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Results On the Existence of Impulsive Neutral Fractional Differential Equations with Infinite Delay

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ABSTRACT
In this article, we inspect the existence and uniqueness mild solutions for the impulsive neutral fractional differential systems with nonlocal initial conditions with infinite time delay. An array of solutions obtained by confining the fixed point theorem combined with an operator semigroup which is strongly continuous.

INTRODUCTION

Fractional differential equation have been seized extensive emphasis because of it's utilization in disparate field. Now a day's partial and ordinary differential equations has been momentous development with the fractional differential equations. In the field of life sciences many researchers approached in fractional differential equations such as anomalous diffusion in cell membranes [8], epidemiological models, etc and also it helps to model in mechanics and engineering [7], medical model Tuck well and Wan for HIV-1 infection in some cells [4], blood flow [9], cancer growth, financial market [3, 5, 6, 10, 11, 15].

Heterogeneous definitions of fractional derivatives suggested by bountiful mathematicians. Onward with the Riemann–Liouville and Caputo multifold summarized derivatives inclined by many researchers equally Caputo fractional derivative [12]. Innumerable researchers have been recognized the neutral integro differential equations for existence solutions with initial and nonlocal conditions. Neutral type fractional differential equation generally peek as models of science and engineering. Many authors showed the non-integer conception is existence of neutral differential equations via Caputo fractional derivative [2]. Most important application in neutral systems of fractional differential equation is problem of control systems [14].

In this article we investigate the existence and uniqueness solution for the given neutral system for fractional differential equations with nonlocal conditions is in the form

$$\begin{cases} D^{\alpha}\left[u(t)-f\left(t,u(t),u'\left(t,u(t)\right)\right)\right]=\mathbb{A}[u(t)-f(t,u(t),u'(t,u(t))] \\ \qquad\qquad +g\left(t,u(t),\int_0^t p\left(t,u(s),u'\left(s,u(s)\right)\right)ds\right) \\ \Delta u|_{t=t_i}=I_i\left(u(t_i^-)\right) \quad i=1,2,3\dots,n \\ u(0)+h(u)=u_0=\vartheta, \qquad \vartheta\in Y_l \end{cases} \tag{1}$$

where $1<\alpha<2$ and D^{α} is the Caputo fractional derivative and $f,g:J\times R\times R\to R$ and $h:J\times R$ are given continous function. The operator $\mathbb{A}$ is a strongly continuous in the semigroup of bounded linear operators. $I_i:X\to X;0=t_0<\cdots<t_m<t_{m+1}=b$, $\Delta u|_{t=t_i}=u(t_i^+)-u(t_i^-)$, $u(t_i^+)=\lim_{h\to 0^+}u(t_i+h)$and $u(t_i^-)=\lim_{h\to 0^-}u(t_i+h)$represent the right and left limits of $u(t)$ at $t=t_i$

Consider $\mathfrak{D}\mathrm{u(t)}=\mathrm{u'(t,u(t))}$. Then **(1)** changes to

$$\begin{cases} D^{\alpha}\left[u(t)-f\left(t,u(t),\mathfrak{D}\mathrm{u(t)}\right)\right]=\mathbb{A}[u(t)-f(t,u(t),u'(t,u(t))] \\ \qquad\qquad +g\left(t,u(t),\int_0^t p\left(t,u(s),\mathfrak{D}\mathrm{u(t)}\right)ds\right) \\ \Delta u|_{t=t_i}=I_i\left(u(t_i^-)\right) \quad i=1,2,3\dots,n \\ u(0)+h(u)=u_0=\vartheta, \qquad \vartheta\in Y_l \end{cases} \tag{2}$$

In the formation of this article is preliminaries and basic definitions, lemmas, theorems are discussed in section 2. In the following section 3 we discussed our main result which is the existence and uniqueness solutions for the neutral system for fractional differential equation with nonlocal conditions.

2 PRELIMINARIES

In this sector, we first introduce some basic definitions, notations, theorems and lemmas which are used to fixed the result.

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Assume that the continuous function $l: (-\infty, 0] \to (0, +\infty)$ satisfying $m = \int_{-\infty}^{0} l(t)dt < +\infty$. Let $(Y_l, \|.\|_{Y_l})$ be the Banach space convinced by the function l is defined as follows.

