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Comparison Of Hybrid-Grey (1,1), Hybrid-Grey (2,1) Methods in Forecasting Nifty Fifty Data Based On Game Theoretical Model

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ABSTRACT

Stock price prediction is one of the emerging fields of research and many methods like technical analysis, time series analysis etc are used for this purpose. Nifty Fifty indices are used as sample data set to validate the concept. The performance of the model is analysed by comparing with GM$(1,1)$ Method and GM$(2,1)$ Method with triangular fuzzy number as parameter. we used three methods that can forecast large sized data: Hybrid-Grey $(1,1)$, Hybrid-Grey $(2,1)$. The results of the analysis show that the model which has the smallest error is Hybrid-Grey $(1,1)$ therefore it is the best model to forecast the nifty fifty stock indices.

Keywords: Stock Value, forecasting, GM $(1,1)$, GM $(2,1)$, triangular fuzzy number, payoff.

1. INTRODUCTION

The present financial world stock market forecasting is considered as one of the most challenging tasks. So, lot of attention has been given to analyze forecast future value and behaviour of financial time series. Ability to predict the direction and correct value of future stock market value is the important factor in financial market to make money. These days because of online trading, stock market has become one of the hot targets where anyone can earn profits. So, forecasting the correct value and behaviour of stock market has become the area of interest.

The Hybrid-Grey Model is a model obtained by combining three models: a forecasting method using the Grey model combined with a triangular fuzzy number and game theory. The Grey model was introduced by Deng Julong and is a forecasting methodology that can overcome the problem of large sized data. Fuzzy logic can be used to predict the next state or condition by triangular fuzzy number. The combination of the Grey model and Fuzzy model with game theory is known as the Hybrid-Grey Model, and can be used to make forecasts based on large sized datasets, as well as coping with very random data fluctuations[1].

The Grey model and the Grey–Markov model have been used by many researchers. Ozdemir and Ozdagoglu conducted a study entitled "Predicting Product Demand from Small-Size Data: Grey Models"[2]. The data used is the demand data of textile products for 12 months. The method used is the modified Grey method and the Grey–Markov method. The purpose of that study was to predict the monthly demand of textile products and to find the model of incorporation of the modified Grey method and Grey–Markov method in the demand prediction problem. The results of the study showed that the Grey–Markov model based on GM $(2,1)$ achieves better prediction performance than other models. Kaylan and Pramanik conducted a study entitled "The Application of Grey System Theory in Predicting the Number of Deaths of Women by Committing Suicide—A Case Study". The data used are female suicide data in India from 2008 to 2013. The method used is the Grey model type GM $(1,1)$, the Grey Verhulst model, and GM $(2,1)$. The purpose of that study was to predict suicides committed by women, given the complexity and uncertainty of factors that affect the suicide. Therefore, it can be analyzed using Grey System Theory. The Grey model requires only a limited amount of data to estimate the behaviour of unknown systems. The results of the study showed that GM model $(1,1)$ is better than the Grey Verhulst model and GM $(2,1)$ for predicting female suicides in India. Rahayu (2017) conducted a study entitled "Comparison of Forecasting Results between Grey System Theory and Grey–Markov type GM $(1,1)$"[3]. In that study, the data used is the inflation rate data in Indonesia from January 2014 to February 2017. The method used is Grey System Theory and Grey–Markov type GM $(1,1)$. The purpose of the study was to forecast an increase in inflation rate

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and to compare the accuracy of the forecasting results of Grey System Theory with those from Grey–Markov type GM $(1,1)$. The results showed that the forecasting results using Grey–Markov type GM $(1,1)$ have smaller error values than those obtained using Grey System Theory.

Unlike any other researches in India that forecast time series data which have limited historical data using a classical model, the present study compares the results of Hybrid-Grey $(1,1)$, and Hybrid-Grey $(2,1)$, as alternative methods for large sized data, in forecasting the Nifty Fifty stock value. The results of the best model that will be used to predict the open value of Nifty Fifty stock indices.

