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The Effect of Different Particle Size on the Rotating Composite Disk Subjected To Thermal Gradient

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Abstract

The effect of different particle size on rotating composite disk in the presence of the thermal gradient is investigated in present study. The mathematical modelling is used to find the value of radial and tangential stress rates and strain rates. The effect of creep is reducing with increasing thermal gradient as compare to uniform temperature for different particle size. In this paper we compare the value of stress and strains for different particle size then concluded that particle size between P=1.7 μm, 14.5μm can be taken for safe design.

.**Keywords:** Stress, strain, thermal gradient, particle size and temperature.

1. Introduction

Composite material is made from two or more materials also called composition material having significantly diverse physical properties as well as chemical properties when they combined these materials then produce a material with characteristics different from individual components for e.g. concrete is made from cement, stone, water and stuns. In 1500s B.C. Egyptians and Mesopotamian settler's used composites to make strong and durable houses by using mixture of mud and straw. In 1200 A.D. Mongols invented the first composite bow by using mixture of wood, animal glue and bone. Today, composite are used to manufacture ladder rails, pipes, tools handle, arrow shafts, train floors, medical devices.

Composite material have many advantages that are high strength, corrosion resistance, light weight, high impact strength, non conductive, radar transparent, design flexible, non magnetic, low thermal conductivity and durable. Composite material have many applications in different fields that are in appliance, manufacturing, electrical circulation, power, marine, sanitary or plumbing and transportations resister's etc [13,15,16,21,22]. Chinese wall is best example of ancient composite structure which has great strength and durability. There are some problem like corrosion and high temperature which can shrink life, decrease its strength and increase the creep rate of the materials [9].

Materials with changing microstructure, composition, porosity across the volume of the material is called FGM materials. FGM is mainly manufactured to work in high temperature environment like light weight, temperature resistance materials for aerospace vehicles [7]. Teeth, bones are named as natural functionally graded material. FGM is used in many engineering devices like steam and gas turbine rotors, turbo generators and brake discs it also provide attractive materials for wide range of engineering applications, medicines and tissue engineering [18].FGMs can be manufactured by different techniques, chemical vapors deposition, centrifugal casting, powder metallurgy etc[10]. FGMs having many applications in components advanced aircraft, dental cores, implants, aerospace engines, thermal barrier coating and computer circuit boards etc [4].

Rotating disk is generally used components in many engineering, structural devices such as flywheels, turbo generators, steam, internal combustion engine, gas turbine rotors, automotive brakers, turbojet engines etc [1,8,12]. In many applications disc brakes and turbine rotors, disk is subjected to high temperature and serve automatic loading to vulnerable creep [17]. Rotating disks are used in wide range of instruments which can either be mechanical and electronic instruments. Rotating disk gives a wide region of research because of their immense usage in rotating machinery like compressors, pumps, flywheels, gas turbine rotors,

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computer disc drive, automotive baking system [11]. We are able to reduce creep deformation by varying thickness, properties of disk by using FGM also we are able to reduce cost and increases efficiency of disk.

Gupta et al. [6] investigate the stress distribution along radial and tangential in a rotating disk for combination of material parameters size and particle content at constant temperature 623K. They investigate that tangential stress increases as move from internal to external radii. The radial stress can't vary possibly for different combinations of material parameters. The radial strain rates and tangential strain rates in rotating disk reducing with decreasing particle size and increasing particle content.

Gupta et al. [8] investigate Sherby's model give better results as compare to Norton's to describe creep behavior of composite material in isotropic rotating disk at steady state. They compare experimental result of Whal's with present study. They concluded that radial and tangential strain rates decreases particle size, decreasing temperature and increasing particle content.

Bose et al. [19] investigate the stable-state creep investigation of the thermally graded rotating disk made from varying particle contraction. They calculated the stress and strain rates from internal to external radius by using Von Mises criteria. They observed that radial stress start increases as we move from internal radii then start decreases at middle of disk that traces parabolic path. They observed that with thermal gradient radial stress decreases as compare to uniform temperature and tangential stress increases at internal radii but start decreases ae move from internal to external radii but increasing thermal gradient then tangential stress and strain rates decreases as compare to uniform temperature.

