**JOURNAL OF ALGEBRAIC STATISTICS**
Volume 13, No. 3, 2022, p. 1415-1422
https://publishoa.com
ISSN: 1309-3452

# A Study on Ordering in Generalized Regular Intuitionistic Fuzzy Matrices

**P. Jenita [1] E. Karuppusamy [2] M. Princy Flora [3]**

**1.** Assistant Professor, Post Graduate and Research, Department of Mathematics, Government Arts College, Coimbatore-641018.
E.mail: (sureshjenita@yahoo.co.in)
**2.** Assistant Professor, Sri Krishna College of Engineering and Technology, Coimbatore-641008.
E.mail: (samy.mathematics@gmail.com)
**3.** Assistant Professor, Kumaraguru College of Technology, Coimbatore-641049.
E.mail: (princyfloram@gmail.com)

---

**ABSTRACT**

As a generalization of the minus ordering for regular fuzzy matrices, the minus ordering for k-regular intuitionistic fuzzy matrices is described in this study, and some of its features related to k-g inverses are examined.

**Keywords:** IFMs, Ordering, k-regular, k-g inverses.

---

**Introduction**

We work with fuzzy matrices, which are matrices with entries that are fuzzy algebra $F = [0,1]$ is defined by the max - min operation $a + b = max\{a, b\}$ and $a \cdot b = min\{a, b\}$ for all $a, b \in F$ . Let $F_{m\times n}$ be the collection of all $m \times n$ fuzzy matrices in the fuzzy algebra $\{F: F = [0,1]\}$. If there exist $X$ such that $AXA = A$, then the matrix $A \in F_{m\times n}$ is said to be regular, $X$ is known as a generalized $(g^-)$ inverse of $A$. Kim and Roush created a fuzzy matrices theory that is similar to Boolean matrices in [13]. The regularity of intuitionistic fuzzy matrices was examined by Meenakshi and Gandhimathi [17]. In [27], Riyaz Ahmad Padder and Murugadas discusses several features of idempotent intuitionistic fuzzy matrices and T-type idempotent intuitionistic fuzzy matrices. As a generalization of fuzzy sets, Atanassov introduced and developed the idea of intuitionistic fuzzy sets in [1]. The concept of generalized inverses was discussed by Ben Israel and Greville in [2]. Meyar proposed the idea of generalized inverses of block triangular matrices in [20]. In [14], Kim and Roush presented inverse Boolean matrices. Pal and Khan derived basic features of intuitionistic fuzzzy matrices as a generalization of the work on fuzzy matrices in [21]. In [28], the topic of reducing intuitionistic fuzzy matrices is investigated, and several helpful properties for nilpotent intuitionistic fuzzy matrices are discovered. Some qualities of a transitive fuzzy matrix are investigated in [15], and the canonical form of the transitive fuzzy matrix is found using the properties. A canonical form of the transitive intuitionistic fuzzy matrix is also obtained using the properties. Szpilrajn's ordering theorem is extended to intuitionistic fuzzy orderings in [29]. Minus ordering on fuzzy matrices was discussed by Sriram and Murugadas in [25]. Cen presented T-ordering in fuzzy matrices and looked into the relationship between T-ordering and generalised inverses in [3]. As a generalization of the negative partial ordering for regular fuzzy matrices, Poongodi, Padmavathi, Vinitha, and Hema presented a particular sort of ordering for k-regular Interval Valued Fuzzy Matrix in [21]. To examine the criteria for convergence of intuitionistic fuzzy matrices, Riyaz Ahmad Padder and Murugadas established the max-max operations on intuitionistic fuzzy matrices in [30]. Cho talked about the consistency of fuzzy matrix equations in [4]. As a generalisation of regular fuzzy matrix, Meenakshi and Jenita recently presented the notion of k - regular fuzzy matrix [19]. Khan and Paul [12] offer the notion of inverse of intuitionistic fuzzy matrices as a generalisation of inverse of intuitionistic fuzzy matrices. Pradhan and Pal [22] present a method for determining the inverse of an intuitionistic fuzzy matrix using the generalized inverses of blocks of the original matrix. Meenakshi and Jenita talked about numerous k-g inverses of k-regular fuzzy matrices in [18]. Jenita and Karuppusamy talked about the k-regularity of fuzzy and block fuzzy matrices in [6]. The idea of generalized regular block intuitionistic fuzzy matrices was presented in [11]. Special types of inverses and its characterization was respectively discussed in [10, 9]. The rank of intuitionistic fuzzy matrices was developed by Pradhan and Pal in [24]. [16, 26] can be used to learn more about fuzzy matrix theory and applications. As a generalization of regular intuitionistic fuzzy matrices, Jenita, Karuppusamy, and Thangamani developed the notion of k - regular intuitionistic fuzzy matrices in [7]. [8] discusses many inverses of k-regular intuitionistic fuzzy matrices. As a generalization of the minus ordering for regular fuzzy matrices, the minus ordering for k-regular intuitionistic fuzzy matrices is described in this study, and some of its features related to k-g inverses are examined.

