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Solving Assignment Problem in Fuzzy Environment by Using New Ranking Technique in Triangular Fuzzy Number

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ABSTRACT

The subject of assignment difficulties is fascinating, and students are always addressing engineering and management science challenges. To select the best answer, we suggested a novel ranking approach for triangular fuzzy numbers in this study. The right answer is fully determined by the ranking system.In anuncertaindecision-making situation, the ranking of fuzzy numbers is extremely vital to make a decision. This ranking approach is very dependable, simple to use, and applicable to all sorts of assignment problems. Our objective is to use a novel ranking algorithm to transform muddled data into clearer ones.The Fuzzy Hungarian Method is employed to attain the best solution forthe fuzzy assignment problem. The numerical examples demonstrate that the proposed ranking measure is straightforward to estimate, works effectively in computing optimal value.

Keywords: *Fuzzyset, Fuzzynumber, triangularfuzzynumber, Fuzzyassignmentproblem, New Ranking Technique.*

1 INTRODUCTION

The Assignment Problem is a subset of linear programming. Real-world problems are solved with assignment tasks. Assignment issues are an astonishing subject that is constantly used in the solution of engineering and management science problems and is being widely implemented in various fields. In an assignment problem, n jobs have to be completed by n people based on their competence.Cij signifies the cost of allocating the jth job to the ith individual in the assignment issue. We suppose that each individual can only do one job and that each person can only do one job. The challenge is to select the smallest assignment possible such that the total cost of doing all activities is as low as possible or the total profit is as high as possible. L.A. Zadeh (1965) established the concept of fuzzy sets, which provided us with a new mathematical model for capturing impreciseness and ambiguity. Following then, several writers proposed alternative techniques to tackling FLP issues. Bortolan and Degani examined and contrasted a few of these ranking methods (1985).Chen and H Wang (1992) analyzed existing methods for rating fuzzy numbers and found that each strategy has shortcomings in some areas, such as indiscrimination and finding that is difficult to comprehend. Chan (1985) presented the notion of generalized fuzzy numbers after stating that in many instances it is not viable to confine the membership function to the standard form. The fuzzy variables in decision making are presented by Jain R (1976). Lee and Chen (2008) investigated the various forms of fuzzy numbers as well as the various deviations of fuzzy numbers. Wang YJ (2008) revises fuzzy number ranking techniques. Kalaiarasi and colleagues (2014) optimized a fuzzy assignment model using triangular fuzzy numbers. Because the suggested ranking technique is very straightforward and simple, it is very easy to grasp and use in order to identify the fuzzy optimum solution to real-life fuzzy assignment issues.

The following is how this document is structured. Section 2 introduces some fundamental concepts and ranks triangular intuitionistic fuzzy numbers. Section 3 offered a system for ranking. Section 4 introduces the fuzzy assignment issue and the fuzzy Hungarian technique. A numerical example is solved to demonstrate the suggested approach. Finally, the report comes to a close with a conclusion.

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2. PRELIMINARIES

2.1 Definition - Fuzzy set

A fuzzy set $\tilde{A}$ is a subset of a universe of discourse *X*, which is characterized by a membership function $\mu_{\tilde{A}}(x)$ representing a mapping $\mu_{\tilde{A}}: X \to [0,1]$. The function value of $\mu_{\tilde{A}}(x)$ is called the membership value, which represents the degree of truth that *x* is an element of fuzzy set $\tilde{A}$.

2.2 Definition - Fuzzy Number

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number and its membership function $\tilde{A}: R \to [0,1]$ has the following characteristics,

(i) $\tilde{A}$ is convex.

$\mu_{\tilde{A}}\left(\lambda x_1 + (1-\lambda)x_2\right) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)),\ \forall x \in [x_1, x_2], \lambda \in [0,1].$

(ii) $\tilde{A}$ is normal if $\max \mu_{\tilde{A}}(x) = 1$.

(iii) $\tilde{A}$ is piecewise continuous.

