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New Type of Open Sets and Decomposition of Continuity Via Picture Fuzzy Topological Spaces

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ABSTRACT
Picture fuzzy set gained attention in the field of decision making because many situation may have more than one choice and investigators are looking for optimum choice/decision. Elements in general topological space deals only one case while picture fuzzy topological space deals the case which has three components. This motivates to introduced new type of open sets as well as closed sets are in picture fuzzy topological space. Also, picture fuzzy continuous functions and some properties of picture fuzzy continuous functions are introduced and studied in this paper.

Keywords: Picture fuzzy sets, picture fuzzy topology, picture fuzzy open and closed sets, picture fuzzy functions, picture fuzzy continuous functions.

1. Introduction

Atanassov's [1,2,3] intuitionistic fuzzy set theory (IFS) is a generalizations of Zadeh's [24] fuzzy set theory(FS). D. Coker [5] developed new topological space called intuitionistic topological space. Intuitionistic fuzzy set theory failed to handle some cases. For instance, an event may be successful or not successful or neutral. To handle these case Smarandache [19. 20, 21] introduced neutrosophic sets(NSs), which is the generalized form of FSs and IFSs. NSs are able to effectively express the informations of inconsistency, incompleteness, and indeterminacy. NS has been applied in science and engineering field. Salama [17] introduced neutrosophic topological space and its operation. Later, Salama [18] introduced and investigated neutrosophic closed sets and its continuous functions. Properties of closed and continuous functions of the same are presented in his work. Parimala et al. [10 – 16] studied open sets, continuity, homeomorphism and several other concepts of topological spaces in neutrosophic environment.

One of extension of IFS is picture fuzzy set (PFS) and it is developed by Cuong [6]. In PFS theory, there are three components namely, positive, neutral and negative terms and sum of three components is less than or equal to one. simultaneously, PFS has been applied in science and engineering field. The development of PFS can be found in many fields [6-9, 22, 23]. In real life, there are more than one opinion for every decision. For instance, a person wants to purchase a motor bike or a car or both or neither. Membership function related to purchase a motor bike, negative membership function related to purchase a car because the person didn't choose a motor bike. Neutrality is related to purchase both car and motor bike. Refusal related to purchase neither a car nor a motorbike. PFS deals these kind of cases. Thao and Dinh [9], ad-vented the concept of picture fuzzy topological spaces (PFTS).

1.1 Motivation and Objective

The notion of PFS [4] , PFTSs [9] motivates to propose this novel notion of different types of weaker forms of open sets and closed sets in PFTS. The expanded and hybrid motivation and goal work is given in the entire manuscript, step by step. We make sure other hybrid systems of FS are special PFS cases, under some necessary circumstances. The robustness, durability, superiority and simplicity of our proposed model and algorithms are discussed. This model is the most common type and is used to collect large-scale data in Artificial Intelligence, Engineering and medical applications. Similar research can be easily replicated in the future with other methods and different forms of hybrid structure.

The preliminary definitions of PFS, PFTS and some operations related to PFTS are presented in section 2. Novel idea of some open and closed sets in PFTSs and established some of its properties and relations with the help of illustrations in section 3. In Section 4, We proposed an the continuous functions of PFTS using PFPs. In Section 5, the conclusion of this work is essentially summed up and future research scope directions in this novel notion are presented.

2. Background Work

The definitions from [6, 8, 9] are used in sequel.

Definition 2.1. [6] A PFS P on the universe U is defined by:

$$P = \{\zeta,: m(\zeta), a(\zeta), n(\zeta) \vee \zeta \in P\}$$

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where $m,a,n : P \rightarrow [0,1]$ are the membership, abstinence and non-membership functions respectively. The condition for a PFS is that $m + a + n \leq [0,1]$ and the degree of refusal is defined as $r(\zeta) = 1 \quad (m(\zeta) + a(\zeta) + n(\zeta))$. A triplet set (m,a,n) is said to be a picture fuzzy number (PFN).

