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Odd Fibonacci edge irregular labelling for some simple graphs

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ABSTRACT: Let G be a graph with p vertices and q edges and $f : V(G) \to \{0,1,2,...,k\}$ be an injective function, where k is a positive integer. If the induced edge labeling $f^*: E(G) \to \left\{F_2, F_4, F_5, F_7, F_8, F_{10}, ..., F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}\right\}$ defined by f*(uv) = f(u)+f(v), for each $uv \in E(G)$, is a bijection, then the labeling f is called an odd Fibonacci edge irregular labeling of G. A graph which admits an odd Fibonacci edge irregular labeling is called an odd Fibonacci edge irregular graph. The odd Fibonacci edge irregularity strength ofes(G) is the minimum k for which G admits an odd Fibonacci edge irregular labeling. The odd Fibonacci edge irregularity strength for P_n, $K_{1,n}$, $P_n \Theta K_1$, B(m,n) and the non existence of an odd Fibonacci edge irregular labeling for the graphs K_p, $K_{m,n}$ have been determined.

Keywords: odd Fibonacci sequence, edge irregular labeling, odd Fibonacci edge irregular labeling

1. INTRODUCTION

By a graph, we mean a finite undirected graph without loops or multiple edges with p vertices and q edges. A graph labeling is an assignment of integers to the vertices or edges or both. Rosa[7] introduced the concept of graceful labeling. The Fibonacci numbers can be defined by the linear recurrence $F_n = F_{n-1}+F_{n-2}$, $n \geq 3$. This generates an infinite sequence of integers $F_1=1$, $F_2=1$, $F_3=2$, $F_4=3$, $F_5=5$, $F_6=8$, $F_7=13$ etc. In 2020, G.Chitra et al. [3] have introduced the concept of odd Fibonacci mean labeling.

Motivated by this, we have introduced an odd Fibonacci edge irregular labeling (OFEIL) which is an injective function $f : V(G) \to \{0,1,2,...,k\}$, k being a positive integer if the induced edge labeling $f^*: E(G) \to \left\{F_2, F_4, F_5, F_7, F_8, F_{10}, ..., F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}\right\}$ defined by f*(uv) = f(u)+f(v), for each $uv \in E(G)$, is bijection. If such a labeling exists, then G is called an odd Fibonacci edge irregular graph (OFEIG) and the minimum possible k is called the odd Fibonacci edge irregularity strength ofes(G). In this paper, the odd Fibonacci edge irregularity strength for P_n, $K_{1,n}$, $P_n \Theta K_1$, B(m,n) and the non existence of an odd Fibonacci edge irregular labeling for the graphs K_p, $K_{m,n}$ have been determined.

2. Main Results

Theorem 2.1. Every path P_n, $(n \geq 2)$ is an OFEIG and $$ofes(P_n) = \begin{cases} \left\lceil \frac{1}{2} F_{\frac{3n-2}{2}} \right\rceil, & \text{if } n \text{ is even} \\ F_{n+\left\lfloor \frac{n}{2} \right\rfloor - 1}, & \text{if } n \text{ is odd.} \end{cases}$$

Proof. Let G = P_n. In G, q = n-1.
Let V(G) = $\{v_1, v_2, \ldots, v_n\}$ and $E(G) = \{v_i v_{i+1} : 1 \leq i \leq n-1\}$.
Case (i) n is even

Define $f : V(G) \to \left\{0,1,2,\ldots,\left\lceil \frac{1}{2} F_{\frac{3n-2}{2}} \right\rceil\right\}$ as follows:

$$f(v_i) = \begin{cases} \left\lfloor \frac{1}{2} F_{\frac{3i+1}{2}} \right\rfloor, & 1 \le i \le n \text{ and i is odd} \\ \left\lceil \frac{1}{2} F_{\frac{3i-2}{2}} \right\rceil, & 1 \le i \le n \text{ and i is even.} \end{cases}$$

Then f* is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} F_{\frac{3i+1}{2}}, 1 \le i \le n-1 \text{ and i is odd} \\ F_{\frac{3i+2}{2}}, 1 \le i \le n-1 \text{ and i is even.} \end{cases}$$

Since n is even, $f(v_n) = \left\lceil \frac{1}{2} F_{\frac{3n-2}{2}} \right\rceil$ and $f(v_{n-1}) = \left\lfloor \frac{1}{2} F_{\frac{3n-2}{2}} \right\rfloor$.