**2. Preliminaries**

**JOURNAL OF ALGEBRAIC STATISTICS**
Volume 13, No. 3, 2022, p. 1415-1422
https://publishoa.com
ISSN: 1309-3452

We're talking about fuzzy matrices here, which are matrices over a fuzzy algebra FM(FN) with support [0,1], under maxmin (minmax) operations and standard real-number ordering. Let $(IF)_{m\times n}$ be the collection of all intuitionistic fuzzy matrices of order $m\times n$, $F^M_{m\times n}$ be the set of all fuzzy matrices of order $m\times n$, under the maxmin composition and $F^N_{m\times n}$ be the set of all fuzzy matrices of order $m\times n$, under the minmax composition. In short $(IF)_n$ signifies the order's fuzzy intuitionistic matrix $n\times n$.

If $A=(a_{ij})_{m\times n}\in(IF)_{m\times n}$, then $A=\left(\langle a_{ij\mu},a_{ij\vartheta}\rangle\right)_{m\times n}$, where $a_{ij\mu}$ and $a_{ij\vartheta}$ are the membership and non membership values of $a_{ij}$ in $A$ with regard to the fuzzy sets $\mu$ and $\vartheta$ respectively, while retaining the condition $0\le a_{ij\mu}+a_{ij\vartheta}\le 1$.

The matrix operations on intuitionistic fuzzy matrices as stated in [16] will be followed.For $A,B\in(IF)_{m\times n}$, then

$$A+B=\left(\langle\max\{a_{ij\mu},b_{ij\mu}\},\min\{a_{ij\vartheta},b_{ij\vartheta}\}\rangle\right)$$

$$AB=\left(\left\langle\max_k\min\{a_{ik\mu},b_{kj\mu}\},\min_k\max\{a_{ik\vartheta},b_{kj\vartheta}\}\right\rangle\right)$$

Define the order relation on $(IF)_{m\times n}$ as follows,

$A\le B\Leftrightarrow a_{ij\mu}\le b_{ij\mu}$ and $a_{ij\vartheta}\ge b_{ij\vartheta}$, for all $i$ and $j$.

We will represent $A\in(IF)_{m\times n}$ as a cartesian product of fuzzy matrices in this paper.

For $A=(a_{ij})_{m\times n}\in(IF)_{m\times n}$. Let $A=(a_{ij})_{m\times n}=\left(\langle a_{ij\mu},a_{ij\vartheta}\rangle\right)_{m\times n}\in(IF)_{m\times n}$.

The membership part of $A$ is defined as $A_\mu=(a_{ij\mu})_{m\times n}\in F^M_{m\times n}$ and the non-membership part is defined as $A_\vartheta=(a_{ij\vartheta})_{m\times n}\in F^N_{m\times n}$.As a result $A$ is expressed as the cartesian product of $A_\mu$ and $A_\vartheta$, $A=<A_\mu,A_\vartheta>$ with $A_\mu\in F^M_{m\times n},A_\vartheta\in F^N_{m\times n}$. $A\in(IF)_{m\times n},R(A)(C(A))$ signifies the space created by the rows (columns) of $A$ and $A^T$ denotes the transpose of $A$.

**Definition 2.1 [7]**

If there exist the matrix $X\in(IF)_n$, such that $A^kXA=A^k$, for some positive integer $k$ then the matrix $A\in(IF)_n$ is said be right k-regular. Right k-g-inverse of $A$ is called $X$.

Let $A_r\{1^k\}=\{X/A^kXA=A^k\}$.

**Definition 2.2 [7]**

If there exists a matrix $Y\in(IF)_n$ such that $AYA^k=A^k$, for some positive integer $k$ then the matrix $A\in(IF)_n$, is said be left k-regular. Left k-g-inverse of $A$ is called $Y$.