$$Y_l = \{\sigma: (-\infty, 0] \to X: for\ any\ e > 0, \sigma(\Lambda)\ is\ a\ bounded\ measurable\ function\ on\ [-e, 0], and\ \int_{-\infty}^{0} l(t) \sup_{s\le\Lambda\le 0} |\sigma(\Lambda)|\, ds < +\infty \}$$

equipped along the norm $\|\sigma\|_{Y_l} = \int_{-\infty}^{0} l(t) \sup_{s\le\Lambda\le 0} |\sigma(\Lambda)|\, ds$. Now we specify the space

$$Y_l{}' = \{\sigma: (-\infty, b] \to X: \sigma_i \in C(J_i, U), i = 0,1,2,\dots,n, and\ \ \exists\ \sigma(t_i^-)\ and\ \sigma(t_i^+)\ with\ \sigma(t_i) = \sigma(t_i^-), \sigma_0 = \sigma(0) + h(\sigma) = \sigma \in Y_l\}$$

Where σ is the constraint of σ to $J_i, J_0 = [0, t_1]$, $J_i = (t_i, t_{i+1}]$, $i = 1,2,\dots,n$. A seminorm denoted by $\|.\|_{Y_l'}$ in the space Y_l' is defined by

$$\|\sigma\|_{Y_l'} = \|\sigma\|_{Y_l} + \max\{\|\sigma_i\|_{J_i}, i = 1,2,\dots,n\}\ where\ \|\sigma_i\|_{J_i} = \sup_{s\in J_i} |\sigma_i(s)|.$$

Definition: 2.1 A function $u: (-\infty, b] \to X$ is said to be a mild solution of the equation if $u(0) + h(u) = u_0 = \vartheta \in Y_l$, the impulsive condition $\Delta u|_{t=t_i} = I_i(u(t_i^-))$, $i = 1,2,3\dots,n$ is documented, the stipulation of $u(.)$ to the interval $J_i (i = 0,1,2,\dots,n)$ is continuous and for $t \in J$ the below integral conditions holds

$$u(t) = \mathbb{T}(t)[\vartheta(0) - h(u) - f(0, \vartheta, \mathfrak{D}\vartheta)] + f(t, u(t), \mathfrak{D}u(t)) + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\alpha)^{\alpha-1} \mathbb{T}(t-s)\, g(s, u(s), \int_0^t p(s, u(\omega), \mathfrak{D}u(\omega)) d\omega)) + \sum_{0<t_i<t} \mathbb{T}(t-t_i) I_i(u(t_i^-))$$

Theorem: 2.2 Let Y be a convex, bounded and closed subset of a Banach space X and $B: Y \to Y$ be a condensing map. Then B has a fixed point in Y.

Lemma:2.3 Assume that $u \in Y_l'$ then for $t \in J, u_t \in Y_l$ Moreover

$$m|u(t)| \le \|u(t)\|_{Y_l} \le \|\vartheta\|_{Y_l} + m \sup_{s\in[0,t]} |u(t)|$$

3 Existence Results

We define $\hat{\vartheta}$ for $\vartheta \in Y_l$,

$$\hat{\vartheta} = \begin{cases} \vartheta(t), & t \in (-\infty, 0] \\ \mathbb{T}(t)\vartheta(0), & t \in J \end{cases}$$

Then $\hat{\vartheta} \in Y_l'$.

Let $u(t) = x(t) + \hat{\vartheta}(t), -\infty < t < b$. It is fact that x satisfies $x_0 = 0$, $t \in (-\infty, 0]$, and

$$x(t) = \mathbb{T}(t)\left[-h(x + \hat{\vartheta}) - f(0, \vartheta, \mathfrak{D}\vartheta)\right] + f\left(t, x_t + \hat{\vartheta}_t, \mathfrak{D}(\mathrm{x_t} + \hat{\vartheta}_t)\right) + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathbb{T}(t-s) \times g\left(s, (x_s + \hat{\vartheta}_s), \int_0^s p\left(s, (x_\omega + \hat{\vartheta}_\omega), \mathfrak{D}(x_\omega + \hat{\vartheta}_\omega)\right) d\omega\right) ds + \sum_{0<t_i<t} \mathbb{T}(t-t_i) I_i(x(t_i^-) + \hat{\vartheta}\,(\mathrm{t_i^-}))$$

iff u satisfies $u(t) = \vartheta(t)$, $t \in (-\infty, 0]$ and $t \in J$

$$u(t) = \mathbb{T}(t)[\vartheta(0) - h(u) - f(0, \vartheta, \mathfrak{D}\vartheta)] + f(t, u(t), \mathfrak{D}u(t)) + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\alpha)^{\alpha-1} \mathbb{T}(t-s)\, g(s, u(s), \int_0^t p(s, u(\omega), \mathfrak{D}u(\omega)) d\omega)) + \sum_{0<t_i<t} \mathbb{T}(t-t_i) I_i(u(t_i^-))$$

We need the following Assumptions to get our main results.