In this case, $f(v_n) - f(v_{n-1}) = 1$ and $f(v_{n-1}) + f(v_n) = F_{\frac{3n-2}{2}} = F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$. So $f(v_n)$ is the minimum k with the required property.

Therefore, $ofes(G) = \left\lceil \frac{1}{2} F_{\frac{3n-2}{2}} \right\rceil$.

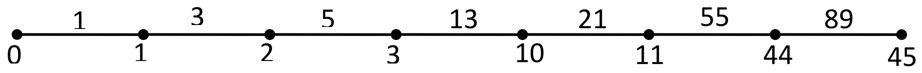

Figure 1:ofes(P_8) = 45

Case (ii) n is odd

Define $f : V(G) \to \left\{0,1,2,\ldots,F_{n+\left\lfloor \frac{n}{2} \right\rfloor-1}\right\}$ as follows:

$f(v_1) = F_{n+\left\lfloor \frac{n}{2} \right\rfloor-1}$,

$f(v_2) = F_{n+\left\lfloor \frac{n}{2} \right\rfloor-2}$ and

$$f(v_i) = \begin{cases} \left\lfloor \frac{1}{2} F_{\frac{3i-5}{2}} \right\rfloor, & 3 \le i \le n \text{ and i is odd} \\ \left\lceil \frac{1}{2} F_{\frac{3i-8}{2}} \right\rceil, & 4 \le i \le n \text{ and i is even.} \end{cases}$$

Then f* is obtained as follows:

$f^*(v_1 v_2) = F_{n+\left\lfloor \frac{n}{2} \right\rfloor}$,

$f^*(v_2 v_3) = F_{n+\left\lfloor \frac{n}{2} \right\rfloor-2}$ and

$$f^*(v_i v_{i+1}) = \begin{cases} F_{\frac{3i-5}{2}} & ,3 \le i \le n-1 \text{ and i is odd} \\ F_{\frac{3i-4}{2}} & ,4 \le i \le n-1 \text{ and i is even.} \end{cases}$$

In pursuance of obtaining the edge label $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$, we may choose the labels of its end vertices as $F_{q+\left\lfloor \frac{q}{2} \right\rfloor-1}$ and $F_{q+\left\lfloor \frac{q}{2} \right\rfloor}$. If the number *r* and *s* such that $F_{q+\left\lfloor \frac{q}{2} \right\rfloor-1} < r \le s < F_{q+\left\lfloor \frac{q}{2} \right\rfloor}$ are assigned to the central vertices, then there is an edge which assigns the label greater than $F_{q+\left\lfloor \frac{q}{2} \right\rfloor-1}$. But $F_{q+\left\lfloor \frac{q}{2} \right\rfloor}$ is no longer an edge value. So such *r* and *s* are impossible. Since $f(v_2) = F_{q+\left\lfloor \frac{q}{2} \right\rfloor-2}$, the value $f(v_1)$ is minimum k with the required property.

Therefore, $ofes(G) = F_{n+\left\lfloor \frac{n}{2} \right\rfloor-1}$. □

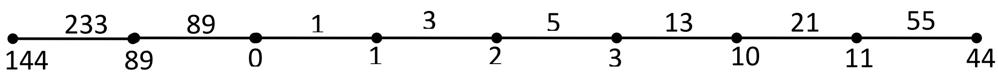

Figure 2: ofes(P_9) = 144

Theorem 2.2 Every star graph $K_{1,n}$ ($n \ge 1$) is an OFEIG and
$ofes(K_{1,n}) = F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - 1.$

Proof. Let $G = K_{1,n}$. In G, q =n.
Let $V(G) = \{u, v_1, v_2, \ldots, v_n\}$ and $E(G) = \{uv_i : 1 \le i \le n\}$.