2.3 Definition – Non-Negative Fuzzy Number

A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only $\mu_{\tilde{A}}(x) = 0,\ \forall\ x < 0$

2.4 Definition – Triangular Fuzzy Number

A triangular fuzzy number $\tilde{N}$ can be defined as a triplet (*l*, *m*, *r*) and the membership function $\mu_{\tilde{N}}(x)$ is defined as:

$$\mu_{\tilde{N}}(x) = \begin{cases} 0 & x < l \\ \left(\dfrac{x-l}{m-l}\right) & l \leq x \leq m \\ \left(\dfrac{r-x}{r-m}\right) & m \leq x \leq r \\ 0 & x > r \end{cases}$$

Where *l*, *m*, *r* are real numbers and $l \leq m \leq r$.

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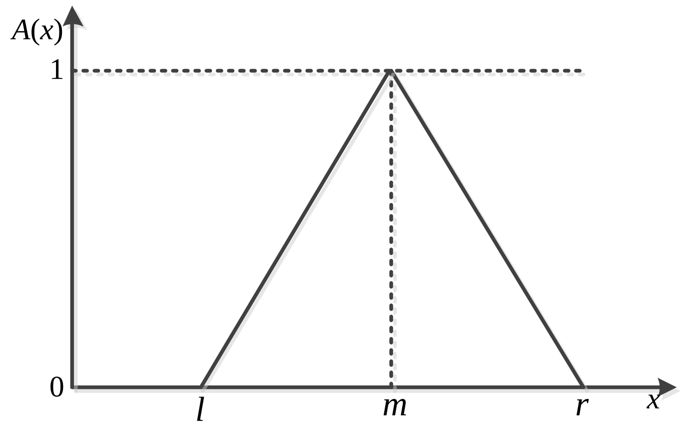

Figure: 1 Membership function of Trapezoidal fuzzy number $\tilde{A}$

Triangular fuzzy number is $\tilde{A} = (a_1, a_2, a_3)$

2.5 Definition – *α*–cut

The α-cut of the fuzzy set $\tilde{A}$ of the universe of discourse *X* is defined as $\tilde{A}_\alpha = \{x \in X \,/\, \mu_{\tilde{A}}(x) \ge \alpha\}$, where $\alpha \in [0,1]$.

2.6 Fuzzy Arithmetical Operations Under Function Principle

Shan-Huo Chen (1985) defined function principle as the operation of fuzzy number addition, subtraction, multiplication, and division.

2.6.1 Arithmetic Operations on Triangular Fuzzy Number

Assume $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1,\ b_2,\ b_3)$ are 2 - triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} + \tilde{B} = (a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3)$ where $a_1,\ a_2,\ a_3,\ b_1,\ b_2,\ b_3$ are real numbers.

(ii) The product of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \times \tilde{B} = (c_1, c_2, c_3)$ where $T = \{a_1 b_1,\ a_1 b_3,\ a_3 b_1,\ a_3 b_3\}$ $c_1 = \min T,\ c_2 = a_2 b_2,\ c_3 = \max T$

If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non-zero positive real numbers, then $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$

(iii) If, $-B = (-b_3,\ -b_2,\ -b_1)$ then the subtraction of from is $\tilde{A} - \tilde{B} = (a_1 - b_3,\ a_2 - b_2,\ a_3 - b_1)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

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(iv) If $1/\tilde{B} = \tilde{B}^{-1} = (1/b_3,\ 1/b_2,\ 1/b_1)$ where b_1, b_2, b_3 are all non zero real numbers then $\tilde{A}/\tilde{B} = (a_1/b_3,\ a_2/b_2,\ a_3/b_3)$

(v) Let $\alpha \in R$, then $\alpha\tilde{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3) & if \quad \alpha \geq 0 \\ (\alpha a_3, \alpha a_2, \alpha a_1) & if \quad \alpha < 0 \end{cases}$