Let $A_\ell\{1^k\}=\{Y/AYA^k=A^k\}$.

In general, right k-regular is different from left k-regular.

**Lemma 2.3 [6]**

For $A,B\in(IF)_n$, and a positive integer k, then

(i) If $A$ is right k - regular and $R(B)\subseteq R(A^k)$ then $B=BXA$ for each right k - g inverse $X$ of $A$.

(ii) If $A$ is left k - regular and $C(B)\subseteq C(A^k)$ then $B=AYB$ for each left k - g inverse $Y$ of $A$.

**Lemma 2.4 [17]**

For $A,B\in(IF)_{m\times n},R(B)\subseteq R(A)\Leftrightarrow B=XA$ for some $X\in(IF)_m,C(B)\subseteq C(A)\Leftrightarrow B=AY$ for some $Y\in(IF)_n$.

**Lemma 2.5 [17]**

For $A\in(IF)_{mn}$ and $B\in(IF)_{np}$, $R(AB)\subseteq R(B),C(AB)\subseteq C(A)$.

**Theorem 2.6 [8]**

Let $A\in(IF)_n$ and $k$ be a positive integer, then $X\in A_r\{1^k\}\Leftrightarrow X^T\in A^T_\ell\{1^k\}$.

**Remark 2.7 [7]**

Each member of the set $A\{1^k\}$ is referred to as a k-g inverse of $A$. For any integer $q\ge k$ if $A$ is k-regular then $A$ is q-regular. For $k=1$, $A\{1^k\}$ reduces to the set of all g-inverse of a regular matrix $A$.

**Definition 2.8 [10]**

If $A$ is an intuitionistic fuzzy matrix, suppose $A^{k+d}=A^k$ for some positive integer $k,d>0$. Then the least $k>0$ such that $A^{k+d}=A^k$ for some $d$ is called the index of $A$. The least $d>0$ such that $A^{k+d}=A^k$ for some $k$ is called the period of A.

**Definition 2.9 [14]**

If every row and column of a square intuitionistic fuzzy matrix includes exactly one $\langle 1,0\rangle$ and all the other entries are $\langle 0,1\rangle$ is called the intuitionistic fuzzy permutation matrix. $P_n$ be the collection of all $n\times n$ permutation matrices in $(IF)_n$.

**Definition 2.10 [13]**

For $A\in F^-_{m,n}$ and $B\in F_{mn}$, the minus ordering denoted as $\bar{<}$ is defined as $A\ \bar{<}\Leftrightarrow A^-A=A^-B$ and $AA^-=BA^-$ for some $A^-\in A\{1\}$.

**JOURNAL OF ALGEBRAIC STATISTICS**
Volume 13, No. 3, 2022, p. 1415-1422
https://publishoa.com
ISSN: 1309-3452

### 3. Ordering on Generalized Regular Intuitionistic Fuzzy Matrices

In this section, we look at k-minus ordering for k-regular IFMs, which is a minus ordering for generalized regular IFMs including k-g inverses. Here, $(IFM)_n^- = \{A \in (IFM)_n |\ A\ has\ a\ k-g\ inverse\}$

**Definition 3.1**
For $A \in (IFM)_n^-$ and $B \in (IFM)_n$, the k-minus ordering denoted as $A <_k^- B$ and is defined by $A <_k^- B \Leftrightarrow A^k U = B^k U$ for some $U \in A\{1_r^k\}$ and $VA^k = VB^k$ for some $V \in A\{1_\ell^k\}$.

**Remark 3.1**
For $k = 1$, Definition (3.1), reduces to the definition of minus ordering for regular fuzzy matrices, which is given in Definition (2.10)
Also from Definition (3.1), and Definition (2.10) it to be noted that, $A <_k^- B \Leftrightarrow A^k <^- B^k$. But in general if $X$ is a k-g inverse of $A$ need not be a g-inverse of $A^k$.
This is demonstrated in the following example. (3.2)

**Example 3.1**
Let $A = \begin{bmatrix} \langle 0.5,0.1\rangle & \langle 0.2,0.3\rangle \\ \langle 0.3,0.2\rangle & \langle 0.1,0.5\rangle \end{bmatrix}$

$A^2 = \begin{bmatrix} \langle 0.5,0.1\rangle & \langle 0.2,0.3\rangle \\ \langle 0.3,0.2\rangle & \langle 0.2,0.3\rangle \end{bmatrix} \neq A$

For the permutation matrices $P_1 = \begin{bmatrix} \langle 1,0\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 1,0\rangle \end{bmatrix}$ and $P_2 = \begin{bmatrix} \langle 0,1\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 0,1\rangle \end{bmatrix}$,

$AP_1A \neq A$ and $AP_2A \neq A$. Hence $A$ is not regular.