Definition 2.2. [8] Two objects $S_1 = \{(\zeta, \ m_{S_1}(\zeta), a_{S_1}(\zeta), n_{S_1}(\zeta) \ ) : \zeta \in U\}$ and $S_2 = \{(\zeta, \ m_{S_2}(\zeta), a_{S_2}(\zeta), n_{S_2}(\zeta) \ ) : \zeta \in U\}$ are two PFSs defined on U, the universe of discourse, and their union and intersection are defined and denoted as follows

1. The union of S_1 and S_2 is

$$S_1 \cup S_2 = \{(\zeta, \ m_{S_1}(\zeta) \vee m_{S_2}(\zeta), a_{S_1}(\zeta) \wedge a_{S_2}(\zeta), n_{S_1}(\zeta) \wedge n_{S_2}(\zeta) \ ) : \zeta \in U\},$$

2. The intersection of S_1 and S_2 is

$$S_1 \cap S_2 = \{(\zeta, \ m_{S_1}(\zeta) \wedge m_{S_2}(\zeta), a_{S_1}(\zeta) \vee a_{S_2}(\zeta), n_{S_1}(\zeta) \vee n_{S_2}(\zeta) \ ) : \zeta \in U\},$$

3. The symmetric difference of S_1 and S_2 is

$$S_1 \quad S_2 = \{\left(\zeta, \ m_{S_1 \quad S_2}(\zeta), a_{S_1 \quad S_2}(\zeta), n_{S_1 \quad S_2}(\zeta) \ \right) : \zeta \in U\}, where$$

$$m_{S_1 \quad S_2}(\zeta) = 0 \vee m_{S_1} \quad m_{S_2}, n_{S_1 \quad S_2}(\zeta) = 0 \vee n_{S_1} \quad n_{S_2},$$

$$a_{S_1 \quad S_2}(\zeta) = \begin{cases} 1 \quad m_{S_1 \quad S_2}(\zeta) \quad n_{S_1 \quad S_2}(\zeta), if a_{S_1}(\zeta) > a_{S_2}(\zeta) \\ \{1 + a_{S_1}(\zeta) \quad a_{S_2}(\zeta)\} \wedge \{1 \quad m_{S_1 \quad S_2}(\zeta) \quad n_{S_1 \quad S_2}(\zeta)\}, if a_{S_1}(\zeta) \leq a_{S_2}(\zeta) \end{cases}$$

4. $S_1 \subseteq S_2$ if and only if

$$m_{S_1}(\zeta) \leq m_{S_2}(\zeta), a_{S_1}(\zeta) \geq a_{S_2}(\zeta) and n_{S_1}(\zeta) \geq n_{S_2}(\zeta), \forall \zeta \in U.$$

Definition 2.3. [9] Let P be a family of PFS on $U \neq \ \phi$. Then (U,T) is called a PFTS if the following:

- 0_S and 1_S are member of T.
- Arbitrary union of PFS S in T if each S in T
- Finite intersection of PFS S in T if each S in T

are satisfied.

The ordered pair (U,T) is called a PFTS. Here, $1_S = (1,0,0), 0_S = (0,0,1)$ are the null set and whole set respectively. Each PFS in P is called a picture fuzzy open set (briefly PFOS). The complement S of a PFOS S in U is called a picture fuzzy closed set (briefly PFCS) in picture fuzzy.

Definition 2.4. [9] Let $(P;T)$ be any PFTS and let $D \in \ P$. Then the picture fuzzy interior of D and picture fuzzy closure of D are defined by

1. $int_{pf}(D) = D^{pfo} = \{G : G$ is a PFOS in P and $G \subseteq D\}$,

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2. $cl_{pf}(D) = D^{pf} = \{G : G$ is a PFCS in P and $G \supseteq D\}$.

Throughout this segment, we will explore several essential definitions from [14, 13, 15]. We use these essential components in sequel.

3. Some new notions in picture fuzzy topological spaces

Definition 3.1. Let D be any PFS of a PFTS $(P;T)$. Then D is said to be a picture fuzzy

1. regular open (PFROS) set, if $D = \left(\left(D^{pf}\right)^{pfo}\right)$,
2. pre open (PFPOS) set, if $D \subseteq \left(\left(D^{pf}\right)^{pfo}\right)$,
3. semi open (PFSOS) set, if $D \subseteq \left(\left(D^{pfo}\right)^{pf}\right)$,
4. α-open (PFαOS) set, if $D \subseteq \left(\left(\left(D^{pfo}\right)^{pf}\right)^{pfo}\right)$,
5. semipre or β-open (PFβOS) set, if $D \subseteq \left(\left(\left(D^{pf}\right)^{pfo}\right)^{pf}\right)$,
6. b-open (PFbOS) set, if $D \subseteq \left(\left(D^{pf}\right)^{pfo}\right) \cup \left(\left(D^{pfo}\right)^{pf}\right)$,

Definition 3.2. Let D be any PFS of a PFTS $(P;T)$. Then D is said to be a picture fuzzy

1. regular closed (PFRCS) set, if D^c is a PFROS,
2. pre closed (PFP CS) set, if D^c is a PFPOS,
3. semi closed (PFSCS) set, if D^c is a PFSOS,
4. α-closed (PFαCS) set, if D^c is a PFαOS,
5. semipre or β-closed (PFβCS) set, if D^c is a PFβOS,
6. b-closed (PFbCS) set if D^c is a PFbOS.