Define $f : V(G) \to \left\{0,1,2,\ldots,F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - 1\right\}$ as follows:

f(u) =1 and
$$f(v_i) = F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1} - 1 \ ,1 \le i \le n.$$

Then f* is obtained as

$$f^*(uv_i) = F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1} \ ,1 \le i \le n.$$

To obtain F_2 as an edge label, it is necessary to assign 0 and 1 to a pair of adjacent vertices. So either 0 is a vertex label of central vertex and 1 is a label of a pendant vertex of $K_{1,n}$ or 0 is a vertex label of pendant vertex and 1 is a label of the central vertex. If 0 is assigned to the central vertex, $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$ is to be assigned as a label of a pendant vertex in pursuance of obtaining the edge label $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$. If 1 is assigned to the central vertex, $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - 1$ is to be assigned as a label of a pendant vertex in pursuance of obtaining the edge label $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$.

Hence $f(v_n) = F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - 1$ is the minimum k with the required property.

Therefore, $ofes(G) = F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - 1.$

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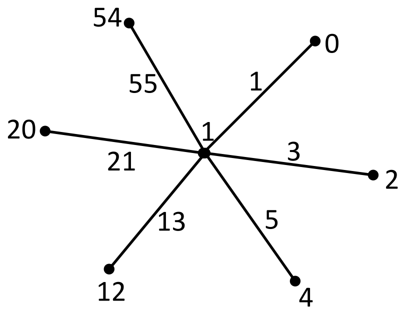

Figure 3: ofes($K_{1,6}$) = 54

Theorem 2.3 $P_n \Theta K_1$ ($n \geq 2$) is an OFEIG and $\mathrm{ofes}(G) = \left\lceil \frac{1}{2} F_{\frac{6n-2}{2}} \right\rceil$.

Proof. Let G = $P_n \Theta K_1$. In G, q = 2n-1.
Let $V(G) = \{v_1, v_2, ..., v_n, u_1, u_2, ..., u_n\}$ and $E(G) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \bigcup \{v_i u_i / 1 \leq i \leq n\}$.
Define $f : V(G) \rightarrow \left\{0,1,2,..., \left\lceil \frac{1}{2} F_{\frac{6n-2}{2}} \right\rceil \right\}$ as follows:

$$f(v_i) = \begin{cases} \left\lceil \frac{1}{2} F_{\frac{6i-2}{2}} \right\rceil, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ \left\lfloor \frac{1}{2} F_{\frac{6i-2}{2}} \right\rfloor, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases} \quad \text{and}$$

$$f(u_i) = \begin{cases} \left\lfloor \frac{1}{2} F_{\frac{6i-2}{2}} \right\rfloor, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ \left\lceil \frac{1}{2} F_{\frac{6i-2}{2}} \right\rceil, & 1 \leq i \leq n \text{ and } i \text{ is even}. \end{cases}$$

Then f* is obtained as follows:
$f^*(v_i v_{i+1}) = F_{3i+1}$, $1 \leq i \leq n-1$ and
$f^*(v_i u_i) = F_{3i-1}$, $1 \leq i \leq n$.

If n is odd, $f(v_n) = \left\lceil \frac{1}{2} F_{\frac{6n-2}{2}} \right\rceil$ and $f(u_n) = \left\lfloor \frac{1}{2} F_{\frac{6n-2}{2}} \right\rfloor$.

If n is even, $f(v_n) = \left\lfloor \frac{1}{2} F_{\frac{6n-2}{2}} \right\rfloor$ and $f(u_n) = \left\lceil \frac{1}{2} F_{\frac{6n-2}{2}} \right\rceil$.

In both cases, $f(v_n) - f(u_n) = 1$ and $f(v_n) + f(u_n) = F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$. So either $f(v_n)$ or $f(u_n)$ is the minimum k with the required property when n is odd or even respectively.

Therefore, $\mathrm{ofes}(G) = \left\lceil \frac{1}{2} F_{\frac{6n-2}{2}} \right\rceil$.

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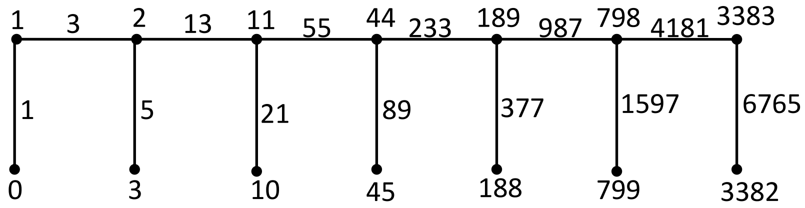

Figure 4: ofes($P_7 \Theta K_1$) = 3383

Theorem 2.4 A Bistar graph B(m, n) with m ≥ n, is an OFEIG and ofes(G) = $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2}$.