For $X = \begin{bmatrix} \langle 0.5,0.1\rangle & \langle 0.1,0.5\rangle \\ \langle 0.1,0.2\rangle & \langle 0.2,0.3\rangle \end{bmatrix}, A^2XA = A^2$.

Hence $A$ is 2-regular and $X$ is a 2-g-inverse of $A$.

For $B = \begin{bmatrix} \langle 0.6,0.1\rangle & \langle 0.2,0.3\rangle \\ \langle 0.3,0.2\rangle & \langle 0.5,0.3\rangle \end{bmatrix}, B^2 = \begin{bmatrix} \langle 0.6,0.1\rangle & \langle 0.2,0.3\rangle \\ \langle 0.3,0.2\rangle & \langle 0.5,0.3\rangle \end{bmatrix}$.

Here, $A^2X = B^2X = \begin{bmatrix} \langle 0.5,0.1\rangle & \langle 0.2,0.3\rangle \\ \langle 0.3,0.2\rangle & \langle 0.2,0.3\rangle \end{bmatrix}$.

For $Y = \begin{bmatrix} \langle 0.5,0.1\rangle & \langle 0.2,0.5\rangle \\ \langle 0.1,0.2\rangle & \langle 0.2,0.3\rangle \end{bmatrix}$, $AYA^2 = A^2$, $Y$ is a left 2-g inverse of $A$.

Also, $YA^2 = YB^2 = \begin{bmatrix} \langle 0.5,0.1\rangle & \langle 0.2,0.5\rangle \\ \langle 0.2,0.2\rangle & \langle 0.2,0.5\rangle \end{bmatrix}$.

Hence $A <_k^- B$.

**Example 3.2**
From Example (3.1),

$$A^2XA = \begin{bmatrix} \langle 0.5,0.1\rangle & \langle 0.2,0.3\rangle \\ \langle 0.3,0.2\rangle & \langle 0.2,0.3\rangle \end{bmatrix} = A^2$$

Hence $A$ is 2- regular and $X$ is a 2-g inverse of $A$.

$$A^2XA^2 = \begin{bmatrix} \langle 0.5,0.1\rangle & \langle 0.2,0.3\rangle \\ \langle 0.3,0.3\rangle & \langle 0.2,0.3\rangle \end{bmatrix} \neq A^2$$

Hence $X$ is not a g-inverse of $A^2$.

**Lemma 3.1**
For $A \in (IFM)_n^-$ and $B \in (IFM)_n$, the following are equivalent.
$(i)$ $A <_k^- B$
$(ii)$ $A^k = B^k UA = AVB^k$ for some $U, V \in A\{1^k\}$

**Proof:**
$(i) \Rightarrow (ii)$

**JOURNAL OF ALGEBRAIC STATISTICS**
Volume 13, No. 3, 2022, p. 1415-1422
https://publishoa.com
ISSN: 1309-3452

$A <_k^- B \Rightarrow A^kU = B^kU$ for some $U \in A\{1_r^k\}$ and $VA^k = VB^k$ for some $V \in A\{1_\ell^k\}$.

$$A^k = A^kUA = (A^kU)A = B^kUA$$
$$A^k = AVA^k = A(VA^k) = AVB^k$$
$$A^k = B^kUA = AVB^k$$

$(ii) \Rightarrow (i)$
Let $X = UAU, U \in A\{1_r^k\}$

$$A^kXA = A^k(UAU)A = (A^kUA)UA = A^kUA = A^k$$

Hence $X \in A\{1_r^k\}$
Similarly, $AYA^k = A^k, Y = VAV$ for $V \in A\{1_\ell^k\}$
Now, $A^kX = A^k(UAU)$
$= (A^kUA)U$
$= A^kU$
$= (B^kUA)U$
$= B^k(UAU)$
$= B^kX$
Hence $A^kX = B^kX$ for some $X \in A\{1_r^k\}$Similarly, $YA^k = YB^k$ for some $Y \in A\{1_\ell^k\}$.
Hence the proof.