Pandey et al. [3] investigate the size reinforcement and volume fraction on the stable state creep behavior of $AlSiC_P$in the temperature range 623-723K. They observed that threshold stress is independent of particle size but varying with volume fraction of reinforcement. They suggested the model based on the applied stress independent load transfer is requiring explaining origin such as threshold stress.

Singh et al. [5] investigate that the secondary state creep response in particle reinforced isotropic FGM disk they compare the linear variation particle distribution with uniform particle distribution with Norton's law. They observed that the radial stresses decreases with increasing gradient in particle distribution as compare to uniform but same average particle content and tangential stress increases near the internal radii but as move from internal to external radii tangential stress goes decreases. The radial strain rate also decreases with increases particle gradient also tangential strain rate with increases particle gradient as move from internal to external radius decreases therefore creep rate decreases with increases particle gradient.

In the current study, the secondary state creep response of rotating disk assumed for three cases for different particle P= 1.7μm,14.5μm and 45.3μm. In case first disk is consider to operate at uniform temperature 635K while in case second disk is operate at thermal gradient 75K by taking temperature at internal radii 560K and external radii 635K. In case third disk is consider to operate at thermal gradient 175K temperature at internal radii 460K and external radii 635K. The internal and outer radii of disk is taken as 30mm and 150mm respectively.

2.Mathematical Modeling

2.1 Thermal Gradient

For present study the disk is operate under thermal gradient, as given by

$$T(r) = H - Lr,\ r_i \le r \le r_0 \quad (1)$$

Where,

$$H = \frac{r_o T_m - r_i T_n}{r_o - r_i} \quad (2)$$

$$\boldsymbol{L} = \frac{\boldsymbol{T_m - T_n}}{\boldsymbol{r_o - r_i}} \quad (3)$$

where T_m and $T_n$are temperature at internal and external radius respectively.

2.2 Creep Law

The stable state creep response of the $Al\!-\!SiC_P$composite of varying composition have been described in the given form terms of Sherby's law [2] :

$$\dot{\epsilon} = [M_a(\bar{\sigma} - \sigma_0)]^8 \quad (4)$$

where effective strain rate is $\dot{\epsilon}$, effective stress rate is $\bar{\sigma}$and thershold stress is σ_0.

2.3 Creep Specification

Creep specification is given below:

,
$$M_a = \frac{1}{E}\left[\frac{AD_L\lambda^3}{|\vec{b}_r^5|}\right]^{1/8} \quad (5)$$

where M_a material creep constant, A is constant, young's modulus is E, magnitude of Burger's vector is $\left|\overrightarrow{b_r}\right|$, lattice diffusivity is D_L.

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2.4 Reinforcement Distribution in the Disk

The composition variation in term of the volume percent of SiC, along the radial distance, $V(R)$, is given by:

$$V(r) = N - Sr, \; r_i \le r \le r_0$$

Where,

$$N = \frac{r_o V_m - r_i V_m}{r_o - r_i} \quad (7)$$

And,

$$S = \frac{V_m - V_n}{r_o - r_i} (8)$$

where V_m=35% and V_n=15% are particle content at internal and external radii. By mixture of law, density variation in composite is given as,

$$\rho(r) = \rho_p + (\rho_m + \rho_p)\frac{V(r)}{100} \quad (9)$$

where density of matrix alloy is ρ_m= 3210kg/m^3 and density of silicon carbide particles ρ_p= 2698.9kg/m^3. Put the value $V(R)$ from eq^n(6) in eq^n(7) and (8), we get

$$\rho(r) = \rho_p + (\rho_m + \rho_p)\frac{N - Sr}{100} \quad (10)$$

If average particle content is V_{avg}in FGM disk then,

$$\int_{r_i}^{r_0} 2\pi r V(r) dr = V_{avg}(r_0^2 - r_i^2)$$

(11)

By using value of the $V(r)$ from equation (6) into the equation (11), we get following relation:

$$V_{avg} = N - \frac{2}{3}\frac{S(r_0^3 - r_i^3)}{(r_0^2 - r_i^2)}$$

(12)

The creep results reported for the $Al - SiC_p$composite [3] used for obtanied values of creep specification M_a and σ_0 and these can be fitted by the following regression equation as a function of temperature is T, P is particle size and V is volume.

