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Edge Domination in Signed Graphs

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ABSTRACT
A signed graph is defined by an ordered pair $\Sigma=(G,\sigma)$ where $G=(V,E)$ is an underlying graph of Σ and $\sigma:E\to\{+,-\}$ is a function called a signature function. In this paper we define the edge domination parameters like positive edge domination number and negative edge domination number in signed graph. Further investigate the properties and bound of these domination parameters. In addition explain the theorem with examples.

Keywords: Signed graph, positive edge domination and negative edge domination.

INTRODUCTION

A signed graph is defined by an ordered pair $\Sigma=(G,\sigma)$ where $G=(V,E)$ is an underlying graph of Σ and $\sigma:E\to\{+,-\}$ is a function called a signature function.

The positive degree of the vertex u in the signed graph is defined by number of positive edges are incident in the vertex u and it is denoted by $d_+(u)$ .The negative degree of the vertex u in the signed graph is defined by number of negative edges are incident in the vertex u and it is denoted by $d_-(u)$.

The maximum positive degree of the signed graph Σ is maximum positive degree along the vertices in Σ it is denoted by $\Delta_+(G)$. The maximum negative degree of the signed graph Σ is maximum negative degree along the vertices in Σ it is denoted by $\Delta_-(G)$.

The minimum positive degree of the signed graph Σ is minimum positive degree along the vertices in Σ it is denoted by $\delta_+(G)$. The minimum negative degree of the signed graph Σ is minimum negative degree along the vertices in Σ it is denoted by $\delta_-(G)$.

Note that the sum of positive degree and negative degree of a vertex in $u\in\Sigma$ is the degree of vertex in underlying graph $G=(V,E)$.

A subset $P\subseteq V$ is a positive dominating set of Σ if every vertex $v\in V-P$ is positively dominated by at least one vertex $u\in P$. A positive dominating set is called minimal positive dominating set if no proper subset of P is a positive dominating set of Σ. The positive dominating number of Σ is the number of vertices in minimum positive dominating set of Σ. It is denoted by $\gamma_+(\Sigma)$.

A subset $N\subseteq V$ is a negative dominating set of Σ if every vertex $v\in V-N$ is negatively dominated by at least one vertex $u\in N$. A negative dominating set is called minimal negative dominating set if no proper subset of N is a negative dominating set of Σ. The negative dominating number of Σ is the number of vertices in minimum negative dominating set of Σ. It is denoted by $\gamma_-(\Sigma)$

In this paper we define the edge domination parameters like positive edge domination number and negative edge domination number in signed graph. Further investigate the properties and bound of these domination parameters. In addition explain the theorem with examples.

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1. POSITIVE EDGE DOMINATING SET IN SIGNED GRAPHS

In this section, we define the domination parameter named as positive edge dominating number. Further investigate the bounds and properties of the positive edge domination parameters.

Definition 2.1: A vertex v is positively edge dominated by an edge e there is an edge uv in Σ having positive sign such that $\sigma(uv) = +$.

Definition 2.2: A subset $E_+ \subseteq E$ is a positively edge dominating set of Σ if every vertex $v \in V$ is positive edge dominated by at least one edge $e \in E_+$. A positive edge dominating set is called minimal positive edge dominating set if no proper subset of E_+ is a positive edge dominating set of Σ. The positive edge dominating number of Σ is the number of edges in the least minimal positive edge dominating set of Σ. It is denoted by $\gamma_{E+}(\Sigma)$.

Theorem 2.1: Let $E_+ \subseteq E$ is a minimal positive edge dominating set of a signed graph Σ, then $uv \in E_+$ where uv is an edge having maximum positive degree in Σ.

Proof: Let $E_+ \subseteq E$ is a minimal positive edge dominating set of a signed graph Σ, Assume $uv \in E_+$ is a vertex uv having maximum positive degree in a signed graph Σ such that $d_+(uv) = \Delta_+(\Sigma)$.This implies there exist an edge $xy \in E - E_+$ is positively edge dominated by an edge uv . Therefore $E_+ - \{uv\}$ is not a positive edge dominating set of a signed graph Σ, Since $xy \in E - E_+$ is positively edge dominated by an edge uv. Hence $uv \in E_+$.