**Remark 3.2**
In general, $B$ does not have to be k-regular in the concept of k-minus ordering $A <_k^- B$. The following example demonstrates this

**Example 3.2**
In example (3.1), it is observed that $B^2 = B$. Therefore, $B$ is regular. Hence, $B$ need not be a k-regular matrix.

**Lemma 3.2**
For $A, B \in (IFM)_n^-$
$(i)$ If $B$ is right k-regular and $R(A^k) \subseteq R(B^k)$ then $A^k = A^kUB$ for each right k-g inverse $U$ of $B$.
$(ii)$ If $B$ is left k-regular and $C(A^k) \subseteq C(B^k)$ then $A^k = BVA^k$ for each left k-g inverse $V$ of $B$.

**Proof:**
$(i)$ Since $R(A^k) \subseteq R(B^k)$, by Lemma (2.4), there exists $Z$ such that $A^k = ZB^k = ZB^kUB$ for each $U \in B\{1_r^k\}$
$= A^kUB$
$(ii)$ Can be proved in the similar manner.

**Theorem 3.1**
For $A, B \in (IFM)_n^-$, if $A <_k^- B$ then $R(A^k) \subseteq R(B^k)$, $C(A^k) \subseteq C(B^k)$ and $A^kUA = A^k = BVA^k$ for each $U \in B\{1_r^k\}$ and for each $V \in B\{1_\ell^k\}$

**Poof:**
By Lemma (3.1),

$$A <_k^- B \Rightarrow A^k = AVB^k = B^kUA$$

By Lemma (2.5), $R(A^k) = R(AVB^k) \subseteq R(B^k)$
$C(A) = C(B^kUA) \subseteq C(B^k)$.
By Lemma (2.3), it follows that
$A^k = A^kUB = BVA^k$ for each $U \in B\{1_r^k\}$ and $V \in B\{1_\ell^k\}$
Hence the proof.

**Theorem 3.2**
For $A, B \in (IFM)_n^-$ the following hold
$(i)$ $A <_k^- A$
$(ii)$ $A <_k^- B$and $B <_k^- A$, then $A^k = B^k$
$(iii)$ $A <_k^- B$ and $B <_k^- C$, then $A <_k^- C$

**Proof:**
$(i)$ $A <_k^- A$ is obvious.
Hence $<_k^-$ is reflexive.
$(ii)$ From Lemma (3.1), $A <_k^- B \Rightarrow A^k = B^kUA$ for some $U \in A\{1_r^k\}$ and
$B <_k^- A \Rightarrow B^k = BVA^k$ for some $V \in A\{1_\ell^k\}$
Now, $A^k = B^kUA$
$= (BVA^k)UA$
$= BV(A^kUA)$
$= BVA^k$
$= B^k$

**JOURNAL OF ALGEBRAIC STATISTICS**
Volume 13, No. 3, 2022, p. 1415-1422
https://publishoa.com
ISSN: 1309-3452

$(iii)$ From Theorem(3.1),

$$A <_k^- B \Rightarrow A^k = A^k B^- B$$
$$= BB^- A^k,\ B^- \in B\{1^k\}$$

From Lemma (3.1)

$$B <_k^- C \Rightarrow B^k = C^k B^- B$$
$$= BB^- C^k \text{ for } B^- \in A\{1^k\}$$

Let $U' = B^- BX$ for $B^- \in B\{1_r^k\}$ and $X \in A\{1_r^k\}$

Then, $A^k U' A = A^k (B^- BX) A$

$$= (A^k B^- B) XA$$
$$= A^k XA = A^k$$

Therefore $U' \in A\{1_r^k\}$

Let $V' = YBB^-$ for $B^- \in B\{1_r^k\}$ and $Y \in A\left\{1_\ell^k\right\}$

Thus $AV' A^k = A(YBB^-) A^k$

$$= AY(BB^- A^k)$$
$$= AYA^k = A^k$$

Therefore, $V' \in A\left\{1_\ell^k\right\}$

$$A^k U' = A^k (B^- BX)$$
$$= (A^k B^- B) X$$
$$= A^k X$$
$$= B^k X$$
$$= (C^k B^- B) X$$
$$= C^k (B^- BX)$$
$$= C^k U' \text{ for some } U' \in A\{1_r^k\}$$
$$V' A^k = (YBB^-) A^k$$
$$= Y(BB^- A^k)$$
$$= YA^k$$
$$= YB^k$$
$$= Y(BB^- C^k)$$
$$= (YBB^-) C^k$$
$$= V' C^k \text{ for some } V' \in A\left\{1_\ell^k\right\}$$

Therefore, $A <_k^- B$ and $B <_k^- C \Rightarrow A <_k^- C$

Hence $<_k^-$ is transitive.