(A1) Let $u \in C[0,1]$ and $f: J \times J \times Y_l \to J$ is a piecewise continuous function and $\exists$ a +ve constants ρ, κ_1, κ_2 such that

$$|f(t_1, u_1, v_1) - f(t_2, u_2, v_2)| \le \rho(|t_1 - t_2| + |u_1 - u_2| + |v_1 - v_2|)$$

For each u_1, u_2, v_1, v_2 in Y , $\kappa_2 = \max_{t\in\mathfrak{R}} \|f(t, 0,0)\|$. Let $Y = C[\mathfrak{R}, X]$ be the set of continuous functions on $\mathfrak{R}$ with the Banach space X values.

(A2) Let $u \in C[0,1]$ and $g: J \times J \times Y_l \to J$ is a piecewise continuous function and $\exists$ a +ve constants $\mathfrak{F}_1, \mathfrak{F}_2$ such that

$$|g(t_1, u_1, v_1) - g(t_2, u_2, v_2)| \le \mathfrak{F}_1(|t_1 - t_2| + |u_1 - u_2| + |v_1 - v_2|)$$

For each u_1, u_2, v_1, v_2 in Y , $\mathfrak{F}_2 = \max_{t\in\mathfrak{R}} \|g(t, 0,0)\|$. Let $Y = C[\mathfrak{R}, X]$ be the set of continuous functions on $\mathfrak{R}$ with the Banach space X values.

(A3) Let $u \in C[0,1]$ and $p: J \times J \times Y_l \to J$ is a piecewise continuous function and $\exists$ a +ve constants $\mathfrak{P}, \mathfrak{E}_1, \mathfrak{E}_2$ such that

$$|p(t_1, u_1, v_1) - p(t_2, u_2, v_2)| \le \mathfrak{P}(|t_1 - t_2| + |u_1 - u_2| + |v_1 - v_2|)$$

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For each u_1, u_2, v_1, v_2 in Y , $\mathfrak{E}_2 = \max_{t\in\mathfrak{R}} \|p(t,0,0)\|$. Let $Y = C[\mathfrak{R}, X]$ be the set of continuous functions on $\mathfrak{R}$ with the Banach space X values.

(A4) Let $u' \in C[a,b]$ satisfy the Lipschitz condition. i.e., There exists a positive constants $\ddot{\eta}_1, \ddot{\eta}_2, \ddot{\eta}$ such that

$$\|\mathfrak{D}(t,u_1) - \mathfrak{D}(t,u_2)\| \leq \ddot{\eta}_1(\|u_1 - u_2\|),$$

for all u_1, u_2 in Y. $\ddot{\eta}_2 = \max_{t\in D} \|\mathfrak{D}(t,0)\|$; and $\ddot{\eta} = \max\{\ddot{\eta}_1, \ddot{\eta}_2\}$.

(A5) A semigroup with the strongly continuous of bounded linear operators $\mathbb{T}(t)$ which is generated by $\mathbb{A}$ is compact satisfying $|\mathbb{T}(t)| \leq \mathbb{M}$ for any $\mathbb{M} \geq 1$ when $t \geq 0$.

(A6) Let $I_i: X \to X$ is absolutely continuous , and ∃ continuous non-decreasing functions $P_i: \mathbb{R}^+ \to \mathbb{R}^+$ such that $\forall\, x \in X$

$$|I_i(x)| \leq Q_i(|x|), \quad \lim_{r\to\infty} \frac{Q_i(r)}{r} = \delta_i < \infty, \qquad i = 1,2,\dots,n$$

(A7) Let us define the continuous function $h: Y_l' \to X$ and ∃ some +ve constants ξ_1, ξ_2 such that $|h(u) - h(v)| \leq \xi_1 \|u - v\|_{Y_l'}$, and $|h(u)| \leq \xi_1 \|u\|_{Y_l'} + \xi_2$

(A8) $\mathbb{M}\left[\xi_1 + \frac{b^\alpha \mathfrak{F}_1 m}{\Gamma(\alpha+1)}(1 + b\mathfrak{E}_1) + \sum_{i=1}^n \beta_i\right] + k_2 m < 1$

Lemma : 3.1 If $(\boldsymbol{A1})$-$(\boldsymbol{A2})$ are satisfied, then the estimate $\|\mathfrak{D}u(t))\| \leq t(\ddot{\eta}_1\|u\| + \ddot{\eta}_2)$, $\|\mathfrak{D}u(t) - \mathfrak{D}v(t)\| \leq \ddot{\eta} t \|u-v\|$ are satisfied for any t∈R and u,v ∈ Y. There exists constants $\kappa_1, > 0$ *such that* $\kappa_1 = \rho + \ddot{\eta}t$; $\mathfrak{E}_1 = \mathfrak{P} + \ddot{\eta}t$.

Theorem: 3.2 If suppose our assumptions $(\boldsymbol{A1})$-$(\boldsymbol{A8})$ are holds. Then the system of equation (2) unique and mild solutions.