Theorem 2.2: In a signed graph Σ, then positive edge dominating number $\gamma_{E+}(\Sigma) \leq \frac{S(G)}{2}$

Proof: Let $E_+ \subseteq E$ is a minimal positive dominating set of a signed graph Σ. This implies $E - E_+$ is a positive edge dominating set of a signed graph Σ. Therefore we get

$$\gamma_{E+}(\Sigma) \leq |E - E_+| = |E| - |E_+|$$

$$\gamma_{E+}(\Sigma) \leq S(G) - \gamma_+(\Sigma)$$

$$2\gamma_{E+}(\Sigma) \leq S(G)$$

$$\gamma_{E+}(\Sigma) \leq \frac{S(G)}{2}$$

Hence $\gamma_{E+}(\Sigma) \leq \frac{S(G)}{2}$.

Theorem 2.3: In a signed graph Σ, then positive edge dominating number $\gamma_{E+}(\Sigma) \leq S(G) - \Delta_{E+}(\Sigma)$.

Proof: Let $E_+ \subseteq E$ is a minimal positive edge dominating set of a signed graph Σ. We know that (Theorem 2.1) $uv \in E_+$ where uv is an edge having maximum positive edge degree in Σ. Therefore we get $E - N_{E+}(uv)$ is a positive edge dominating set of a signed graph Σ. This implies

$$E_+ \subseteq (E - N_{E+}(uv))$$

$$|E_+| \leq |E - N_{E+}(uv)|$$

$$|E_+| \leq |E| - |N_{E+}(uv)|$$

$$\gamma_{E+}(\Sigma) \leq S(G) - \Delta_{E+}(\Sigma)$$

Hence $\gamma_{E+}(\Sigma) \leq S(G) - \Delta_{E+}(\Sigma)$.

Illustration 2.1: Signed graph $\Sigma = (G, \sigma)$

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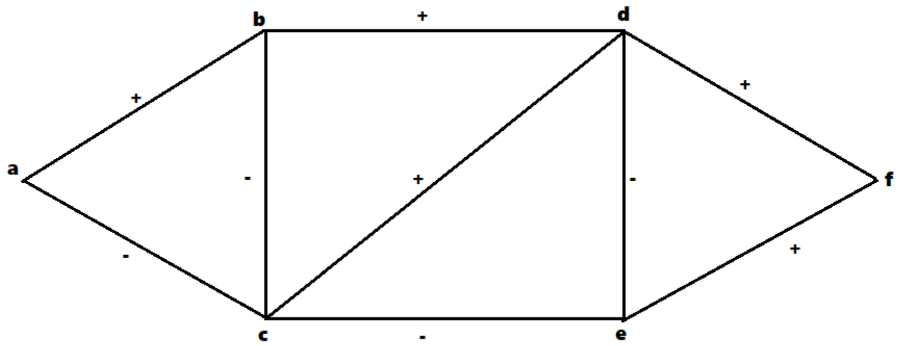

Figure 2.1: Signed graph $\Sigma=(G,\sigma)$

In a signed graph $\Sigma=(G,\sigma)$,

- The size of the underlying graph $G(V,E)$ is $S(G)=9$.
- The positive edge degree of the edges are $d_{E+}(ab)=1$, $d_{E+}(ac)=1$, $d_{E+}(bc)=1$, $d_{E+}(bd)=3$, $d_{E+}(cd)=0$, $d_{E+}(ce)=1$, $d_{E+}(de)=4$, $d_{E+}(df)=3$ and $d_{E+}(ef)=1$.
- The maximum positive edge degree of the signed graph $\Sigma=(G,\sigma)$ is $\Delta_{E+}(\Sigma)=4$.
- The positive edge dominating set of the signed graph $\Sigma=(G,\sigma)$ is $E_+=\{ab,de\}$.
- The positive dominating number of the signed graph $\Sigma=(G,\sigma)$ is $\gamma_{E+}(\Sigma)=2$.
- Note that $\gamma_{E+}(\Sigma)\leq S(G)-\Delta_{E+}(\Sigma)$.

2. NEGATIVE EDGE DOMINATING SET IN SIGNED GRAPHS

In this section, we define the domination parameter named as negative edge dominating number. Further investigate the bounds and properties of the negative edge domination parameters.

Definition 3.1: A vertex v is negative edge dominated by an edge e there is an edge uv in Σ having negative sign such that $\sigma(uv)=-$.

Definition 3.2: A subset $E_-\subseteq E$ is a positively edge dominating set of Σ if every vertex $v\in V$ is negative edge dominated by at least one edge $e\in E_-$. A positive edge dominating set is called minimal negative edge dominating set if no proper subset of E_- is a negative edge dominating set of Σ. The negative edge dominating number of Σ is the number of edges in the least minimal negative edge dominating set of Σ. It is denoted by $\gamma_{E-}(\Sigma)$.