$$\ln(M_a(r)) = -34.91 + 0.2112\ln P + 4.89\ln T(r) - 0.591\ln V(r) \quad (13)$$

$$\sigma_0(r) = -0.02050P + 0.0378T(r) + 1.033V(r) - 4.9695 \quad (14)$$

3.Mathematical Formulation

Consider a$Al - SiC_P$composite disk having internal radii $r_i$and external radii r_0 with angular velocity ω. The following assumptions are made for modeling:

- Consider that stress is at stable state.
- As compared to the creep deformation elastic deformation is small so it neglected.
- At any point of disk the biaxial state of the stress exists.
- The disk show a stable state creep behaviour, which can be described by the

Sherby's constitutive model is given by equation(4)

For an isotropic rotating disk the generalized constitutive eq^{n}are :

$$\dot{\epsilon}_r(r) = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}}[2\sigma_r(r) - (\sigma_\theta(r) + \sigma_z(r))] \quad (15)$$

$$\dot{\epsilon}_\theta(r) = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}}[2\sigma_\theta(r) - (\sigma_z(r) + \sigma_r(r))] \quad (16)$$

$$\dot{\epsilon}_z(r) = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}}[2\sigma_z(r) - (\sigma_r(r) + \sigma_\theta(r))]$$

(6)
(17)

where strain rates are $\dot{\epsilon}_r$,$\dot{\epsilon}_\theta$ and $\dot{\epsilon}_z$ and stress rates σ_r, $\sigma_\theta$and $\sigma_z$corresponding in direction r,θ,z which indicate by subscripts. Forbixial state of stress(σ_z=0) .The effective stress $\bar{\sigma}$is given below where assumed that effective stress is based on the Mises criterion , for biaxial state of stress,

$$\bar{\sigma} = \frac{1}{\sqrt{2}}[\sigma_r^2(r) + \sigma_\theta^2(r) + (\sigma_r(r) - \sigma_\theta(r))^2]^{\frac{1}{2}}$$

(18)

By using equations (17) and (5) in equation (15) and (16), we get [2],

$$\dot{\epsilon}_r(r) = \frac{[M_a(r)(\bar{\sigma} - \sigma_0(r))]^8 \; (2e(r) - 1)}{2[e^2(r) - e(r) + 1]^{\frac{1}{2}}}$$

(19)

$$\dot{\epsilon}_\theta(r) = \frac{[M_a(r)(\bar{\sigma} - \sigma_0(r))]^8 \; (2 - e(r))}{2[e^2(r) - e(r) + 1]^{\frac{1}{2}}}$$

(20)

$$\dot{\epsilon}_z(r) = -(\dot{\epsilon}_r(r) + \dot{\epsilon}_\theta(r))$$

(21)

the ratio of the radial stress and the tangential stress at any radius is given below,

$$e(r) = \frac{\sigma_r(r)}{\sigma_\theta(r)}$$

(22)

The stability of forces in the radial direction of element express as:

$$\frac{d}{dr}[r\sigma_r(r)] - \sigma_\theta(r) + \rho(r)\omega^2 r^2 = 0$$

(23)

Where density of composite is ρ, solving equation (19) and (20) to obtained $\sigma_\theta(r)$ is given below:

$$\sigma_\theta(r) = \frac{\dot{u}_a^{\frac{1}{8}}}{M_a(r)} f_1(r) + f_2(r)$$

(24)

$$\dot{u}_a^{1/8} = \frac{\int_{r_i}^{r_0} M_a(r)\sigma_\theta(r)dr - \int_{r_i}^{r_0} M_a(r) f(r) dr}{\int_{r_i}^{r_0} f_1(r) dr}$$

(25)

$$f_1(r) = \frac{f(r)}{[e^2(r) - e(r) + 1]^{1/2}}$$

(26)

$$f_2(r) = \frac{\sigma_0(r)}{[e^2(r) - e(r) + 1]^{1/2}}$$

(27)

$$f(r) = \left[\frac{2(e^2(r) - e(r) + 1)^{1/2}}{r(2 - e(r))} exp \quad \left(\int_{r_i}^{r_0} \frac{h(r)}{r} dr\right)\right]^{1/8}$$

(28)

And,

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$h(r) = \frac{2e(r)-1}{2-e(r)}$

(29)

After finding value of $\sigma_\theta(r)$ tangential stress and value of radial stress $\sigma_r(r)$ can be calculated from equation (23)