Proof: Let us define $\Pi: Y_l' \to Y_l'$ by $\Pi(u(t)) = 0, t \in (-\infty, 0]$

$$\Pi(u(t) = \mathbb{T}(t)\left[-h(u + \hat{\vartheta}) - f(0,\vartheta,\mathfrak{D}\vartheta)\right] + f\left(t, u_t + \widehat{\vartheta_t}, \mathfrak{D}(u_t + \hat{\vartheta}_t)\right) + \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\mathbb{T}(t-s) \times g\left(s, (u_s + \hat{\vartheta}_s), \int_0^s p\left(s, (u_\omega + \hat{\vartheta}_\omega), \mathfrak{D}(u_\omega + \hat{\vartheta}_\omega)\right) d\omega\right) ds + \sum_{0<t_i<t} \mathbb{T}(t - t_i) I_i(u(t_i^-) + \hat{\vartheta}(t_i^-))$$

Obiviously, $\hat{u}$ is a fixed point of Π then the given equation (2) has a mild solution $\hat{u} + \hat{\vartheta}$. We have to prove that the function Π satisfies the hypothesis of the theorem (2.2) .

Now define $(Y_l'', \|.\|_{Y_l'})$ is the Banach space which is induced by Y_l'; $Y_l'' = \{u \in Y_l' : u_0 \in Y_l\}$ with the norm $\|u\|_{Y_l'} = \sup\{|u(s)|: s \in [0,b]\}$.

Let $Y_l = \{u \in Y_l' : \|u\|_{Y_l'} \leq r\}$ for any $r > 0$. Then $Y_r, \forall\, r$, is a bounded, closed convex subset in X. For some $u \in Y_r$ by Lemma (2.3) we have

$$\|u_t + \widehat{\vartheta_t}\| \leq \|u_t\|_{Y_l} + \|\widehat{\vartheta_t}\|_{Y_l} \leq m \sup_{s\in[0,t]} |u(s)| + \|u_0\|_{Y_l} + m \sup_{s\in[0,t]} |\hat{\vartheta}(s)| + \|\widehat{\vartheta_0}\|_{Y_l}$$
$$\leq \|\vartheta\|_{Y_l} + m(r + \mathbb{M}|\vartheta(0)|)$$
$$\|u + \hat{\vartheta}\|_{Y_l'} \leq \|u\|_{Y_l'} + \|\widehat{\vartheta_t}\|_{Y_l'} \leq r + \|\widehat{\vartheta_0}\|_{Y_l} + \sup_{s\in[0,b]} |\widehat{\vartheta_t}| \leq r + \|\vartheta\|_{Y_l} + \mathbb{M}|\vartheta(0)|$$
$$\sup_{t\in J} |u(t) + \hat{\vartheta}(t)| \leq m^{-1}\|u_t + \widehat{\vartheta_t}\|_{Y_l} \leq r + \mathbb{M}|\vartheta(0)| + m^{-1}\|\vartheta\|_{Y_l}.$$

Step 1: ∃ a +ve integer $r \in \mathbb{N}$ such that $\Pi(Y_l) \subset Y_l$ ∀ positive number r, Y_r is clearly bounded closed convex set in Y_l' we have to prove that ∃ a positive integer $r \in \mathbb{N}$ such that $\Pi(Y_l) \subset Y_l$.
If suppose the condition is not true, then for all positive integer r, ∃ $u^r \in Y_r$ and $t(r) \in (-\infty, b]$ ∋: $\|\Pi(u^r)(t(r))\| > r$, where $t(r)$ stands for t reliant to r. Moreover, we have

$$r \leq |\Pi(u^r)(t(r))| \leq \left|\mathbb{T}(t(r))\left[-h(u^r + \hat{\vartheta}) - f(0,\vartheta,\mathfrak{D}\vartheta)\right]\right| + \left|f\left(t, u_{t(r)} + \hat{\vartheta}_{t(r)}, \mathfrak{D}(u_{t(r)} + \hat{\vartheta}_{t(r)})\right)\right| + \frac{1}{\Gamma(\alpha)}\int_0^{t(r)} (t(r) - s)^{\alpha-1} |\mathbb{T}(t(r) - s)| \times \left|g\left(s, (u_s + \hat{\vartheta}_s), \int_0^s p\left(s, (u_\omega + \hat{\vartheta}_\omega), \mathfrak{D}(u_\omega + \hat{\vartheta}_\omega)\right) d\omega\right)\right| ds + \left|\sum_{0<t_i<t(r)} \mathbb{T}(t(r) - t_i) I_i(u^r(t_i^-) + \hat{\vartheta}(t_i^-))\right|$$