Proof. Let G = B(m, n). In G, q = m+n+1.

Let V(G) = {v,u, $v_1,v_2,\ldots,v_m$, $u_1,u_2,\ldots,u_n$} and $E(G) = \{vv_i : 1 \le i \le m\} \cup \{vu\} \cup \{uu_i : 1 \le i \le n\}$.

Define $f : V(G) \to \left\{0,1,2,\ldots,F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2}\right\}$ as follows:

$$f(v_i) = F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1}, 1 \le i \le m,$$

$$f(u) = F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2} \text{ and}$$

$$f(u_i) = F_{m+\left\lfloor \frac{m+1+i}{2} \right\rfloor+2+i} - F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2}, 1 \le i \le n.$$

Then f* is obtained as follows:

f*(vv_i) = $F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1}$, $1 \le i \le m$,

f*(vu) = $F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2}$ and

f*(uu_i) = $F_{m+\left\lfloor \frac{m+1+i}{2} \right\rfloor+2+i}$, $1 \le i \le n$.

To obtain F_2 as an edge label, it is necessary to assign 0 and 1 to a pair of adjacent vertices. There are two possibilities to assign 0 and 1. Either assign 0 and 1 to a pair of central vertices or assign 0 and 1 to a pair of central vertex and its adjacent pendant vertex. If 0 and 1 are assigned to the central vertices, then $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$ or $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - 1$ is to be assigned as a label of a pendant vertex in pursuance of obtaining the edge label $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$. If 0 and 1 is assigned to u and its pendant vertex, then it leads to take a larger value for k while
deg v > deg u.

Case (i) Assign 1 to the central vertex v and 0 to its pendant vertex.

In this case, the adjacent vertices of v such as v_1, $v_2,\ldots,v_m$, u are labeled as F_2-1, F_4-1, …, $F_{m+\left\lfloor \frac{m}{2} \right\rfloor+1} - 1$, $F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2} - 1$. Since f(u) = $F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2} - 1$, a pendant vertex of u is to be labeled as $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - \left(F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2} - 1\right)$ in pursuance of obtaining the edge label $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$.

Case (ii) Assign 0 to the central vertex v and 1 to its pendant vertex.

In this case, the adjacent vertices of v such as $v_1, v_2, \ldots, v_m$, u are labeled as $F_2, F_4, \ldots, F_{m+\left\lfloor \frac{m}{2} \right\rfloor+1}$, $F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2}$. Since f(u) $= F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2}$, a pendant vertex of u is to be labeled as

$F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2}$ in pursuance of obtaining the edge label $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1}$.

Hence $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2}$ is the minimum k with the required property.

Therefore, ofes(G) = $F_{q+\left\lfloor \frac{q}{2} \right\rfloor+1} - F_{m+\left\lfloor \frac{m+1}{2} \right\rfloor+2}$.

□

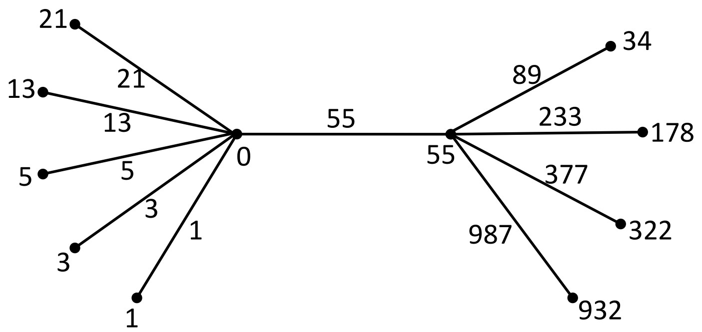

Figure 5: ofes(B(5, 4)) = 987

Theorem 2.5 Every complete graph K_p $(p \geq 3)$, is not an OFEIG.