$\sigma_r(r) = \frac{1}{r}\int_{r_i}^{r} \sigma_\theta dr \; - \frac{\omega^2}{r}\left[\frac{r^3-r_i^3}{3}\left(\rho_p +\right.\right.$

□□−□□□100−□□−□□100□(□4−□□4)4 (30)

4.Discussion and graphical representation

In figure 1 the radial stress is decreases with increases thermal gradient as compare to uniform temperature for particle size P=1.7μm. In figure 2 the radial stress is decreases with increases thermal gradient as compare to uniform temperature for particle size P=14.5μm. In figure 3 the radial stress is decreases with increases thermal gradient as compare to uniform temperature for particle size P=45.3μm. In figure 4 shows the comparison of radial stress for different particle size P=1.7μm,14.5μm and 45.5μm. In figure 5 the tangential stress goes decreases as we move from internal to external radii but increasing thermal gradient the tangential stress goes decreases for particle size P= 1.7μm. In figure 6 the tangential stress goes decreases as we move from internal to external radii but increasing thermal gradient the tangential stress goes decreases for particle size P= 14.5μm. In figure 7 the tangential stress goes decreases as we move from internal to external radii but increasing thermal gradient the tangential stress goes decreases for particle size P= 45.3μm. In figure 8 shows the comparison of tangential stress for different particle size P=1.7μm,14.5μm and 45.5μm. In figure 9 the radial strain rate goes increases with increases thermal gradient as compare to uniform temperature for P=1.7μm. In figure 10 the radial strain rate goes increases with increases thermal gradient as compare to uniform temperature for P=14.5μm. In figure11 the radial strain rate goes increases with increases thermal gradient as compare to uniform temperature for P=45.3μm. In figure 12 shows the comparison of radial strain rate for different particle size P=1.7μm, 14.5μm and 45.5μm. In figure 13 the tangential strain rate goes decreases with increases thermal gradient as compare to uniform temperature for P=1.7μm. In figure 14 the tangential strain rate goes decreases with increases the thermal gradient as compare to uniform temperature for P=14.5μm. In figure 15 the tangential strain rate goes decreases with increases the thermal gradient as compare to uniform temperature for P=45.3μm. In figure 16 shows the comparison of tangential strain rate for different particle size P=1.7μm, 14.5μm and 45.5μm

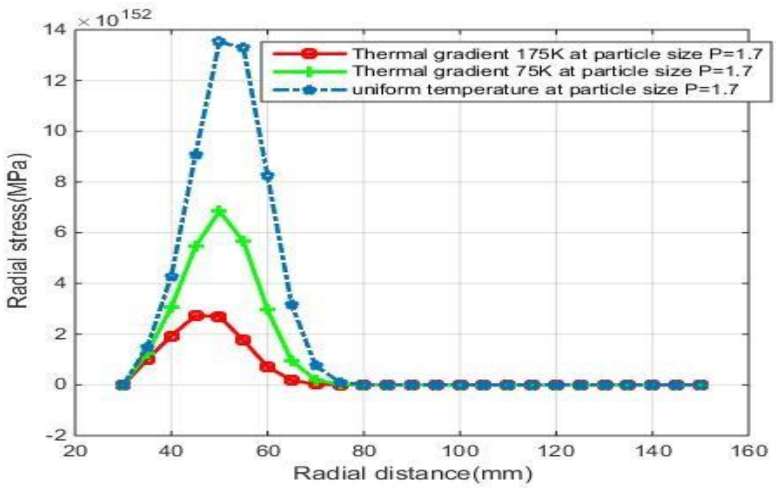

Figure 1: The difference of the radial stress along radius of disk with or without thermal gradient for particle size P=1.7μm

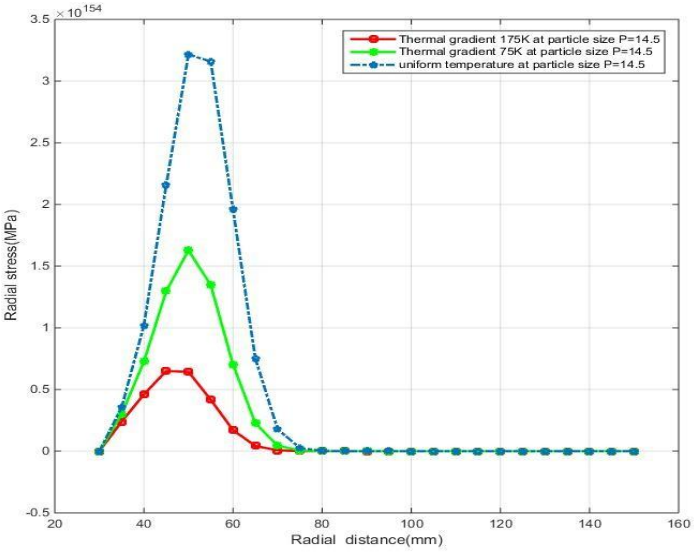

Figure 2: The difference of radial stress along radius of disk with or without thermal gradient for particle size P=14.5μm