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$$\leq \mathbb{M}\left(\xi_1\left\|u^{r}+\hat{\vartheta}\right\|_{Y_l'}+\xi_2+(\rho+\ddot{\eta}t)\left\|\hat{\vartheta}\right\|_{Y_l}+\kappa_2\right)+(\rho+\ddot{\eta}t)\left\|u_{t(r)}+\hat{\vartheta}_{t(r)}\right\|_{Y_l'}+\kappa_2$$
$$+\frac{\mathbb{M}}{\Gamma(\alpha)}\int_0^{t(r)}(t(r)-s)^{\alpha-1}\left|g\left(s,\left(u_s+\hat{\vartheta}_s\right),\int_0^s p\left(s,\left(u_\omega+\hat{\vartheta}_\omega\right),\mathfrak{D}\left(u_\omega+\hat{\vartheta}_\omega\right)\right)d\omega\right)\right|ds$$
$$+\mathbb{M}\sum_{i=1}^{n}\mathcal{Q}_i\left(\left|u^{r}(t_i^-)+\hat{\vartheta}\,(\mathrm{t}_\mathrm{i}^-)\right|\right)$$
$$\leq \mathbb{M}\left[\xi_1\left(r+\mathbb{M}|\vartheta(0)|+\|\vartheta\|_{Y_l}\right)+\xi_2+\kappa_1\left\|\hat{\vartheta}\right\|_{Y_l}+\kappa_2\right]+\kappa_1\left(mr+m\mathbb{M}|\vartheta(0)|+\|\vartheta\|_{Y_l}\right)+k_2$$
$$+\frac{\mathbb{M}b^\alpha}{\Gamma(\alpha+1)}\left[\mathfrak{F}_1\left(m(r+\mathbb{M}|\vartheta(0)|)+\|\vartheta\|_{Y_l}\right)+\mathfrak{F}_1 b\left(\mathfrak{E}_1\left(m(r+\mathbb{M}|\vartheta(0)|)+\|\vartheta\|_{Y_l}\right)+\mathfrak{E}_2\right)+\mathfrak{F}_2\right]$$
$$+\mathbb{M}\sum_{i=1}^{n}\mathcal{Q}_i\left(r+\mathbb{M}|\vartheta(0)|+\|\vartheta\|_{Y_l}\right)$$

Now let us take the limit as $r\to+\infty$ and divide on both sides by r.

$$1\leq \mathbb{M}\left[\xi_1+\frac{b^\alpha\mathfrak{F}_1 m}{\Gamma(\alpha+1)}(1+b\mathfrak{E}_1)+\sum_{i=1}^{n}\beta_i\right]+\kappa_2 m$$

Which is attain a contradiction to **(A8)**.Thus for some positive integer r, $\Pi(Y_r)\subset Y_r$.

Step 2: Now we have to prove that the operator $\Pi=\Pi_1+\Pi_2$ is deflation, Π_1 is a recession and Π_2 is compact. Then the term Π_1 and Π_2 are authentic on Y_l by correspondingly.

$$(\Pi_1 u)(t)=\begin{cases}0, & t\in(-\infty,0],\\ \mathbb{T}(t)\left[-h\left(u+\hat{\vartheta}\right)-f(0,\vartheta,\mathfrak{D}\vartheta)\right]+f\left(t,u_t+\hat{\vartheta}_t,\mathfrak{D}\left(u_t+\hat{\vartheta}_t\right)\right), & t\in J\end{cases}$$

$$(\Pi_2 u)(t)=\begin{cases}0,\quad t\in(-\infty,0],\\ \dfrac{1}{\Gamma(\alpha)}\displaystyle\int_0^t(t-s)^{\alpha-1}\mathbb{T}(t-s)\,g\left(s,\left(u_s+\hat{\vartheta}_s\right),\int_0^s p\left(s,\left(u_\omega+\hat{\vartheta}_\omega\right),\mathfrak{D}\left(u_\omega+\hat{\vartheta}_\omega\right)\right)d\omega\right)ds\\ \qquad+\displaystyle\sum_{0<t_i<t}\mathbb{T}(t-t_i)I_i(u(t_i^-)+\hat{\vartheta}\,(\mathrm{t}_\mathrm{i}^-))\end{cases}$$