Example 3.3. Let $P = \{p,q\}$. Define the PFSs $A = \{\ p,(0.2,0.2,0.5)\ ,\ q,(0.3,0.3,0.3)\ \}$, $B = \{\ p,(0.1,0.1,0.6)\ ,\ q,(0.2,0.2,0.5)\ \}$, and $C = \{\ p,(0.2,0.2,0.6)\ ,\ q,(0.3,0.3,0.3)\ \}$. The family $T = \{1_p, 0_p, A, B, A \cup B, A \cap B\}$ is a topology on the PFS P. Hence, $(P;T)$ is a PFTS. Here C is a PFSOS and PF α OS.

Definition 3.4. Let P be a non-void set. If $m,a,n \in [0,1]$, then the PFS $x_{(m,a,n)}$ is said to be a picture fuzzy point (PFP) in P given by

$$x_{(m,a,n)}(x_p) = \begin{cases} (m,a,n), if x = x_p \\ (0,0,1), if x \neq x_p \end{cases}$$

for $x_p \in P$ is said to be the support of $x_{(m,a,n)}$.

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Theorem 3.5. Let $(P;T)$ be a PFTS and if D be a PF α OPS, then it is a PFSOS.

Theorem 3.6. Let $(P;T)$ be a PFTS and if D be a PF α OPS, then it is a PFPOS.

Theorem 3.7. Let D be a PFS in a PFTS $(P;T)$. If E is a PFSOS such that $E \subseteq D \subseteq \left((E)^{pf}\right)^{pfo}$; then D is a PF α OS.

proof. Since E is a PFSOS, we have $E \subseteq \left((E)^{pfo}\right)^{pf}$. But, $D \subseteq \left((E)^{pf}\right)^{pfo} \subseteq \left(\left(\left((E)^{pfo}\right)^{pf}\right)^{pf}\right)^{pfo} = \left(\left((E)^{pfo}\right)^{pf}\right)^{pfo} = \left(\left((D)^{pfo}\right)^{pf}\right)^{pfo}$, This shows that D is a PF α OS.

Theorem 3.8. Let $(P;T)$ be a PFTS. A subset D of P is picture fuzzy α open iff it is both PFSOS and PFPOS.
Proof. If part: This is obvious.
Else Part: Let D be PFSOS and PFPOS. Then we have $D \subseteq \left((D)^{pf}\right)^{pfo} \subseteq \left(\left(\left((D)^{pfo}\right)^{pf}\right)^{pf}\right)^{pfo} = \left(\left((D)^{pfo}\right)^{pf}\right)^{pfo}$, this proves that D is PF α OS.

Remark 3.9. Arbitrary union of PF α OSs (resp. PFPOSs) is a PF α OSs (resp., PFPOSs).

Theorem 3.10. A PFS D in a PFTS P is PF α OS (resp. PFPOS) iff $\forall$ PFP $x_{(m,\rho,n)} \in D$, there exists a PF α OS (resp. PFPOS) $E_{x_{(m,\rho,n)}}$ such that $x_{(m,\rho,n)} \in E_{x_{(m,\rho,n)}} \subseteq D$.
Proof. If D is a PF α OS (resp. PFPOS), then we can have $E_{x_{(m,\rho,n)}} = D$ for each $x_{(m,\rho,n)} \in D$. Conversely let us assume that, $\forall$ PFP $x_{(m,\rho,n)} \in D$, there exists a PF α OS (resp., PFPOS), $E_{x_{(m,\rho,n)}}$ such that, $x_{(m,\rho,n)} \in E_{x_{(m,\rho,n)}} \subseteq D$. Then, $D = \cup\{x_{(m,\rho,n)} : x_{(m,\rho,n)} \in D\} \subseteq \cup\{E_{x_{(m,\rho,n)}} : x_{(m,\rho,n)} \in D\}$, and so $D = \cup\{E_{x_{(m,\rho,n)}} : x_{(m,\rho,n)} \in D\}$, which is a PF α OS (resp. PFPOS) by Remark 3.1.