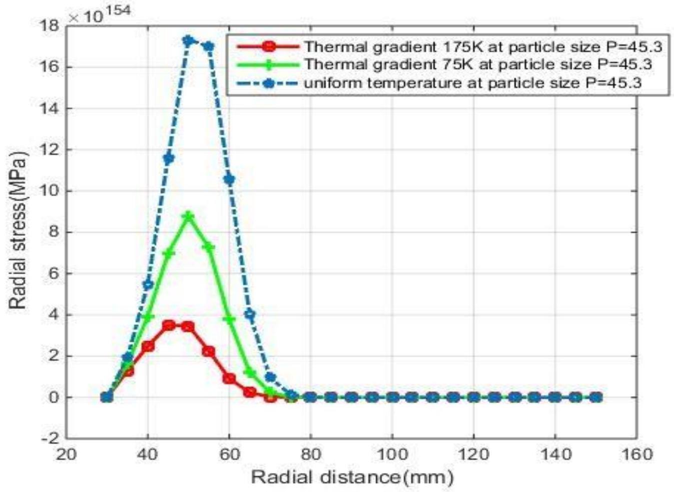

Figure 3: The difference of the radial stress along radius of disk with or without thermal gradient for particle size P=45.3μm

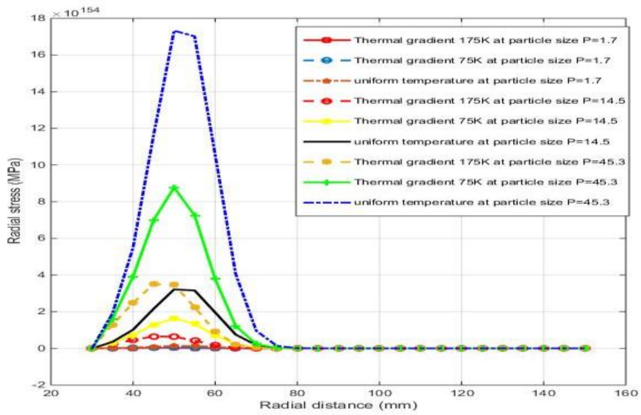

Figure 4: The difference of radial stress along radius of disk with or without thermal gradient for particle size P=1.7μm,14.5μm and 45.3μm

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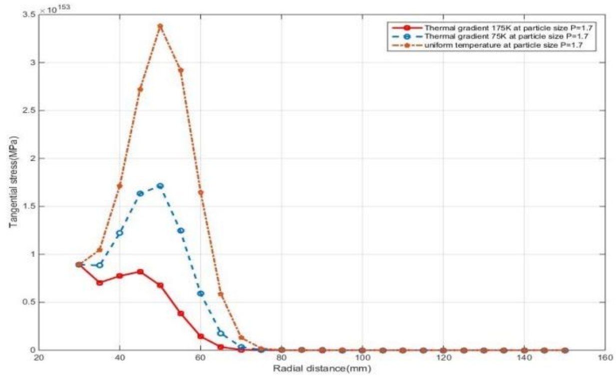

Figure 5: The difference of the tangential stress along radius of disk with or without thermal gradient for particle size P=1.7μm