3. Proposed Ranking Procedure

We introduced a novel method for ranking trapezoidal fuzzy numbers in this chapter. Ranking algorithms convert fuzzy numbers into real-world lines. The following summarizes several phases of the computational approach for getting an optimum solution. A fuzzy number $\tilde{A}$ is defined to be a trapezoidal fuzzy number if its membership functions $\mu_{\tilde{A}} : R \to [0,1]$ is equal to

$$\mu_{\tilde{A}}(\mathrm{x}) = \begin{cases} 0\ if\ a_1 < x \\ \frac{x-a_1}{a_2-a_1}, \quad if\ a_1 < x \leq a_2 \\ 1, \quad if\ x = a_2 \\ \frac{a_3-x}{a_3-a_2} \quad if\ a_2 \leq x < a_3 \\ 0, \quad otherwise \end{cases}$$

triangular fuzzy number defined $\tilde{A} = (a_1, a_2, a_3)$

3.1 Proposed Ranking Function

We use the following membership function

$$\mu_{\tilde{A}}(\mathrm{x}) = \begin{cases} \frac{x-a_1}{a_2-a_1}, \quad if\ a_1 < x \leq a_2 \\ 1, \quad if\ x = a_2 \\ \frac{a_3-x}{a_3-a_2} \quad if\ a_2 \leq x < a_3 \end{cases}$$

By using $\alpha - cut$, where $\alpha \in [0,1]\ and\ 0 \leq \alpha \leq 1, then$

$$\alpha = \frac{x-a_1}{a_2-a_1} \quad \text{and} \quad \alpha = \frac{a_3-x}{a_3-a_2}$$

$$\mathrm{x} = a_1 + \alpha(a_2 - a_1) = A_L\alpha \ \text{ and } \ \mathrm{x} = a_3 - \alpha(a_3 - a_2) = A_U\alpha$$

$$\mathrm{R}(\widetilde{A_\alpha}) = \int_0^1 m(A_\alpha)\mathrm{d}\alpha, \quad \text{Where } m(A_\alpha) = \frac{A_L\alpha + A_U\alpha}{2}$$

$$R(\widetilde{A_\alpha}) = \int_0^1 \left(\frac{(a_1 + \alpha(a_2-a_1) + a_3 - \alpha(a_3-a_2))}{2}\right) d\alpha = \frac{a_1 + 2a_{s2} + a_3}{4}$$

4. ASSIGNMENT PROBLEM

The assignment problem is a fundamental combinatorial optimization issue in the discipline of mathematics known as optimization. An assignment issue is a subset of the transportation problem, which is a type of linear programming problem. The goal of an assignment issue is to allocate a quantity of resources to an equal number of activities in order to maximize allocation profit or minimize total expenses. Even if the problems can be handled using the basic technique and the transportation method, the assignment model simplifies the solution. Consider the case when there are n agents and n tasks. Each individual can only do one task, and each task can only be allocated to one person.

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4.1 The general assignment problem can be mathematically stated as follows:

Minimize Z = $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for i} = 1,2,...,\text{n}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for j} = 1,2,...,\text{n}$$

$$x_{ij} = 0 \text{ or } 1$$

4.2 Fuzzy Assignment Problem

Minimize Z = $\sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij} \, x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for i} = 1,2,...,\text{n}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for j} = 1,2,...,\text{n}$$

$$x_{ij} = 0 \text{ or } 1$$

5. HUNGARIAN ASSIGNMENT ALGORITHM

Step 1: Convert fuzzy values to crisp values using the specified ranking function. If the number of rows exceeds the number of columns, or vice versa, a fake row or column with zero cost components must be created.

Step 2: In each row of the cost matrix, find the least cost element and remove it from each element in that row. As a result, each row of this new matrix, dubbed the first reduced cost matrix, will include at least one zero.

Step 3: In each column of the reduced cost matrix, find the lowest element. Subtract the cheapest component from each column's total. As a result, the second reduced cost matrix would include at least one zero in each row and column.

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Step 4:Determine an optimum assignment as follows:

Examine the rows one at a time until you locate one with exactly one zero. Draw a box around the zero elements and cross out all other zeros in its column as an assigned cell. Continue in this manner until you have completed all of the rows. Skip that row and go on to the next if it has more than one zero.

Repeat the procedure for the columns of the lowered cost matrix. If no row or column of the reduced matrix has a single zero, choose the row or column with the fewest zeroes at random. Select a zero at random from the row or column and cross the other zeros in that row or column.