Definition 3.11. Let $(P;T)$ and $(Q;S)$ be two PFTSs and let $H : (P;T) \to (Q;S)$. Then H is defined as

1. a picture fuzzy open function if $H(D)$ is a PFOS in Q $\forall$ PFOS D in P.
2. a picture fuzzy α-open function if $H(D)$ is a PF α OS in Q $\forall$ PFOS D in P.
3. a picture fuzzy pre-open function if $H(D)$ is a PFPOS in Q $\forall$ PFOS D in P.
4. a picture fuzzy semi-open function if $H(D)$ is a PFSOS in Q $\forall$ PFOS D in P.
5. a picture fuzzy β-open function if $H(D)$ is a PF β OS in Q $\forall$ PFOS D in P.
6. a picture fuzzy regular-open function if $H(D)$ is a PFROS in Q $\forall$ PFOS D in P.
7. a picture fuzzy b-open function if $H(D)$ is a PF b OS in Q $\forall$ PFOS D in P.

Theorem 3.12. Let $(P;T)$ and $(Q;S)$ are PFTSs. If $H : (P;T) \to (Q;S)$ is picture fuzzy α-open function, then it is a picture fuzzy semi open function.
Proof. Let H be a picture fuzzy α-open function and D be a PFOS in P. This shows that, $H(D)$ is a PF α OS in Q. By Theorem 3.1, we show that $H(D)$ is a PFSOS, this implies that the function H is a picture fuzzy semi open function.

Theorem 3.13. Let $(P;T)$ and $(Q;S)$ are two PFTSs. If $H:(P;T)\rightarrow(Q;S)$ is picture fuzzy α-open function, then it is a picture fuzzy pre open function.

4. Decomposition of continuity via picture Fuzzy topological spaces

Definition4.1. Let $(P;T)$ and $(Q;S)$ be any two PFTSs. And let H be a function from the PFTS $(P;T)$ to the PFTS $(Q;S)$. Then H is defined as

1. a picture fuzzy continuous function (PFCF), if for each PFOS D in $(Q;S)$, $H^{-1}(D)$ is a PFOS in $(P;T)$.
2. a picture fuzzy regular continuous function (PFRCF), if for each PFOS D in $(Q;S)$, $H^{-1}(D)$ is a PFROS in $(P;T)$.
3. a picture fuzzy pre continuous function (PFPCF), if for each PFOS D in $(Q;S)$, $H^{-1}(D)$ is a PFPOS in $(P;T)$.
4. a picture fuzzy semi continuous function (PFSCF), if for each PFOS D in $(Q;S)$, $H^{-1}(D)$ is a PFSOS in $(P;T)$.
5. a picture fuzzy α-continuous function (PFαCF), if for each PFOS D in $(Q;S)$, $H^{-1}(D)$ is a PFαOS in $(P;T)$.
6. a picture fuzzy β-continuous function (PFβCF), if for each PFOS D in $(Q;S)$, $H^{-1}(D)$ is a PFβOS in $(P;T)$.
7. a picture fuzzy b-continuous function (PFbCF), if for each PFOS D in $(Q;S)$, $H^{-1}(D)$ is a PFbOS in $(P;T)$

Theorem 4.2. Let us consider two PFTSs $(P;T)$ and $(Q;S)$ and let $H:(P;T)\rightarrow(Q;S)$ be a function. The following conditions are equivalent

1. H is a PFCF.
2. $H^{-1}(D)$ is a picture fuzzy closed set in $(P;T)$, for each picture fuzzy closed set D in $(Q;S)$.
3. $H(D^{pf}) \subseteq (H(D))^{pf}$, for each PFS D in $(P;T)$.
4. $(H^{-1}(D))^{pf} \subseteq H^{-1}(D^{pf})$, for each PFS D in $(Q;S)$.