Proof. Let $G = K_p$ and the vertex set of G be $\{v_1, v_2, \ldots, v_p\}$.

In G, $\deg(v_i) = p-1$, for all i. To obtain F_2 as an edge label, it is necessary to assign 0 and 1 to a pair of adjacent vertices. Choose an arbitrary vertex v_i and v_{i+1} such that $f(v_i) = 0$, $f(v_{i+1}) = 1$. To get F_4, either assign 3 to the one of the adjacent vertex of v_i or 2 to the one of the adjacent vertex of v_{i+1}, where the addition in the suffix is taken over addition modulo p. If 3 is assigned to the vertex v_{i-1}, then the edge v_iv_{i-1} have the label F_4. But the label of the edge $v_{i-1}v_{i+1}$ is 4, which is not an odd Fibonacci number. Suppose 2 is assigned to the vertex v_{i-1}, then the edge $v_{i-1}v_{i+1}$ have the label F_4. But the label of the edge $v_{i-1}v_i$ is 2, which is not an odd Fibonacci number.

Hence the graph G is not an OFEIG. □

Theorem 2.6 A graph G with p (≥ 5) vertices having $\deg(v_i) \geq p-2$, for all i is not an OFEIG.

Proof. Let $V(G) = \{v_1, v_2, \ldots, v_p\}$. To obtain F_2, it is necessary to assign 0 and 1 to a pair of adjacent vertices. Choose v_1 and v_2 such that $f(v_1) = 0$ and $f(v_2) = 1$. To obtain F_4, either 3 is assigned to one of the adjacent vertex of v_1 or 2 is assigned to one of the adjacent vertex of v_2.

Choose a vertex v_3 which is adjacent to both v_1 and v_2. Suppose 3 is assigned to the vertex v_3. Then the edge v_1v_3 has the label F_4, but the label of the edge v_2v_3 is 4 which is not an odd Fibonacci number. If 2 is assigned to the vertex v_3, then the edge v_2v_3 has the label F_4. But the label of the edge v_1v_3 is 2 which is not an odd Fibonacci number.

Suppose v_3 is a vertex adjacent to v_1 and non adjacent to v_2. To obtain F_4 as an edge label, either 3 is assigned to the vertex of v_3 or 2 is assigned to one of the adjacent vertex of v_2. By assigning 3 to the vertex v_3, the edge v_1v_3 has the label

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F_4. To obtain F_5 as an edge label, either 2 is assigned to one of the adjacent vertex of v_3 or 4 is assigned to one of the adjacent vertex of v_2. Let the adjacent vertex of v_3 say v_4 which is assigned by 2. Then the edge v_3v_4 has the label F_5. Since $\deg(v_i) \geq p - 2$, for all i, it is impossible that v_4 is non adjacent to both v_1 and v_2. If v_4 is adjacent to v_1, then the edge v_1v_4 has the label 2 which is not an odd Fibonacci number. Suppose v_4 is adjacent to v_2. Then the edge v_2v_4 has the label 3 which is already an edge label of v_1v_3. Let the adjacent vertex of v_2 say v_4 which is assigned by 4. Then the edge v_2v_4 has the label F_5. Since $\deg(v_i) \geq p - 2$, for all i, it is impossible that v_4 is non adjacent to both v_1 and v_3. If v_4 is adjacent to v_1, then the edge v_1v_4 has the label 4 which is not an odd Fibonacci number. Suppose v_4 is adjacent to v_3. Then the edge v_3v_4 has the label 7 which is not an odd Fibonacci number. Suppose 2 is assigned to one of the adjacent vertex of v_2 say v_3. Then the edge v_2v_3 has the label F_4. To obtain the edge label F_5, either 5 is assigned to one of the adjacent vertex of v_1 or 3 is assigned to one of the adjacent vertex of v_3 or 4 is assigned to one of the adjacent vertex of v_2. Let the adjacent vertex of v_1 say v_4 which is assigned by 5. Then the edge v_1v_4 has the label F_5. It is impossible that v_4 is non adjacent to both v_2 and v_3. If v_4 is adjacent to v_2, then the edge v_4v_2 has the label 6 which is not an odd Fibonacci number. Suppose v_4 is adjacent to v_3. Then the edge v_3v_4 has the label 7 which is not an odd Fibonacci number. Let the adjacent vertex of v_2, say v_4, which is assigned by 4. Then the edge v_2v_4 has the label F_5. It is impossible that v_4 is non adjacent to both v_1 and v_3. If v_4 is adjacent to v_1, then the edge v_4v_1 has the label 4 which is not an odd Fibonacci number. Suppose v_4 is adjacent to v_3. Then the edge v_3v_4 has the label 6 which is not an odd Fibonacci number. If the adjacent vertex of v_3 say v_4 which is assigned by 3, then the edge v_3v_4 has the label F_5. It is impossible that v_4 is non adjacent to both v_1 and v_2. If v_4 is adjacent to v_1, then the edge v_1v_4 has the label 3 which is already an edge label of v_2v_3. Suppose v_4 is adjacent to v_2. Then the edge v_2v_4 has the label 4 which is not an odd Fibonacci number.