2. METHODOLOGY

The methods used in this paper are Hybrid-Grey $(1,1)$ and Hybrid-Grey $(2,1)$.The theoretical basis for each method is presented in this section.

2.1 Hybrid-Grey $(1,1)$

The steps of GM$(1,1)$ model are shown as follows [4],

Step 1: Original time sequence with n samples is define as

$$O^{(0)} = \left(o^{(0)}(1), o^{(0)}(2), \ldots . o^{(0)}(n)\right) \quad (1)$$

Step 2: Construct monotonic increasing sequence $O^{(1)}$ by a one-time accumulated generating operation (I-AGO) expressed as

$$O^{(1)} = \left(o^{(1)}(1), o^{(1)}(2), \ldots . o^{(1)}(n)\right) \quad (2)$$

Where $o^{(1)}(k) = \sum_{i=1}^{k} o^{(0)}(i), \quad k = 1,2, \ldots . n$

Step 3 From the GM(1,1) model by establishing a first order grey differential equation

$$o^{(0)}(k) + az^{(1)}(k) = b \quad (3)$$

Where $z^{(1)}(k) = \frac{1}{2}(o^{(1)}(k) + o^{(1)}(k-1))$. In Equation (3), k (k=2,3,.....,n) is a time point, a is called the development coefficient and b is called grey action coefficient. Using least mean square estimation technique coefficients, $[a, b]^T$ can be estimated as

$$[a, b]^T = (A^T A)^{-1} A^T B \quad (4)$$

Where $B = \begin{bmatrix} O^{(0)}(2) \\ O^{(0)}(3) \\ \vdots \\ O^{(0)}(n) \end{bmatrix}$, $A = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}$

Step 4: According to the estimated coefficients a and b, the grey prediction equation can be obtained by solving differential equation in equation (3)

$$o^{(1)}(k+1) = \left(o^{(1)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \quad (5)$$

Where $o^{(1)}(k)$ denotes the prediction of o at k time point.

Step 5: To obtain the forecasting values of $o^{(0)}(k), \ k = 2,3, \ldots \ldots .n$, the inverse accumulated generating operation (I-AGO) is to establish the following grey model

$$\begin{cases} o^{(1)}(k+1) = \left(o^{(1)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \\ o^{(0)}(k+1) = o^{(1)}(k+1) - o^{(1)}(k) \end{cases} \quad (7)$$

We can also simplify Equation (7) as

$$o^{(0)}(k+1) = (1 - e^{a})\left(o^{(1)}(1) - \frac{b}{a}\right) e^{-ak} \quad (8)$$

The advantages of grey prediction model GM(1,1) is easy to calculate and suitable for many problems of approximate exponential change.

2.2 TRIANGULAR FUZZY NUMBER

The main purpose triangular fuzzy number is to increase the forecasting value. The residual time series as the difference between real value and predicted value is obtained

$$\varepsilon^{(0)} = \{\varepsilon^{(0)}(2), \varepsilon^{(0)}(3), \ldots \ldots, \varepsilon^{(0)}(n)\} \quad (9)$$

Where $\varepsilon^{(0)}(k) = O^{(0)}(k) - \hat{O}^{(0)}(k), \ k = 2,3,3, \ldots \ldots, n$ (10)

Let a, b and c be real numbers with a < b < c. Then the **Triangular Fuzzy Number** (TFN) A = (a, b, c) is the FN with membership function:

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$$y = \varepsilon^{(0)}(k) = \begin{cases} 0 & k < a \\ \frac{k-a}{b-a} & a < k < b \\ \frac{c-k}{c-b} & b < k < c \\ 0 & k > c \end{cases} \quad (11)$$

2.3 GAME THEORY

Game theory is a multi-person decision making process or decision analysis. It is assumed that players have knowledge about situation, opponent player's strategies ,and preferences in game theory. But most of the times, it is not possible to formulate all strategies implicitly because of the complexity of real situation (Law and Pan, 2008). A strategic game consists of set of players, strategies for each player and payoff for each strategy combination. If there is an interactive situation described as a game, a formal analysis should find optimal strategies for players and determine an expected outcome of game. A solution to a game is a certain combination of strategies.