**Remark 3.3**

In the set of all regular intuitionistic fuzzy matrices, minus ordering is a partial ordering.

**Theorem 3.3**

For $A, B \in (IFM)_n^-$, we have the following

$(i)$ $A <_k^- B \Leftrightarrow A^T <_k^- B^T$

$(ii)$ $A <_k^- B \Leftrightarrow PAP^T <_k^- PBP^T$ for some permutation matrix $P$.

**Proof:**

$(i)$ $A <_k^- B \Leftrightarrow A^k A^- = B^k A^-$ and $A^- A^k = A^- B^k$ for some $A^- \in A\{1^k\}$

By Theorem (2.6), $A^- \in A\{1_r^k\} \Leftrightarrow (A^-)^T \in A^T\left\{1_\ell^k\right\}$

$$A^k A^- = B^k A^- \Leftrightarrow (A^k A^-)^T = (B^k A^-)^T$$
$$\Leftrightarrow (A^-)^T (A^k)^T = (A^-)^T (B^k)^T$$
$$\Leftrightarrow (A^T)^- (A^T)^k = (A^T)^- (B^T)^k$$

Similarly, $A^- A^k = A^- B^k \Leftrightarrow (A^T)^k (A^T)^- = (B^T)^k (A^T)^-$

Hence $A <_k^- B \Leftrightarrow A^T <_k^- B^T$.

$(ii)$ $W = PA^- P^T, A^- \in A\{1^k\}$

$$(PAP^T)^k W (PAP^T) = (PA^k P^T)(PA^- P^T)(PAP^T)$$
$$= PA^k A^- AP^T$$
$$= PA^k P^T$$
$$= (PAP^T)^k$$
$$(PAP^T) W (PAP^T)^k = (PAP^T)(PA^- P^T)(PA^k P^T)$$
$$= PAA^- A^k P^T$$
$$= PA^k P^T$$
$$= (PAP^T)^k$$

Hence $W = PA^- P^T \in PAP^T\{1^k\}$

**JOURNAL OF ALGEBRAIC STATISTICS**
Volume 13, No. 3, 2022, p. 1415-1422
https://publishoa.com
ISSN: 1309-3452

$$(PAP^T)^-(PAP^T)^k = PA^-P^TPA^kP^T$$
$$= PA^-A^kP^T$$
$$= P(A^-A^k)P^T$$
$$= P(A^-B^k)P^T$$
$$= PA^-(P^TP)B^kP^T$$
$$= (PAP^T)^-(PBP^T)^k.$$

Similarly, $(PAP^T)^k(PAP^T)^- = (PBP^T)^k(PAP^T)^-$.
Therefore, $(PAP^T) <_k^- (PBP^T)$
Conversely, $A = P^T(PAP^T)P <_k^- P^T(PBP^T)P = B$
Hence the proof.

**Theorem 3.4**
For $A, B \in (IFM)_n^-$, if $A <_k^- B$ with $B^k$ is idempotent, then $A^k$ is idempotent.

**Proof:**
From Lemma (3.1), $A <_k^- B \Rightarrow A^k = AVB^k = B^kUA, U, V \in A\{1^k\}$

$$A^{2k} = A^kA^k$$
$$= (AVB^k)(B^kUA)$$
$$= AV(B^{2k})UA$$
$$= AVB^kUA$$
$$= A^kUA = A^k$$

Hence the proof.

**Remark 3.4**
In the above Theorem, the converse need not be true. That is, if $A <_k^- B$ with $A$ is idempotent then $B$ need not be idempotent.
This is illustrated in the following.

**Example 3.3**
Let $A = \begin{bmatrix} \langle 0.5,0.3\rangle & \langle 0.3,0.3\rangle \\ \langle 0.3,0.3\rangle & \langle 0.3,0.3\rangle \end{bmatrix}$, $A^2 = \begin{bmatrix} \langle 0.5,0.3\rangle & \langle 0.3,0.3\rangle \\ \langle 0.3,0.3\rangle & \langle 0.3,0.3\rangle \end{bmatrix} = A$, $A$ is idempotent.