Hence G is not an OFEIG. □

Theorem 2.7 The graph $K_{m,n}$ ($m \geq 2$, $n \geq 4$) is not an OFEIG.

Proof. Let $G = K_{m,n}$.

Let $V_1 = \{u_1, u_2, \ldots, u_m\}$, $V_2 = \{v_1, v_2, \ldots, v_n\}$ be the partitions of G and assume that $m \leq n$.

$E(G) = \{u_iv_j : 1 \leq i \leq m, 1 \leq j \leq n\}$. To obtain F_2 as an edge label, it is necessary to assign 0 and 1 to a pair of adjacent vertices. Choose an arbitrary vertex, say u_1 in V_1 and v_1 in V_2 such that $f(u_1) = 0$, $f(v_1) = 1$. To get F_4 as an edge label, either 3 is assigned to one of the adjacent vertex of u_1 or 2 is assigned to one of the adjacent vertex of v_1.

Choose a vertex v_2 which is adjacent to all u_i's, $1 \leq i \leq m$. If 3 is assigned to the vertex v_2, then the edge u_1v_2 has the label F_4. To get the edge label F_5, either 5 is assigned to one of the adjacent vertex of u_1 or 4 is assigned to one of the adjacent vertex of v_1 or 2 is assigned to one of the adjacent vertex of v_2. If 4 is assigned to the vertex u_2, then the edge u_2v_1 has the label F_5 but the label of the edge u_2v_2 is 7 which is not an odd Fibonacci number. Suppose 2 is assigned to the vertex u_2. Then the edge u_2v_2 has the label F_5 but the label of the edge u_2v_1 is 3 which is already an edge label of u_1v_2. Therefore, the only way to obtain F_5 is the assignment of the label 5 to one of the adjacent vertex of u_1. Thus choose a vertex v_3 and label as 5. To obtain F_7, either 13 is assigned to one of the adjacent vertex of u_1 or 12 is assigned to one of

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the adjacent vertex of v_1 or 10 is assigned to one of the adjacent vertex of v_2 or 8 is assigned to one of the adjacent vertex of v_3. If 12 is assigned to the vertex u_2, then the edge u_2v_1 has the label F_7 but the label of the edge u_2v_2 is 15 and u_2v_3 is 17 respectively which are not an odd Fibonacci numbers. Suppose 10 is assigned to the vertex u_2. Then the edge u_2v_2 has the label F_7 but the label of the edges u_2v_1 and u_2v_3 are 11 and 15 respectively which are not odd Fibonacci numbers. Suppose 8 is assigned to the vertex u_2. Then the edge u_2v_3 has the label F_7 but the label of the edges u_2v_1 and u_2v_2 are 9 and 11 respectively which are not odd Fibonacci numbers. Therefore, 13 is assigned to the vertex v_4 which is adjacent to u_1. Thus the label of the edge u_1v_4 is F_7. Proceeding like this the vertices vj can get the label as $F_{j+\left\lfloor \frac{j}{2} \right\rfloor+1}, 1 \le j \le n.$ In pursuance of obtaining $F_{n+\left\lfloor \frac{n}{2} \right\rfloor+2} \left(\text{or } F_{n+\left\lfloor \frac{n}{2} \right\rfloor+3} \right)$ as edge label, if a number $k \le F_{n+\left\lfloor \frac{n}{2} \right\rfloor+2} - 2$ is assigned to one of the vertex u_i's namely u_m, then the edges u_mv_1 and u_mv_2 have the labels k+1 and k+3 respectively which are not odd Fibonacci numbers. Therefore, $F_{n+\left\lfloor \frac{n}{2} \right\rfloor+2} - 1$ is to be assigned to the vertex u_m. But the edge label of u_mv_2 is $F_{n+\left\lfloor \frac{n}{2} \right\rfloor+2} + 2$ is not an odd Fibonacci number as $\left|F_{i+1} - F_i\right| \ge 8$, for all $i \ge 4$.