Steps (i) and (ii) should be repeated until all zeros have been allocated or crossed out.
Step 5: An ideal assignment is attained when the number of allotted cells equals the number of rows (and columns). If a zero cell is randomly chosen, an alternate optimum may exist. If an ideal solution cannot be found (some rows or columns are empty), go to the next stage.
Step 6: Using the following formula, draw the smallest number of horizontal and/or vertical lines across all the zeros:

(i) Add a (√) to any rows that haven't been assigned yet.
(ii) Add a (√) to the columns in the specified rows that have zeros.
(iii) Mark (√) rows with assignments in indicated columns (if not already marked).
(iv) Repeat the process until there are no more rows or columns to examine.
(v) Draw straight lines through all rows and columns that aren't marked.

Step 7: If the least number of lines crossing all zeroes equals the number of rows or columns, the optimal solution is obtained by making random assignments in the zeroes not crossed in step 3. If not, we'll go on to the next stage.
Step 8: Make the following changes to the expense matrix:

(i) Determine which objects a line covers. Choose the smallest of these components and delete it from all uncrossed parts before inserting it where the two lines intersect.
(ii) Other components that the lines intersect are unaffected.

Step 9: Return to Step 4 and continue the process until you've found the best solution.

5.1 NUMERICAL EXAMPLE

A company has four source of persons are M1, M2, M3, M4 and destination of Jobs are J1, J2, J3, J4. The cost matrix$[C_{ij}]$, represented by trapezoidal fuzzy numbers. The objective is to find the optimal assignment with minimum cost.

Persons	Jobs			
	J1	J2	J3	J4
M1	(7,9,10)	(8,10,14)	(6,8,10)	(8,10,15)
M2	(8,9,16)	(5,7,10)	(9,10,11)	(6,8,10)
M3	(2,4,6)	(3,8,16)	(8,11,13)	(4,6,7)
M4	(9,10,12)	(7,10,12)	(5,7,10)	(5,8,10)

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With the help of ranking method for triangular intuitionistic fuzzy number to covert crisp value , $R(\widetilde{A_\alpha}) = \frac{a_1 + 2a_2 + a_3}{4}$

For ranking technique for triangular fuzzy number is (7,9,10) to covert crisp value is 8.75

$$R(\tilde{A}_\alpha) = \frac{1}{4}\left(7 + 2X9 + 10\right) = 8.75$$

Optimal Solution table

Persons	Jobs			
	J1	J2	J3	J4
M1	8.75	10.50	8.00	10.75
M2	10.50	7.25	10.00	8.00
M3	4.00	8.75	10.75	5.75
M4	10.25	9.75	7.25	7.75

Applying Hungarian method, we get the optimal solution.

Minimum Total Cost = Rs.(8+7.25+4+7.75) = Rs. 27.

6. CONCLUSION

This work presented a simple way for tackling fuzzy assignment problems by ordering triangular intuitionistic fuzzy integers. We conclude that the outcomes obtained were satisfactory, as evidenced by numerical examples. Each ranking system reflects a unique perspective on fuzzy numbers. Selecting a technique is usually a question of personal taste. Yet, we hope that the results produced in this research provide us with the optimal cost, which is far lower than previous techniques.

REFERENCES

[1] Angelov. P, Optimization in an intuitionistic fuzzy environment, Fuzzy Sets andSystems,86,(1997),299-306
[2] Annie Varghese, Mand Sunny Kuriakose, Scompare the intuitionistic fuzzy numbers. Applied Mathematical Sciences, 6(47)(200), 2345– 2355.
[3] Bai. X.J, Liu Y.K. and Shen. S.Y. Fuzzy generalized assignment problem with credibility constraints, Proceedings of the Eighth International Conference on Machine Learning and Cybernetics, Baoding, (2009), 657-662.
[4] Bellmann B.E and Zadeh.L.A, Decision making in fuzzy environment management sciences, 17(1970), 141-164.
[5] Bortolan G, Degani R. A review of some methods for ranking fuzzy subsets, Fuzzy Set and Systems. 15,(1985), 1-19.
[6] Chanas. S and Kuchta. D, A concept of the optimal solution of the transportation problem with fuzzy cost coefficients, Fuzzy Sets and Systems, 82(1996), 299-305.
[7] Chanas. S, Kolodziejczyk. W and Machaj. A, A fuzzy approach to the transportation problem, Fuzzy Sets and Systems, 13(1984), 211-221.
[8] Chen C.B and Klein C.M, A simple approach to ranking a group of aggregated fuzzy utilities, IEEETrans, Syst,Man, Cybern. B, SMC-27(1997),26-35.
[9] Chen M.S, On a fuzzy assignment problem, Tamkang J, 22(1985), 407-411.
[10] Chen SJ, Hwang CL, Fuzzy multiple attribute decision making. Berlin: Springer;1992.