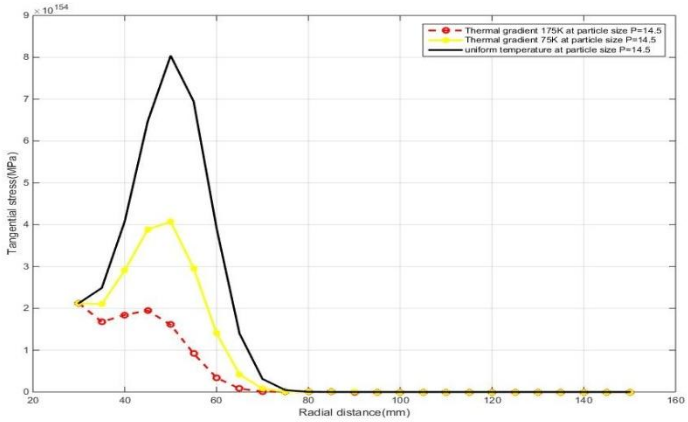

Figure 6: The difference of tangential stress along the radius of disk with or without thermal gradient for particle size P=14.5μm

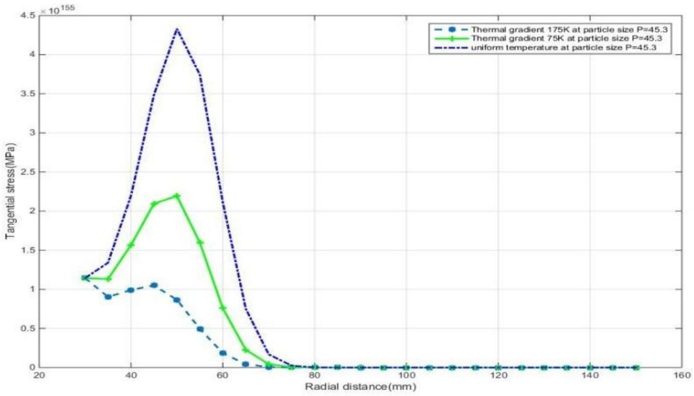

Figure 7: The difference of tangential stress along radius of disk with or without thermal gradient for particle size P=45.3μm

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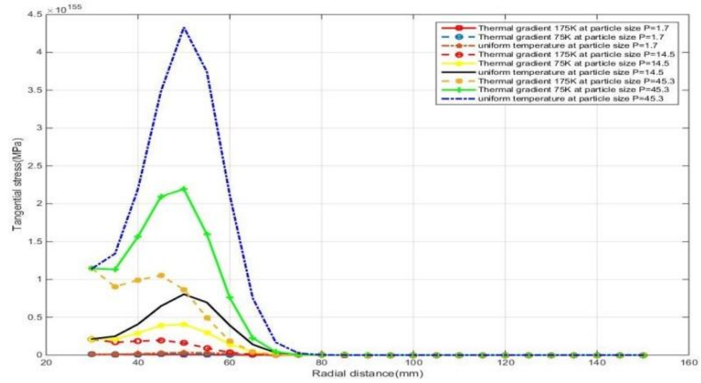

Figure 8: The difference of tangential stress along the radius of disk with or without thermal gradient for particle size P=1.7μm,14.5μm and 45.3μm

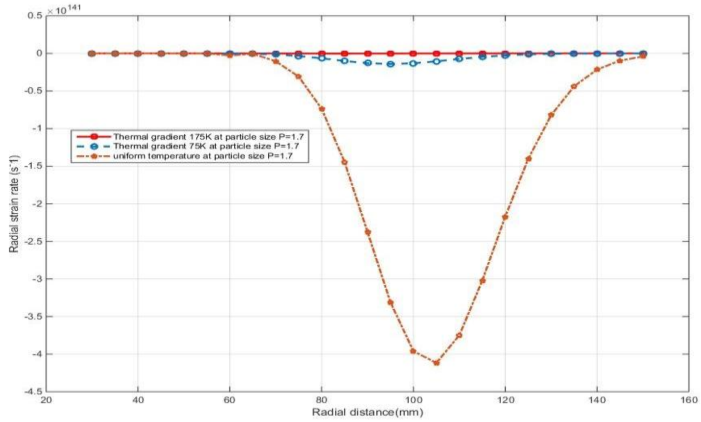

Figure 9: The difference of radial strain rate along the radius of disk with or without thermal gradient for particle size P=1.7μm