Proof: $(i)\Rightarrow(ii)$: The proof is Obvious.
$(ii)\Rightarrow(iii)$: Consider D as a picture fuzzy closed set in Q. Thus $(H(D))^{pf}$ is a picture fuzzy closed set in Q. By (ii), $H^{-1}(H(D))^{pf}$ is a picture fuzzy closed set in P. But we have $H(D)\subseteq (H(D))^{pf}$. Then $H^{-1}(H(D))\subseteq H^{-1}((H(D))^{pf})$. Thus $(H^{-1}(H(D)))^{pf} = (D)^{pf} \subseteq (H^{-1}((H(D))^{pf}))^{pf}$. Therefore $H(D^{pf})\subseteq (H(D))^{pf}$.
$(iii)\Rightarrow(iv)$ Let D be any picture fuzzy closed set in Q. By (iii), $H(H^{-1}(D)^{pf})\subseteq (H(H^{-1}(D)))^{pf}$. Thus $(H^{-1}(D))^{pf} \subseteq H^{-1}(D^{pf})$.
$(iv)\Rightarrow(i)$ Let D be any PFOS in Q. Then D^{-1} is a picture fuzzy closed set in Q. By (iv)

$\left(H^{-1}\left(D^c\right)\right)^{pf} \subseteq H^{-1}\left(\left(D^c\right)^{pf}\right) = H^{-1}\left(D^c\right)$.

Also we know that $\left(H^{-1}\left(D^c\right)\right)^{pf} \supseteq H^{-1}\left(D^c\right)$. Then $\left(H^{-1}\left(D^c\right)\right)^{pf} = H^{-1}\left(D^c\right)$. Thus D^c is a picture fuzzy closed set in P. So D is a PFOS in P. Hence H is a picture fuzzy continuous function.

Theorem 4.3. Let $(P;T)$ and $(Q;S)$ be any two PFTSs. Let $H:(P;T)\rightarrow(Q;S)$ be a function. Then H is a picture fuzzy continuous function if and only if $H^{-1}\left((D)^{pfo}\right) \subseteq \left(H^{-1}(D)\right)^{pfo}$ for each PFS D in $(Q;S)$.

Proof: Assume that H is a picture fuzzy continuous function. Let D be any PFS in Q. Then $(D)^{pfo}$ is a picture fuzzy open set in Q. Since H is a picture fuzzy continuous function, $\left(H^{-1}(D)\right)^{pfo}$ is a PFOS in P.

Also we know that $(D)^{pfo} \subseteq D$, then $H^{-1}\left((D)^{pfo}\right) \subseteq H^{-1}(D)$. Thus $\left(H^{-1}\left((D)^{pfo}\right)\right)^{pfo} = H^{-1}\left((D)^{pfo}\right) \subseteq \left(H^{-1}(D)\right)^{pfo}$. Therefore, $H^{-1}\left((D)^{pfo}\right) \subseteq \left(H^{-1}(D)\right)^{pfo}$. Conversely, suppose $H^{-1}\left((D)^{pfo}\right) \subseteq \left(H^{-1}(D)\right)^{pfo}$ for each PFS D in $(Q;S)$. If D is a PFOS in Q, then by assumption, $H^{-1}(D) \subseteq \left(H^{-1}(D)\right)^{pfo}$ but we know that $H^{-1}(D) \supseteq \left(H^{-1}(D)\right)^{pfo}$. Then $H^{-1}(D) = \left(H^{-1}(D)\right)^{pfo}$. Thus $H^{-1}(D)$ is a picture fuzzy open set in D. So H is a picture fuzzy continuous function.

Theorem 4.4. Let H be a function from a PFTS $(P;T)$ to a PFTS $(Q;S)$, that satisfies $\left(\left(H^{-1}(E)^{pf}\right)^{pfo}\right)^{pf} \subseteq H^{-1}\left(E^{pf}\right)$, for every PFS E in Q. Then H is a picture fuzzy α-continuous function.

Proof. Let E be a PFOS in Q. Then E is a picture fuzzy closed set in Q, which implies that from the assumption that $\left(\left(H^{-1}\left(E^c\right)^{pf}\right)^{pfo}\right)^{pf} \subseteq H^{-1}\left(E^{c\,pf}\right) = H^{-1}\left(E^c\right)$. It follows that

$$\left[\left(\left(H^{-1}(E)^{pf}\right)^{pfo}\right)^{pf}\right]^c = \left[\left(\left(H^{-1}(E)^{pf}\right)^{pfo}\right)^c\right]^{pfo} = \left(\left[\left(H^{-1}(E)^{pf}\right)^c\right]^{pf}\right)^{pfo}$$

$$\left(\left(\left[H^{-1}(E)^c\right]^{pfo}\right)^{pf}\right)^{pfo} = \left(\left(H^{-1}\left(E^c\right)^{pfo}\right)^{pf}\right)^{pfo} \subseteq H^{-1}\left(E^c\right) = \left[H^{-1}(E)\right]^c$$

This implies that $H^{-1}(E) \subseteq \left(\left(H^{-1}(E)^{pfo}\right)^{pf}\right)^{pfo}$. This shows that $H^{-1}(E)$ is a PFαOS in P. Hence, H is a picture fuzzy α-continuous function.