Payoff Matrix and Optimal Strategies

In game theory, payoff matrix method is the most common method to analyzed time series . It is a mathematical representation of situation in which there are two players and one player's outcome is equal to the other's losses. Using above $\varepsilon^{(0)}(k)$ to built game payoff matrix as follows

Payoff Matrix

		Column player				
		Probability	y_1	y_2	⋯	y_n
	Probability	Strategy	1	2	⋯	n
Row player	x_1 x_2 ⋮ x_n .	1 2 ⋮ M	a_{11} a_{21} ⋮ a_{m1}	a_{12} a_{12} ⋮ a_{m2}	⋯ ⋯ ⋮ ⋯	a_{1n} a_{2n} ⋮ a_{mn}

Maximum (Row Minimum) = Minimum (Column Maximum) (12)

2.4 Hybrid-Grey (2, 1)

The Hybrid grey method is a sequence of Grey model and the triangular fuzzy number which can be used to predict future data and information.$GM(2,1)$ is a crucial model which is derived from $GM(1,1)$ [4] and it is defined as follows.

In this model the first parameter denotes the order of the differential equation and the second denotes number of variables.

First, arrange the sequence of original data $p^{(0)}(k), k = 1,2,3....n$

$$p^{(0)}(k) = \left\{p^{(0)}(1), p^{(0)}(2).....p^{(0)}(n)\right\} \quad (13)$$

Then, calculating the smoothing data using AGO as in

$$p^{(1)}(k) = \left\{p^{(1)}(1), p^{(1)}(2),.....p^{(1)}(n)\right\} \quad (14)$$

where $p^{(1)}(1) = p^{(0)}$

$p^{(1)} = \sum_{i=1}^{n} p^{())}(i), k =,2,3...n$ and I-AGO to create a new sequence from original data

using the equation below

$$\alpha^{(1)} p^{(0)}(k) = \left\{\alpha^{(1)} p^{(0)}(2), \alpha^{(1)} p^{(0)}(3),....\alpha^{(1)} p^{(0)}(n)\right\}, k = 2,3,....n \quad (15)$$

Here

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$$\alpha^{(1)} p^{(0)}(k) = \{p(k) - p^{(0)}(k-1)\}, k = 2,3,....n \quad (16)$$

Next, apply the Mean Generating Operation (MGO) to notate by $z^{(1)}(k) = \frac{p^{(0)} + p^{(1)}(k-1)}{2}, k = 2,3,....n.$ (17)

The value of MGO is used to solve the second-order differential equation, given by

$$\frac{d^2 p^{(1)}}{dt^2} + s_1 \frac{dp^{(1)}}{dt} + s_2 p^{(1)} = t \quad (18)$$

where s_1 is the lower order parameter, s_2 is the upper order parameter, and t is the Grey inputs.The values of s_1 ,s_2 and $t$can be determined by evaluating the second order differential equationusing the least square method .The solution is given by

$$(s_1, s_2, t)^T = (C^T C)^{-1} C^T X \quad (19)$$

with

$$X = \{\alpha^{(1)} p^{(0)}(2), \alpha^{(1)} p^{(0)}(3),....\alpha^{(1)} p^{(0)}(n)\}, k = 2,3,....n$$

$$C = \begin{bmatrix} -x^{(0)}(2) & -Z^n(2) & 1 \\ -x^{(0)}(3) & -Z^n(3) & 1 \\ . & . & . \\ . & . & . \\ -x^{(0)}(n) & -Z^n(n) & 1 \end{bmatrix}$$