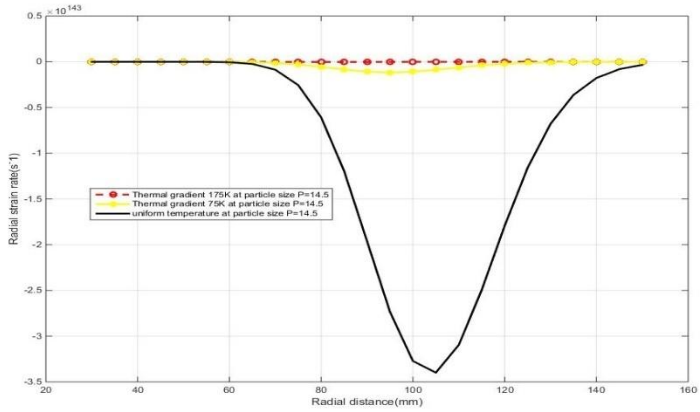

Figure 10: The difference of radial strain rate along the radius of disk with or without thermal gradient for particle size P=14.5μm

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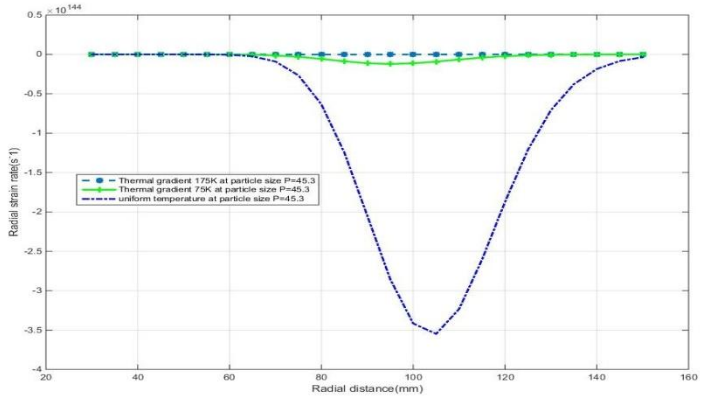

Figure 11: The difference of radial strain rate along the radius of disk with or without thermal gradient for particle size P=45.3μm

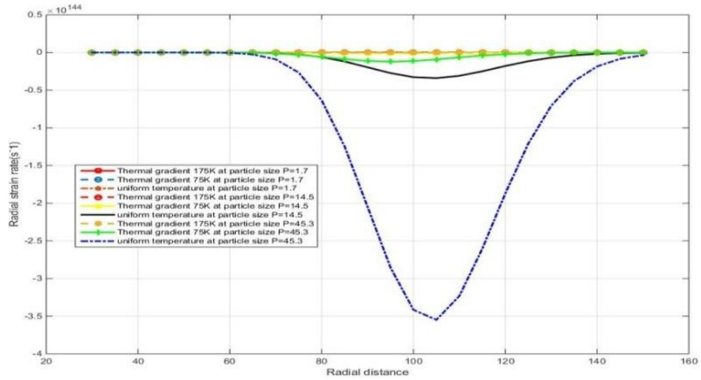

Figure 12: The difference of radial strain rate along the radius of disk with or without thermal gradient for particle size P=1.7μm,14.5μm and 45.3μm

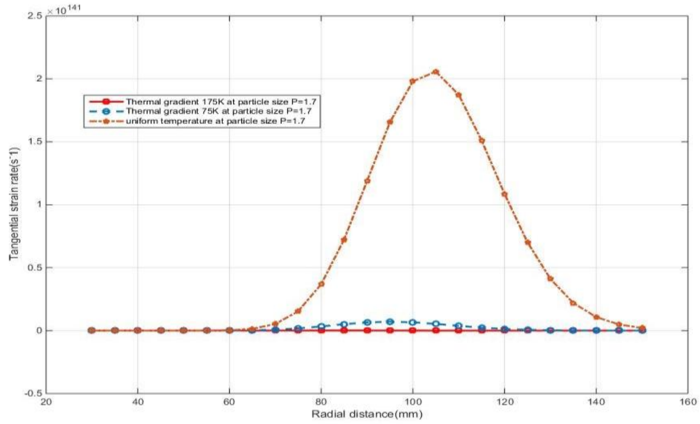

Figure 13: The difference of tangential strain rate along the radius of disk with or without thermal gradient for particle size P=1.7μm