Here we show that the Π_1 is a contraction, let us we take arbitrary value$u_1,u_2\in Y_r$. Then by the assumption (A2)-(A7) for all $t\in(-\infty,b]$, we have

$$|(\Pi_1 u)(t)-(\Pi_2 u)(t)|=\left|\mathbb{T}(t)\left[h\left(u_1+\hat{\vartheta}\right)+h\left(u_2+\hat{\vartheta}\right)\right]+f(t,u_{1_t}+\hat{\vartheta}_t,\mathfrak{D}(u_{1_t}+\hat{\vartheta}_t))-f(t,u_{2_t}+\hat{\vartheta}_t,\mathfrak{D}(u_{2_t}+\hat{\vartheta}_t))\right|$$
$$\leq(\mathbb{M}\xi_1+\kappa_2 m)\|u_1-u_2\|_{Y_l'}.$$

conclude that Π_1 is a contraction. In the following we show that Π_2 is continuous on Y_r. Let $\{u_m\}_{i=0}^{\infty}\subset Y_r$, with $u_m\to u$ in Y_r using the assumption (A1)-(A4), (A6) and Lemma (3.1) we have

$$|(\Pi_2 u)(t)-(\Pi_2 u_m)(t)|$$
$$=\left|\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}\mathbb{T}(t-s)\,g\left(s,\left(u_s+\hat{\vartheta}_s\right),\int_0^s p\left(s,\left(u_\omega+\hat{\vartheta}_\omega\right),\mathfrak{D}\left(u_\omega+\hat{\vartheta}_\omega\right)\right)d\omega\right)ds\right.$$
$$-\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}\mathbb{T}(t-s)\,g\left(s,\left(u_{m_s}+\hat{\vartheta}_s\right),\int_0^s p\left(s,\left(u_{m_\omega}+\hat{\vartheta}_\omega\right),\mathfrak{D}\left(u_{m_\omega}+\hat{\vartheta}_\omega\right)\right)d\omega\right)ds$$
$$\left.+\sum_{0<t_i<t}\mathbb{T}(t-t_i)\left[I_i\left(u(t_i^-)+\hat{\vartheta}\,(\mathrm{t}_\mathrm{i}^-)\right)-I_i(u(t_i^-)+\hat{\vartheta}\,(\mathrm{t}_\mathrm{i}^-))\right]\right|$$
$$\leq\frac{\mathbb{M}b^\alpha\mathfrak{F}_1 m}{\Gamma(\alpha+1)}(1+b\mathfrak{E}_1)\|u-u_m\|_{Y_l'}+\mathbb{M}\sum_{0<t_i<t}\mathbb{T}(t-t_i)\left[I_i\left(u(t_i^-)+\hat{\vartheta}\,(\mathrm{t}_\mathrm{i}^-)\right)-I_i(u(t_i^-)+\hat{\vartheta}\,(\mathrm{t}_\mathrm{i}^-))\right]$$
$$\to 0\ as\ m\to\infty$$

Hence, Π_2 is continuous. Thereafter we show that the group of equicontinuous functions in $\{\Pi_2 u: u\in Y_r\}$. Let $0<\gamma_1<\gamma_2\leq b$ then

$$|(\Pi_2 u)(\gamma_1)-(\Pi_2 u_m)(\gamma_2)|$$

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$$\leq \frac{1}{\Gamma(\alpha)}\int_0^{\gamma_1}[(\gamma_1-s)^{\alpha-1}|\mathbb{T}(\gamma_1-s)-\mathbb{T}(\gamma_2-s)|+|(\gamma_1-s)^{\alpha-1}-(\gamma_2-s)^{\alpha-1}|\,|\mathbb{T}(\gamma_2-s)|]\times$$
$$\left[\mathfrak{F}_1\left(r'+b(\mathfrak{E}_1 r'+\mathfrak{E}_2)\right)+\mathfrak{F}_2\right]ds+\frac{1}{\Gamma(\alpha+1)}\mathbb{M}\left[\mathfrak{F}_1\left(r'+b(\mathfrak{E}_1 r'+\mathfrak{E}_2)\right)+\mathfrak{F}_2\right]\ (\gamma_2-\gamma_1)^{\alpha}$$
$$+\sum_{0<t_i<t}|\mathbb{T}(\gamma_1-t_i)-\mathbb{T}(\gamma_2-t_i)|\mathcal{Q}_i\left(u(t_i^-)+\hat{\vartheta}(t_i^-)\right)+\mathbb{M}\sum_{0<t_i<t_r}\mathcal{Q}_i\left(u(t_i^-)+\hat{\vartheta}(t_i^-)\right)$$

Where $r'=r+\mathbb{M}|\vartheta(0)|+\|\vartheta\|_{Y_l}$. The RHS is free from $u\in Y_r$ and it bears zero as $\gamma_1-\gamma_2\to 0$ because the closeness of $\mathbb{T}(t)(t>0)\Longrightarrow$ the continuity in the topological space and thus $\Pi_2 Y_r$ is equicontinuous. The testimony of the other cases $\gamma_1<\gamma_2<0$or $\gamma_1<0<\gamma_2$ are elementary so we overlooked.