[11] Chi-JenLin, Ue-PyngWen ,A Labelling algorithm for the fuzzy assignment problem, fuzzy Sets and system142(2004), 373-391.
[12] Choobineh F and Li. H, An index for ordering fuzzy numbers, Fuzzy Sets andSystems,54(1993), 287-294.
[13] De ,PK and Das. D, Ranking of trapezoidal intuitionistic fuzzy numbers, 12th International Conference on Intelligent Systems Design and Applications (ISDA),(2012),184 – 188.
[14] Dubois D and Fortemps P, Computing improved optimal solutions to max-min flexible constraint satisfaction problems,EuropeanJ,OperationsResearch,118(1999),95-126.
[15] Fortemps P and Roubens M "Ranking and defuzzification methods based area compensation" fuzzy sets and systems 82,(1996),319-330
[16] Geetha, S and Nair, K.P.K, A variation of the assignment problem, European Journal of Operation Research. 68(3),(1993),422-426.
[17] Hussain. R J and Senthil Kumar. P, Algorithm approach for solving in tuitionistic fuzzy transportation problem, Applied mathematical sciences,6(80),(2012),3981-3989.
[18] Jain R, Decision making in the presence of fuzzy variables, IEEETransactionson Systems, Man and Cybernetics. 6,(1976),698-703.
[19] Kalaiarasi. K, Sindhu. S, and Arunadevi. M, Optimization of fuzzy assignment model with triangular fuzzy numbers IJISET 3, (2014).2348-7968
[20] Klir. G.J and Yuan. B, Fuzzy Sets and Fuzzy Logic,Theory and Applications,Prentice-Hall, International Inc.,1995.
[21] Kuhn H.W, The Hungarian method for assignment problem, Naval Research Logistics Quarterly, 2, (1955), 83-97.
[22] LeeL W, Chen SM, Fuzzy risk an alysis based on fuzzy numbers with different shapes and different deviations, Experts Systems with Applications34,(2008)
[23] M. OLhELigeartaigh, A fuzzy transport ation algorithm, Fuzzy Sets and Systems, 8(1982),235-243.
[24] Nirmala G, Anju R. Cost Minimization assignment problem using fuzzy quantifier, International Journal of Computer Science and InformationTechnologies,5,(2014),7948-7950.
[25] Phani Bushan Rao P, Ravishankar N, Ranking fuzzy numbers with a distance method using circum center of centroids and an Index of Modality, Advances in Fuzzy System,10, (2011), 1155-1161.
[26] Sakawa. M, Nishizaki. I and Uemura. Y, Interactive fuzzy programming for two level linear and linear fractional production and assignment problems: a case study,EuropeanJ. OperationsResearch, 135(2001), 142-157.
[27] Tada, MandIshii. H, An integer fuzzy transportation problem,Comput,Math, Appl,31(1996),71-87.
[28] Valvis. E A new linear ordering of fuzzy numbers onsubsets of F(R). Fuzzy Optimization and Decision Making,8,(2009),141-63.
[29] Wang YJ, Lee HS. The revised method of ranking fuzzy numbers with an area between the centroid and original points.Computer and Mathematics with Applications,55 (2008),2033-2042.
[30] Yager. R.R, Aprocedure for ordering fuzzy sub sets of the unit interval, Information Sciences, 24(1981), 143-161.