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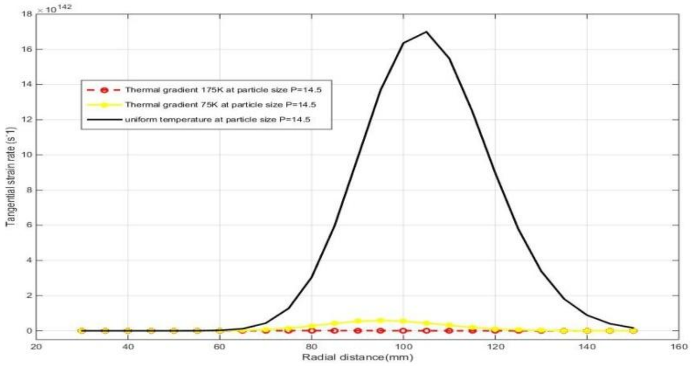

Figure 14: The difference of tangential strain rate along the radius of disk with or without thermal gradient for particle size P=14.5μm

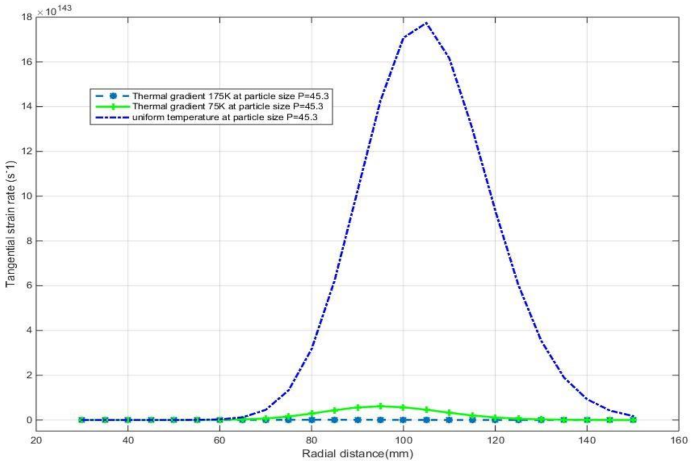

Figure 15: The difference of tangential strain rate along the radius of disk with or without thermal gradient for particle size P=45.3μm

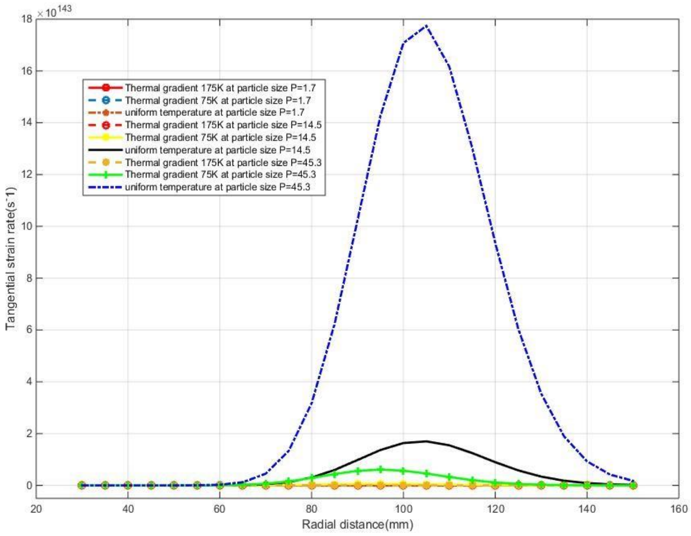

Figure 16: The difference of tangential strain rate along the radius of disk with or without thermal gradient for particle size P=1.7μm, 14.5μm, 45.3μm

5. Conclusions

The study reported following conclusions based on the graphical representation.

The radial stress start increases as we move from internal radii it become maximum at the middle of disk then it started decreases near the external radii.

But increases thermal gradient the radial goes decreases as compare to uniform temperature as shown in figure 1,2,3 for different particle size P=1.7μm, 14.5μm and 45.3μm. We concluded that with increasing particle size the radial stress start increasing as show in figure 4.

The tangential stress start decreases as move from internal to external radii. With increases thermal gradient the tangential stress decreases as compare to uniform temperature as shown in figure 5,6,7. We concluded that increasing particle size the tangential stress start increasing as shown in figure 8.

The radial strain rate start increases with increasing thermal gradient as shown in figure 9, 10,11 as compare to uniform temperature. We conclude that increasing particle size the radial strain starts decreasing.

The tangential strain rate decreases with increasing thermal gradient as shown in figure 13,14,15. We concluded that increasing particle size the tangential strain rate increasing as shown in figure 16.

We concluded the result that the particle size P=1.7μm and 14.5μm gives almost same result to reduce deformation in disk take the particle size between P=1.7μm and 14.5μm.

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