Further we prove that $\Pi_2 Y_r$ is compact. We fixed $0<t\leq b$ and let us take any arbitrary real number μ which is in $0<\mu<b$ whereas $u\in Y_r$, we exemplify

$$(\Pi_2^{\mu}u)(t)=\frac{\mathbb{T}}{\Gamma(\alpha)}\int_0^{t-\mu}\left(t-s-\mu\right)^{\alpha-1}\mathbb{T}\left(t-s-\mu\right)$$
$$g\left(s,(u_s+\hat{\vartheta}_s),\int_0^s p\left(s,(u_\omega+\hat{\vartheta}_\omega),\mathfrak{D}(u_\omega+\hat{\vartheta}_\omega)\right)d\omega\right)ds+\mathbb{T}(\varepsilon)\sum_{0<t_i<t-\varepsilon}\mathbb{T}(t-t_i-\varepsilon)I_i(u(t_i^-)+\hat{\vartheta}(t_i^-))$$

Subsequently $\mathbb{T}(t)(t>0)$ is compact, the set $L_\varepsilon(t)=\{(\Pi_2^\varepsilon u)(t): u\in Y_r\}$ is relatively compact in Y_l' for every $\mu, 0<\mu<b$. Furthermore, for every $u\in Y_r$,

$$\left|(\Pi_2 u)(t)-(\Pi_2^{\mu}u)(t)\right|$$
$$=\frac{1}{\Gamma(\alpha)}\left|\int_{t-\mu}^{t}(t-s)^{\alpha-1}\mathbb{T}(t-s)\,g\left(s,(u_s+\hat{\vartheta}_s),\int_0^s p\left(s,(u_\omega+\hat{\vartheta}_\omega),\mathfrak{D}(u_\omega+\hat{\vartheta}_\omega)\right)d\omega\right)ds\right|$$
$$+\sum_{t-\varepsilon<t_i<t}\left|\mathbb{T}(t-t_i)I_i\left(u(t_i^-)+\hat{\vartheta}(t_i^-)\right)\right|$$
$$\leq\frac{1}{\Gamma(\alpha)}\left|\int_{t-\mu}^{t}(t-s)^{\alpha-1}\mathbb{T}(t-s)\,g\left(s,(u_s+\hat{\vartheta}_s),\int_0^s p\left(s,(u_\omega+\hat{\vartheta}_\omega),\mathfrak{D}(u_\omega+\hat{\vartheta}_\omega)\right)d\omega\right)ds\right|$$
$$+\mathbb{M}\sum_{t-\mu<t_i<t}\mathcal{Q}_i\left|\left(u(t_i^-)+\hat{\vartheta}(t_i^-)\right)\right|$$

Compassionate $\mu\to 0$, we conclude that there are relatively compact sets $L_\varepsilon(t)$ arbitrarily close to the set $L(t)$, thus it is relatively compact in Y_l'.

On the point of follow up the existing steps and by the Arzela-Ascoli theorem, we consummate that the operator is Π_2 is a compact. From those arguments we capacitate us to wrap up that $\Pi=\Pi_1+\Pi_2$ is a reduction map on Y_r and by the Theorem 2.1 accord that the conclusion of the given equation (2) has the mild solution.

Now we show that the uniqueness of the solutions of the given equation (2). Assume that u_1,u_2 are the system (2) of two mild solutions with $u_1\neq u_2,\hat{t}=\sup\{t\in(0,+\infty)\ for\ each\ s\in\in(-\infty,t],u_1(s)=u_2(s)\}$ and that the scalar ε is adequately modest $\ni$: $t_i\notin(\hat{t},\hat{t}+\mu)$ for every positive integer i.

Let $0<\zeta<\frac{\mu}{2}, t\in[\hat{t},\hat{t}+\zeta]$

$$|u_1(t)-u_2(t)|$$
$$=\left|\mathbb{T}(t)[-h(u_1)+h(u_2)]+f\left(t,u_{1_t},\mathfrak{D}u_{1_t}\right)-f\left(t,u_{2_t},\mathfrak{D}u_{2_t}\right)\right.$$
$$+\frac{1}{\Gamma(\alpha)}\int_{\hat{t}}^{t}(t-s)^{\alpha-1}\mathbb{T}(t-s)\left\{g\left(s,u_{1_s},\int_0^s p(s,u_{1_\omega},\mathfrak{D}u_{1_\omega})d\omega\right)ds\right.$$
$$\left.\left.-\ g\left(s,u_{2_s}\int_0^s p(s,u_{2_\omega},\mathfrak{D}u_{2_\omega})d\omega\right)ds\right\}\right|$$
$$\leq\mathbb{M}\xi_1\|u_1(t)-u_2(t)\|_{Y_l'}+\left(k_1+\frac{\mathbb{M}b^\alpha\mathfrak{F}_1}{\Gamma(\alpha+1)}(1+b\mathfrak{E}_1)\right)\|u_1(t)-u_2(t)\|_{Y_l'}$$