After calculating s_1 ,s_2 and t values of the second-order differential equation (18) can be obtained as below

$$\hat{p}(k+1) = \bar{p}^{(1)}(k+1) + \frac{t}{s_2} \quad (20)$$

where $\bar{p}^{(1)}(k+1)$ is the broad solution of the similar homogeneous equation in(18).The value of $\bar{p}^{(1)}(k+1)$ can be planned depending on the sign of the equation

$$\beta^2 + s_1\beta + s_2 = 0$$

with $\psi = s_1^{\,2} - 4s_2$

There are three situations to establish the value of $\bar{p}^{(1)}(k+1)$

a. If $\psi > 0$, then

$$\bar{p}^{(1)}(k+1) = g_1 \ell^{\beta_1 k} + g_2 \ell^{\beta_2 k} \quad (21)$$

where $\beta_{1,2} = \frac{-s_1 \pm \sqrt{s_1^{\,2} - 4s_2}}{2}$

$$g_1 = \frac{1}{\beta_2 - \beta_1}(\beta_2 p^{(0)}(1) - \frac{1}{2}(p^{(0)}(3) - p^{(0)}(1))) - \beta_2 \frac{t}{s_2}$$

$$g_2 = \frac{1}{\beta_1 - \beta_2}(\beta_1 p^{(0)}(1) - \frac{1}{2}(p^{(0)}(3) - p^{(0)}(1))) - \beta_1 \frac{t}{s_2}$$

b. If $\psi = 0$,then

$$\bar{p}^{(1)}(k+1) - \ell^{\frac{s_1}{2}k}(g_1 + g_2 k) \quad (22)$$

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with $g_1 = p^{(0)}(1) - \frac{t}{s_2}$

$$g_2 = \frac{1}{2}(p^{(0)}(3) - p^{(0)}(1)) - \beta(p^{(0)}(1) - \frac{t}{s_2})$$

c. If $\psi < 0$, then

$$\overline{p}^{(1)}(k+1) = \ell^{\frac{s_1}{2}k}(g_1 \cos(\sqrt{\frac{4s_2 - s_1^2}{2}})k + g_2 \sin(\sqrt{\frac{4s_2 - s_1^2}{2}})k) \quad (23)$$

with $g_1 = p^{(0)}(1) - \frac{t}{s_2}$

$$g_2 = \frac{\frac{1}{2}((p^{(0)}(3) - p^{(0')}(1)) - \alpha(p^{(0)}(1) - \frac{t}{s_2})}{\beta}$$

After calculating the value of $\overline{p}^{(1)}(k+1)$, predict the data using I-AGO

$$\hat{p}^{(0)}(k+1) = \hat{p}^{(1)}(k+1) - \hat{p}^{(1)}(k) \quad (24)$$

The next step is to predict the upcoming data by using triangular fuzzy number which was discussed in Hybrid - Grey$(1,1)$ [12]

2.5 Model Evaluation

This model achieves a analytical accuracy, and also increase the coefficient, that takes the error test in grey model. Then the error can be obtained as follows

The average absolute error (AAE): $\tilde{\varepsilon} = \frac{1}{n}\sum_{i=0}^{n} \tilde{o}^{(0)}(i) - o^{(0)}(i)$ (25)

The average relative error (ARE): $\varepsilon_r = \frac{1}{n}\sum_{i=0}^{n} \left| \frac{\tilde{o}^{(n)}(i) - o^{(0)}(i)}{o^{(0)}(i)} \right|$ (26)

RESULTS AND DISCUSSION

Data used in this study, is obtained actual value of Nifty Fifty Stock Indices from 1st February 2019 to 9th April 2019 as shown in table 1.

TABLE:1 ACTUAL DATAOF NIFTY FIFTY STOCK INDICES

Date	01-Feb19	04-Feb19	05-Feb19	. . ..	.. .. ..	05Apr-19	08Apr-19	09Apr-19
Open	10851. 35	10876. 75	10908. 65	. . ..	.. .. ..	11638. 4	11704. 35	11612. 05

Using the data in Table 1, we forecast the values of the variables using GM (1, 1) and GM (2, 1).