Theorem 4.5. Let H be a function from a PFTS $(P;T)$ to a PFTS $(Q;S)$. Then the following assertions are equivalent.

1. H is a picture fuzzy α-continuous function.
2. For each picture fuzzy point $x_{m,a,n} \in P$ and every picture fuzzy neighbourhood D of $H\left(x_{m,a,n}\right)$, there exists a picture fuzzy α-open set E in P such that $x_{m,a,n} \in E \subseteq H^{-1}(D)$.
3. For each picture fuzzy point $x_{m,a,n} \in P$ and every picture fuzzy neighbourhood D of $H\left(x_{m,a,n}\right)$, there exists a PFαOS E in P such that $x_{m,a,n} \in E$ and $H(E) \subseteq D$.

Proof. $(i) \Rightarrow (ii)$ Let $x_{m,a,n}$ be a picture fuzzy point in P and let D be a picture fuzzy neighbourhood of $H\left(x_{m,a,n}\right)$. Then there exists a PFOS E in Q such that $H\left(x_{m,a,n}\right) \in E \subseteq D$. Since H is picture fuzzy α-continuous, we know that $H^{-1}(x_{m,a,n})$ is a PF α OS in P and $x_{m,a,n} \in H^{-1}(H(x_{m,a,n})) \subseteq H^{-1}(E) \subseteq H^{-1}(D)$. Hence (ii) is valid. $(ii) \Rightarrow (iii)$ Let $x_{m,a,n}$ be a picture fuzzy point in P and let D be a picture fuzzy neighbourhood of $H(x_{m,a,n})$. The condition (ii) implies that there exists a PF α OS E in P such that $x_{m,a,n} \in E \subseteq H^{-1}(D)$ so that $x_{m,a,n} \in E$ and $H(E) \subseteq H(H^{-1}(D)) \subseteq D$. Consequently (iii) is true. $(iii) \Rightarrow (i)$ Let E be a PFOS in Q and let $x_{m,a,n} \in H^{-1}(E)$. Then $H(x_{m,a,n}) \in E$ and so E is a picture fuzzy neighbourhood of $H(x_{m,a,n})$ since E is PFOS. It follows from (iii) that there exists a PF α OS D in P such that $x_{m,a,n} \in D$ and $H(D) \in E$ so that $x_{m,a,n} \in D \subseteq H^{-1}(H(D)) \subseteq H^{-1}(E)$. Applying Theorem 3.1 induces that $H^{-1}(E)$ is a PF α OS in P. Therefore, H is a picture fuzzy α-continuous function.

Theorem 4.6. Let H be a function from a PFTS $(P;T)$ to a PFTS $(Q;S)$. If H is picture fuzzy α-continuous, then it is picture fuzzy semi continuous.
Proof. Let E be a PFOS in Q. Since H is picture fuzzy α-continuous, $H^{-1}(E)$ is a PFSOS in P. It follows from Theorem 3.1 that $H^{-1}(E)$ is a PFSOS in P so that H is a picture fuzzy semi-continuous function

Theorem 4.7. Let H be a function from a PFTS $(P;T)$ to a PFTS $(Q;S)$. If H is picture fuzzy α-continuous, then it is picture fuzzy pre continuous.
Proof. The proof is an immediate consequence from Theorem 3.2.

Theorem 4.8. Let H be a function from a PFTS $(P;T)$ to a PFTS $(Q;S)$. If H is PF α CF, iff it is PFPCF and PFSCF.
Proof. The proof is an immediate consequence from Theorem 3.4.

Theorem 4.9. Let $(P;T)$, $(Q;S)$ and $(R;R)$ be any three PFTSs. Let $H:(P;T) \to (Q;S)$ and $G:(Q;S) \to (R;R)$ be two functions. If H is a PF α CF and G is a PFCF, then $G \circ H$ is a picture fuzzy α-continuous function.