$B = \begin{bmatrix} \langle 0.5,0.3\rangle & \langle 0.3,0.3\rangle \\ \langle 0.3,0.3\rangle & \langle 0.4,0.5\rangle \end{bmatrix}$, $B^2 = \begin{bmatrix} \langle 0.5,0.3\rangle & \langle 0.3,0.3\rangle \\ \langle 0.3,0.3\rangle & \langle 0.3,0.3\rangle \end{bmatrix} \neq B$. $B$ is not idempotent.

Let $X = \begin{bmatrix} \langle 0.5,0.3\rangle & \langle 0.3,0.3\rangle \\ \langle 0.3,0.3\rangle & \langle 0.3,0.5\rangle \end{bmatrix}$

$$A^2X = B^2X = \begin{bmatrix} \langle 0.5,0.3\rangle & \langle 0.3,0.3\rangle \\ \langle 0.3,0.3\rangle & \langle 0.3,0.3\rangle \end{bmatrix}.$$

For $Y = \begin{bmatrix} \langle 0.6,0.3\rangle & \langle 0.3,0.3\rangle \\ \langle 0.3,0.2\rangle & \langle 0.3,0.5\rangle \end{bmatrix}$ Also, $YA^2 = YB^2 = \begin{bmatrix} \langle 0.5,0.3\rangle & \langle 0.3,0.3\rangle \\ \langle 0.3,0.3\rangle & \langle 0.3,0.3\rangle \end{bmatrix}$.

Hence $A <_k^- B$.

**Theorem 3.5**
For $A \in (IFM)_n^-, B \in (IFM)_n$ if $A <_k^- B$ and $B^k = 0$ then $A^k = 0$.

**Proof:**
From Lemma (3.1), $A <_k^- B \Rightarrow A^k = AVB^k = 0$ and $A^k = B^kUA = 0$ for some $U, V \in A\{1^k\}$

**Theorem 3.6**
For $A \in (IFM)_n^-$ and $B \in (IFM)_n$, then we have
$(i)$ $R(B) \subseteq R(A^k) \Rightarrow C(B^T) \subseteq C(A^T)$
$(ii)$ $C(B) \subseteq C(A^k) \Rightarrow R(B^T) \subseteq R(A^T)$

**Proof:**
$(i)$ By Lemma (2.3),
$A$ is right k-regular and $R(B) \subseteq R(A^k)$, then $B = BUA$ for each right k-g inverse $U$ of $A$.

$$B = BUA \Rightarrow B^T = (BUA)^T$$
$$= A^TU^TB^T$$

By Lemma (2.4), $C((B^T) \subseteq C(A^T)$
$(ii)$ By Lemma (2.3),
$A$ is left k-regular and $C(B) \subseteq C(A^k)$, then $B = AVB$ for each left k-g inverse $V$ of $A$.

$$B = AVB \Rightarrow B^T = (AVB)^T$$

$= B^T V^T A^T$

By Lemma (2.4), $R(B^T) \subseteq R(A^T)$

## 4. Conclusion

This article provides the way to identify a special type of ordering which is called k-minus ordering for k-regular IFMs. As minus ordering has close relationship with g-inverse, it has some special role in fuzzy relational equations. Thus the study of k-minus ordering on intuitionistic fuzzy matrices has some future.

## References

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and System 20(1) (1986), 87-96.

[2] A. Ben Israel and T. N. E. Greville Generalized inverses: Theory and Applications John Wiley,(1974) New York.

[3] Cen Jianmiao, Fuzzy Matrix Partial Orderings and Generalized Inverses John Wiley,(1974) New York. Fuzzy Sets and System 105(1) (1999), 453-458.

[4] H. H. Cho, Regular fuzzy matrices and fuzzy equations, Fuzzy Sets and Systems, 105 (1999), 445-451.

[5] T. Gandhimathi, Characterization of various g-inverse of intuitionistic fuzzy matrices, Global Journal of Mathematical Science: Theory andPractical, Vol 4, No 4(2012), 389-396.

[6] P. Jenita and E.Karuppusamy, Fuzzy Relational Equations of k-regular Intuitionistic Fuzzy and Block Fuzzy Matrices Advances in Research, 11(2) (2017), 1-10.

[7] P. Jenita, E. Karuppusamy and D. Thangamani, k - Pseudo similar intuitionistic fuzzy matrices, Annals of Fuzzy Mathematics and Informatics, (2017) 1 - 11.