3.1 GM (1,1) and GM (2,1)

The forecasts using this method are divided into two main results: the first is the result of GM (1,1) and the second is the result of the GM (2,1) model. The forecast values using GM (1,1) and GM(2,1) are shown in Table 2, which also shows the value of the relative error between the actual value and the predicted value using GM (1,1) (Eq. (10)) and GM(2,1) (Eq.(25) which will be used to predict the states defined in the Hybrid-Grey (1,1) and Hybrid-Grey(2,1) model.

Table 2: GM(1,1) and GM(2,1) Model value and predication error of stock market

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Date	Open	GM(1,1)	GM(2,1)	relative error of GM(1,1) $\varepsilon_k^{(0)}$	**Relative error of** GM(2,1)$\boldsymbol{\varepsilon_k^{(0)}}$
01-Feb-19	10851.35	10851.35	10851.35	0	0
04-Feb-19	10876.75	10656.26	10580.52	2.027133	2.723516
05-Feb-19	10908.65	10678.76	10639.89	2.107446	2.463733
06-Feb-19	10965.1	10704.18	10680.75	2.379504	2.593228
07-Feb-19	11070.45	10723.88	10854.92	3.130568	1.946895
08-Feb-19	11023.5	10746.52	10769.07	2.512664	2.308069
....	.....	.....	....	.....	
.....	.....	.....	....	.....	
.....	.....	.....	....	.....	
....	....	....	....	....	
....	....	....	....	....	
....	....	....	....	....	
....	....	....	.....	....	
04-Apr-19	11660.2	11662.44	11436.09	-0.01924	
05-Apr-19	11638.4	11642.97	11327.81	-0.03922	2.544594
08-Apr-19	11704.35	11667.54	10851.35	0.314503	2.291968
09-Apr-19	11612.05	11692.17	10580.52	-0.68994	2.447802

Now, applying triangular fuzzy number to improve our prediction value based on GM(1,1)and GM(2,1) residuals $\varepsilon_k^{(0)}$ as shown in table 2. Next corresponding triangular fuzzy membership function of GM(1,1) Model is

$$y = \varepsilon^{(0)}(k) = \begin{cases} 0 & k < -\ 2.1454 \\ \dfrac{k + 2.1454}{2.387969} & -2.1454 < k < 0.242569 \\ \dfrac{2.630568 - k}{2.387999} & 0.242569 < k < 2.630568 \\ 0 & k > 2.630568 \end{cases}$$

Here we use triangular fuzzy number as the entries in the payoff matrix, consider a fuzzy game between X and Y the fuzzified payoff matrix is given by A as follows.

$$A = \begin{bmatrix} 0.333 & 0.4444 & 0.2222 & 0 \\ 0.2941 & 0.52941 & 0.1765 & 0 \\ 0.0556 & 0.2222 & 0.6667 & 0.0556 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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In this game if X choose third row then Y will choose fourth column then X wins an value x∈ 0.0556 is a triangular fuzzy number and Y losses the same value. The triangular fuzzy membership function of GM(2,1) Model is

$$y=\varepsilon^{(0)}(k)=\begin{Bmatrix}0,k<1.846089\\ \dfrac{k-1.846089}{0.450684},1.846089<k<2.296773\\ \dfrac{2.296773-k}{0.450684},2.296773<k<2.747457\\ 0,k>2.747457\end{Bmatrix}$$

Here we apply triangular fuzzy number as the entries in the payoff matrix. Consider a fuzzy game between X and Y then the fuzzified payoff matrix is given by

$$A=\begin{bmatrix}0 & 0.0222 & 0.0222 & 0.0444\\ 0.0222 & 0 & 0.0889 & 0\\ 0.0444 & 0.0667 & 0.2889 & 0.1778\\ 0.0222 & 0.0222 & 0.1778 & 0\end{bmatrix}$$

In this game if player A choose third row then Player B will choose fourth column then Player A wins an value x∈0.0444 and Player B suffers the same value.

