzzy values are minimum compared to the crisp values.

3. Conclusion

This research work involves an inventory model without shortage in which the demand and the storage are assumed as triangular fuzzy number. The Graded mean integration technique is then introduced to defuzzify the objective function and the order quantity and total cost values are calculated for both precise and imprecise sense. The estimated lot size and annual cost are determined with an example which indicates that the fuzzy values are minimum than the crisp values.

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