Choose a vertex u_2 which is adjacent to all v_j's, $1 \le j \le n$. By assigning 2 to the vertex u_2, the edge u_2v_1 has the label F_4. To obtain F_5 as an edge label, either 5 is assigned to one of the adjacent vertex of u_1 or 3 is assigned to one of the adjacent vertex of u_2 or 4 is assigned to one of the adjacent vertex of v_1. If 5 is assigned to the vertex v_2, then the edge u_1v_2 has the label F_5 but the label of the edge u_2v_2 is 7 which is not an odd Fibonacci number. Suppose 3 is assigned to the vertex v_2. Then the edge u_2v_2 has the label F_5 but the label of the edge u_1v_2 is 3 which is already an edge label of u_1v_2. Therefore, 4 is to be assigned to the vertex u_3 which is adjacent to v_1. Thus the label of the edge u_3v_1 is F_5. To obtain F_7 as an edge label, either 13 is assigned to one of the adjacent vertex of u_1 or 11 is assigned to one of the adjacent vertex of u_2 or 9 is assigned to one of the adjacent vertex of u_3 or 12 is assigned to one of the adjacent vertex of v_1. If 13 is assigned to the vertex v_2, then the edge u_1v_2 has the label F_7 but the label of the edges u_2v_2 and u_3v_2 are 15 and 17 respectively which are not odd Fibonacci numbers. Suppose 11 is assigned to the vertex v_2. Then the edge u_2v_2 has the label F_7 but the label of the edges u_1v_2 and u_3v_2 are 11 and 15 respectively which are not odd Fibonacci numbers. If 9 is assigned to the vertex v_2, then the edge u_3v_2 has the label F_7 but the label of the edges u_1v_2 and u_2v_2 are 9 and 11 respectively which are not odd Fibonacci numbers. Therefore, 12 is to be assigned to the vertex u_4 which is adjacent to v_1.

Thus the label of the edge u_4v_1 is F_7. Proceeding like this, the vertices of u_i can get the label as $F_{i+\left\lfloor \frac{i}{2} \right\rfloor+1} - 1,\ 1 \le i \le m$.

In order to obtain $F_{m+\left\lfloor \frac{m}{2} \right\rfloor+2} \left(\text{or } F_{m+\left\lfloor \frac{m}{2} \right\rfloor+3} \right)$ as an edge label, if a number $k \le F_{m+\left\lfloor \frac{m}{2} \right\rfloor+2} - 1$ is assigned to the vertex, then the edges u_1v_2 and u_2v_2 have the labels k and k+2 respectively which are not odd Fibonacci numbers. Therefore, $F_{m+\left\lfloor \frac{m}{2} \right\rfloor+2}$ is to be assigned to the vertex v_2. But the edge label of u_2v_2 is $F_{m+\left\lfloor \frac{m}{2} \right\rfloor+2} + 2$ is not an odd Fibonacci number as $\left|F_{i+1} - F_i\right| \ge 8$, for all $i \ge 4$.

Similarly, odd Fibonacci edge irregular labeling does not exist if we choose $f(u_1)=1$ and $f(v_1)=0$. Hence the graph $K_{m,n}$ is not an OFEIG. □

Observation 2.8: The graphs $K_{2,2}$, $K_{2,3}$ and $K_{3,3}$ are not OFEIG. From Theorem 2.2, Theorem 2.7 and Observation 2.8, it can be concluded that $K_{m,n}$ is an OFEIG only when it is a star graph.

Conjecture: A cyclic graph is not an OFEIG.

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