5. Conclusion

This paper contributed to the study the structure of new notion of open sets and continuity by using the general topological structure in terms of picture fuzzy environment. Several interesting properties were investigated and some examples were shown. Because PFSs are a generalization of FSs and IFSs, the results in this paper can be considered as a generalization of FTSs and IFTSs For future research, it is interesting to study PFSs in other types of topological structures.

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References

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87-96.

[2] K. Atanassov, Review and new results on Intuitionistic fuzzy sets, Preprint IM-MFAIS1-88, So_a, 1988.

[3] K. Atanassov and S. Stoeva, Intuitionistic fuzzy sets, in: Polish Syrup. on Interval & Fuzzy Mathematics, Poznan,(August 1983) 23-26.

[4] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182-190.

[5] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems 88(1997), no. 1, 81-89.

[6] B. C. Cuong. Picture fuzzy sets, Journal of Computer Science and Cybernetics, 30(4)(2014), 409-420.

[7] N. V. Dinh, N. X. Thao. Some measures of picture fuzzy sets and their application in multi-attribute decision making, I.J. Mathematical Sciences and Computing, 3, (2018), 23-41.

[8] H. Garg. Some Picture Fuzzy Aggregation Operators and Their Applications to Multi criteria Decision-Making, Arab. J. Sci. Eng,42, (2017), 5275-5290.

[9] N. X. Thao, N. V. Dinh. Rough Picture Fuzzy Set and Picture Fuzzy Topologies, Journal of Computer Science and Cybernetics, 31(3), (2015), 245 - 253. DOI: 10.15625/18139663/31/3/5046

[10] M. Parimala, R. Jeevitha, S. Jafari, F. Smarandache and R. Udhayakumar. Neutrosophic αψ-Homeomorphism in Neutrosophic Topological Spaces,Information,9,187,(2018), 1-10. ; doi:10.3390/info9080187.

[11] M. Parimala, R. Jeevitha, A. Selvakumar. A New Type of Weakly Closed Set in Ideal Topological Spaces, International Journal of Mathematics and its Applications, 5(4-C), (2017), 301-312.

[12] M. Parimala, M. Karthika, R. Dhavaseelan, S. Jafari. On neutrosophic supra precontinuous functions in neutrosophic topological spaces, New Trends in Neutrosophic Theory and Applications , 2, (2018), 371-383.

[13] M. Parimala, M. Karthika, S. Jafari, F. Smarandache , R. Udhayakumar. Neutrosophic nano ideal topological structures, Neutrosophic sets and systems, 24(2019), 70-77.

[14] M. Parimala, M. Karthika, S. Jafari, F. Smarandache, R. Udhayakumar. Decision Making via Neutrosophic Support Soft Topological Spaces, Symmetry, 10(2018), 1-10. doi:10.3390/sym10060217

[15] M. Parimala and R. Perumal. Weaker form of open sets in nano ideal topological spaces, Global Journal of Pure and Applied Mathematics, 12(1), (2016), 302-305.

[16] M. Parimala, F. Smarandache, S. Jafari, R. Udhayakumar. On Neutrosophic αψ-Closed Sets, Information, 9, (2018), 103, 1-7.

[17] A. A. Salama, S. A. Alblowi. Neutrosophic Set and Neutrosophic Topological Spaces, IOSR J. Math., 3, (2012), 31-35. 11

[18] A. A. Salama. F. Smarandache, Valeri Kromov. Neutrosophic Closed Set and Neutrosophic Continuous Functions, Neutrosophic Sets and Systems, 4, (2014),4-8.

[19] F. Smarandache. Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic Set, Probability and Statistics; University of New Mexico, Gallup, NM, USA, (2002).

[20] F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Research Press, Rehoboth, NM, USA, (1999).

[21] F. Smarandache. Extension of Hyper Graph to n-Super Hyper Graph and to Plithogenic n-Super Hyper Graph, and Extension of Hyper Algebra to n-ary (Classical-/Neutro/Anti-)Hyper Algebra, Neutrosophic Sets and Systems, 33,(2020), 290-296.

[22] G. W. Wei. Some cosine similarity measures for picture fuzzy sets and their applications to strategic decision making, Informatica, 28(3), (2017), 547-564.

[23] G. W. Wei. Some similarity measures for picture fuzzy sets and their application, Iranian Journal of Fuzzy Systems, 15(1), (2018), 77-89.

[24]L. A. Zadeh. Fuzzy Sets, Information and Control, 18, (1965), 338-353