Hybrid-Grey (1, 1) and Hybrid-Grey (2,1) Model

Table 3: Predicted Values of Nifty Fifty Stock Indices

Date	Open	Hybrid-GM(1,1)	Hybrid-GM(2,1)
01-Feb-19	10851.35	10851.35	10846.14
04-Feb-19	10876.75	10656.26	10872.52
05-Feb-19	10908.65	10678.76	10905.07
06-Feb-19	10965.1	10704.18	10959.89
07-Feb-19	11070.45	10723.88	11066.22
08-Feb-19	11023.5	10746.52	11019.92
....	.....	.....	
.....	.....	.....	
.....	.....	.....	
....	....	....	
....	....	....	
....	....	....	
....	....	....	
04-Apr-19	11660.2	11662.44	11654.99
05-Apr-19	11638.4	11642.97	11634.17
08-Apr-19	11704.35	11667.54	11700.77
09-Apr-19	11612.05	11692.17	11606.84

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Figure: 1 Comparison of Hybrid-Grey(1,1) and Hybrid Grey (2,1)

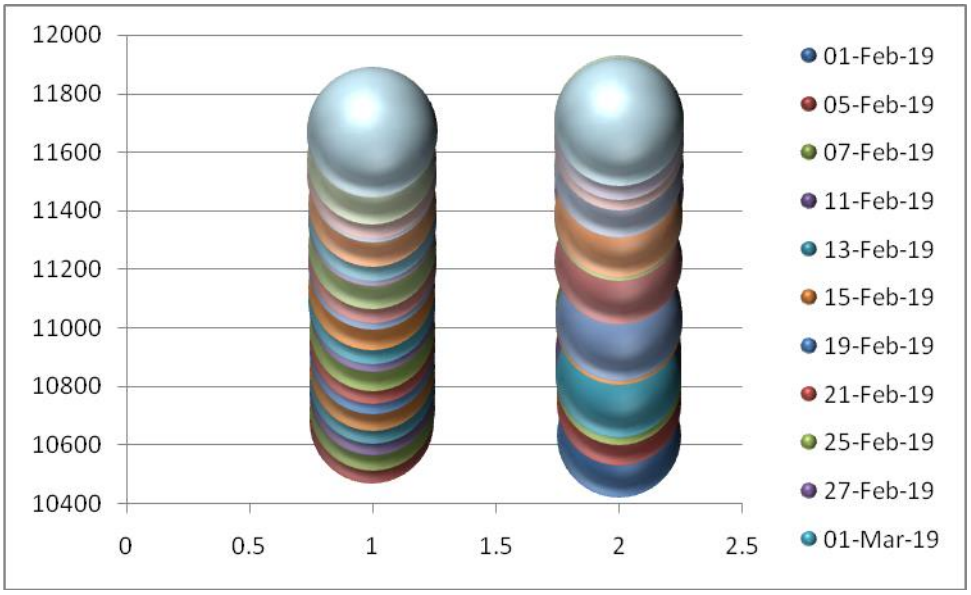

Figure 1 shows the comparison of the predicted values of Nifty Fifty stock indices using Hybrid-Grey (1,1) and Hybrid-Grey (2,1). From the figure we can see that the predicted values using Hybrid-Grey (1,1) are the closed to the actual value compared to the other methods (the predicted value of Hybrid-Grey (1,1) quite coincides with the actual value).

Table: 4 Model Evaluation

Model	AAE	ARE
Hybrid-Grey (1,1)	0.03	0.04
Hybrid-Grey (2,1)	0.04%	0.05%

Based on the model evaluation shown in Table 4, we can see that the model which has the smallest values of AAE, and ARE is Hybrid-Grey (1,1). Therefore, we can conclude that Hybrid-Grey (1, 1) is the best method to predict the Nifty Fifty stock values.

CONCLUSION

We have predicted the value of stock market using two methods which can overcome the problems associated with small sized data, which are Hybrid-Greys (1,1) and Hybrid-Grey(2,1). Based on the results, Hybrid-Grey (1,1) has the smallest value of AAE and ARE. Therefore, we can conclude that Hybrid-Grey (1,1) is the best method to predict the value of stock indices

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