From the definition of $\hat{t}$, we having that $u_1(t)=u_2(t),\forall\ t\in(-\infty,\hat{t}]$, and

$$\left\|u_{1_t}-u_{2_t}\right\|_{Y_l'}=\int_{-\infty}^{0}l(s)\left\|u_{1_t}-u_{2_t}\right\|_{[s,0]}ds\leq\int_{-\infty}^{0}l(s)\,ds\,max\{|u_1(t)-u_2(t)|,t\in[\hat{t},\hat{t}+\zeta]\}$$

$$\leq m\, max\{|u_1(t)-u_2(t)|, t\in[\hat{t},\hat{t}+\zeta]\}$$

Hence,

$$|u_1(t)-u_2(t)| \leq \mathbb{M}\xi_1|u_1(t)-u_2(t)| + \left(k_1+\frac{\mathbb{M}b^\alpha\mathfrak{F}_1}{\Gamma(\alpha+1)}(1+b\mathfrak{E}_1)\right) m\, max\{|u_1(t)-u_2(t)|\}\; t\in[\hat{t},\hat{t}+\zeta]$$

Accordingly,

$$max\{|u_1(t)-u_2(t)|, t\in[\hat{t},\hat{t}+\zeta]\} \leq \left(\mathbb{M}\xi_1+k_1m+\frac{\mathbb{M}b^\alpha\mathfrak{F}_1 m}{\Gamma(\alpha+1)}(1+b\mathfrak{E}_1)\right) max\{|u_1(t)-u_2(t)|, t\in[\hat{t},\hat{t}+\zeta]\}$$

Beyond the assumption (A8) we conclude that $u_1(t)=u_2(t)$ for $t\in[\hat{t},\hat{t}+\zeta]$. Which is controvert with the definition of $\hat{t} \Longrightarrow u_1(t)=u_2(t)$ for $t\in(-\infty,b]$.

In the equation consider $f=0, g=g(t,u_t,\mathfrak{D}u(t))$ then the equation (2) becomes

$$\begin{cases} D^\alpha[u(t)] = \mathbb{A}[u(t)] + g\left(t,u(t),\int_0^t p\big(t,u(s),\mathfrak{D}\mathrm{u}(\mathrm{t})\big)ds\right) \\ \Delta u|_{t=t_i} = I_i\big(u(t_i^-)\big) \quad i=1,2,3\ldots,n \\ u(0)+h(u)=u_0=\vartheta, \quad \vartheta\in Y_l \end{cases} \tag{3}$$

Here we define the upcoming assumptions

(A9) Let $u\in C[0,1]$ and $g: J\times J\times Y_l\to J$ is a piecewise continuous function and ∃ a +ve constants $\mathfrak{F}_1^{'},\mathfrak{F}_2^{'}$ such that

$$|g(t_1,u_1,v_1)-g(t_2,u_2,v_2)| \leq \mathfrak{F}_1^{'}(|t_1-t_2|+|u_1-u_2|+|v_1-v_2|)$$

For each u_1,u_2,v_1,v_2 in Y , $\mathfrak{F}_2^{'} = \max_{t\in\mathfrak{R}}\|g(t,0,0)\|$. Let $Y=C[\mathfrak{R},X]$ be the set of continuous functions on $\mathfrak{R}$ with the Banach space X values.

(A10) $\mathbb{M}\left[\xi_1+\frac{b^\alpha\mathfrak{F}_1' m}{\Gamma(\alpha+1)}+\sum_{i=1}^n\beta_i\right]<1$

(A11) $\mathbb{M}\left[\frac{b^\alpha\mathfrak{F}_1 m}{\Gamma(\alpha+1)}+\sum_{i=1}^n\beta_i\right]<1$

Then we have the following corrollary

Corollary 3.3 Suppose that the assumption (A5), (A6), (A7), (A9), (A10) are satisfied. Then there is a unique mild solution for the system of equation (3).

Leftout the nonlocal conditions the given equation becomes

$$\begin{cases} D^\alpha[u(t)] = \mathbb{A}[u(t)] + g\left(t,u(t),\int_0^t p\big(t,u(s),\mathfrak{D}\mathrm{u}(\mathrm{t})\big)ds\right) \\ \Delta u|_{t=t_i} = I_i\big(u(t_i^-)\big) \quad i=1,2,3\ldots,n \\ u(0)=u_0=\vartheta, \quad \vartheta\in Y_l \end{cases} \tag{4}$$

Hence from the above corollary we have the consecutive corollary.

Corollary 3.4 Persume that the conditions (A5), (A6), (A7), (A9), (A11) are satisfied. Then there is a unique mild solution for the system of equation (4).

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