[8] P. Jenita and E.Karuppusamy, Inverses of k-regular Intuitionistic Fuzzy Matrices International Journal of Pure and Applied Mathematics, Volume 119 (12); 2341-2359, 2018.

[9] E.Karuppusamy and P. Jenita, Characterization of the set of all k-g inverses of k-regular intuitionistic fuzzy matrices International Journal of Advanced Science and Technology, 28(16), 1157-1161, 2019.

[10] P. Jenita and E.Karuppusamy, Special Types of Inverses of k-regular Intuitionistic Fuzzy Matrices TEST Engineering and Management, 83, 20035-20049, 2020.

[11] P. Jenita and E.Karuppusamy, Generalized Regular Block Intuitionistic Fuzzy Matrices Advances in Maathematics: Scientific Journal, 9(15), 2983-3006, 2020.

[12] S. Khan and Anita paul, The Generalised inverse of intuitionistic fuzzy matrices, Journal of Physical Science, Vol II (2007), 62-67.

[13] K. H. Kim and F. W. Roush, On generalized fuzzy matrices, Fuzzy Sets and Systems, 4 (1980), 293-375.

[14] K. H. Kim and F. W. Roush, Inverse of Boolean matrices, Journal of Linear Algebra and Applications, 22 (1978), 247-262.

[15] H. Y. Lee and N. G. Jeong, Canonical form of a transitive intuitionistic fuzzy matrices, Honam Mathematical Journal, Vol. 27, No. 4 (2005), 543-550

[16] AR Meenakshi, Fuzzy Matrix Theory and Applications, MJP Publishers, Chennai, 2008.

[17] AR Meenakshi and T. Gandhimathi, On regular intuitionistic fuzzy matrices, International Journal of Fuzzy Mathematics, Vol. 19, No. 2 (2011), 599-605.

[18] A.R Meenakshi and P. Jenita, Inverses of k-regular Fuzzy Matrices, International Journal of Mathematical Science and Engineering Applications, Vol 4 (2010), 187-195.

[19] AR Meenakshi and P. Jenita, Generalized regular fuzzy matrices, Iranian Journal of Fuzzy Systems, Vol. 8, No. 2 (2011), 133-141.

[20] C. D. Meyar , Generalized inverses of block triangular matrices, Journal of Applied Mathematics, (19), (1970), 741-750.

[21] M. Pal, S. K. Khan and A. K. Shyamal, Intuitionistic fuzzy matrices, Notes on Intuitionistic Fuzzy Sets, Vol. 8, No. 2 (2002), 51-62.

[22] P. Poongodi, C. Padmavathi, R. Vinitha and G. Hema Orderings on Generalized Regular Interval Valued Fuzzy Matrices, International Journal of Engineering and Advanced Technology, Vol 10, (2020), 194-198.

[23] R. Pradhan and M. Pal, The Generalised inverse of Block intuitionistic fuzzy matrices, International Journal of Applications of Fuzzy Sets and Artificial Intelligence, Vol 3 (2013), 23-38.

[24] R. Pradhan and M. Pal, Some Result on Generalized inverse of intuitionistic fuzzy matrices, Fuzzy Information and Engineering, (6) (2014), 133-145.

[25] S. Sriram and P. Murugadas The Moore penrose inverses of intuitionistic fuzzy matrices International Journal of Mathematics Analysis (4) (2010), 1779-1786.

[26] C. R. Rao and S. K. Mitra Generalized inverses: Theory and its Applications John Wiley,(1971) New York.

[27] Riyaz Ahmad Padder and P. Murugadas, On idempotent intuitionistic fuzzy matrices, of T-type, International Journal of Fuzzy Logic and Intelligent Systems, Vol. 16, No. 3 (2016), 181-187.

[28] Riyaz Ahmad Padder and P. Murugadas, Reduction of a nilpotent intuitionistic fuzzy matrix using implication operator, Application of Applied Mathematics, Vol. 11, Issue 2 (2016), 614-631.
[29] Riyaz Ahmad Padder and P. Murugadas, Generalization of Szpilrajn's theorem on intuitionistic fuzzy matrix, Journal of Mathematics and Informatics, Vol. 6, (2016), 7-14.
[30] Riyaz Ahmad Padder and P. Murugadas, Max-max operations on intuitionistic fuzzy matrix, Annals of Fuzzy Mathematics and Informatics Vol. 12, No. 6 (2016), 757-766.