Theorem 3.1: Let $E_-\subseteq E$ is a minimal negative edge dominating set of a signed graph Σ, then $uv\in E_+$ where uv is an edge having maximum negative degree in Σ.

Proof: Let $E_-\subseteq E$ is a minimal negative edge dominating set of a signed graph Σ, Assume $uv\in E_-$ is a vertex uv having maximum negative degree in a signed graph Σ such that $d_-(uv)=\Delta_-(\Sigma)$.This implies there exist an edge $xy\in E-E_-$ is negative edge dominated by an edge uv . Therefore $E_- -\{uv\}$ is not a negative edge dominating set of a signed graph Σ, Since $xy\in E-E_-$ is negative edge dominated by an edge uv. Hence $uv\in E_-$.

Theorem 3.2: In a signed graph Σ, then negative edge dominating number $\gamma_{E-}(\Sigma)\leq\dfrac{S(G)}{2}$.

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Proof: Let $E_- \subseteq E$ is a minimal negative dominating set of a signed graph Σ. This implies $E - E_-$ is a negative edge dominating set of a signed graph Σ. Therefore we get

$$\gamma_{E-}(\Sigma) \leq |E - E_-| = |E| - |E_-|$$
$$\gamma_{E-}(\Sigma) \leq S(G) - \gamma_-(\Sigma)$$
$$2\gamma_{E-}(\Sigma) \leq S(G)$$
$$\gamma_{E-}(\Sigma) \leq \frac{S(G)}{2}$$

Hence $\gamma_{E-}(\Sigma) \leq \frac{S(G)}{2}$.

Theorem 3.3: In a signed graph Σ, then negative edge dominating number $\gamma_{E-}(\Sigma) \leq S(G) - \Delta_{E-}(\Sigma)$.

Proof: Let $E_- \subseteq E$ is a minimal negative edge dominating set of a signed graph Σ. We know that (Theorem 3.1) $uv \in E_-$ where uv is an edge having maximum negative edge degree in Σ. Therefore we get $E - N_{E-}(uv)$ is a negative edge dominating set of a signed graph Σ. This implies

$$E_- \subseteq (E - N_{E-}(uv))$$
$$|E_-| \leq |E - N_{E-}(uv)|$$
$$|E_-| \leq |E| - |N_{E-}(uv)|$$
$$\gamma_{E-}(\Sigma) \leq S(G) - \Delta_{E-}(\Sigma)$$

Hence $\gamma_{E-}(\Sigma) \leq S(G) - \Delta_{E-}(\Sigma)$.

Illustration 3.1: Signed graph $\Sigma = (G, \sigma)$

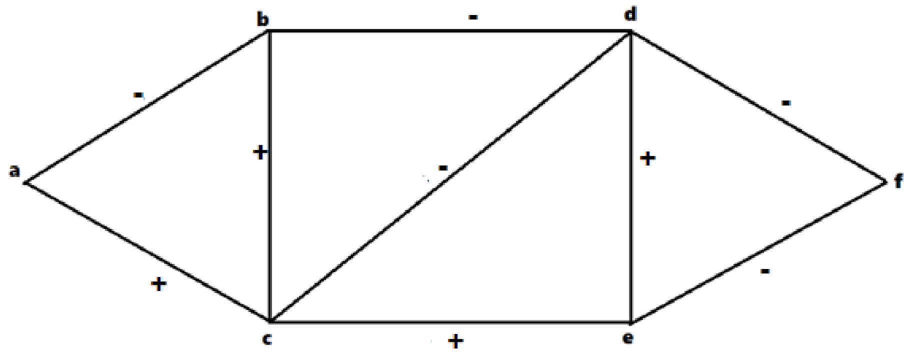

Figure 3.1: Signed graph $\Sigma = (G, \sigma)$

In a signed graph $\Sigma = (G, \sigma)$,

- The size of the underlying graph $G(V, E)$ is $S(G) = 9$.
- The negative edge degree of the edges are $d_{E-}(ab) = 1$, $d_{E-}(ac) = 1$, $d_{E-}(bc) = 1$, $d_{E-}(bd) = 3$, $d_{E-}(cd) = 0$, $d_{E-}(ce) = 1$, $d_{E-}(de) = 4$, $d_{E-}(df) = 3$ and $d_{E-}(ef) = 1$.
- The maximum positive edge degree of the signed graph $\Sigma = (G, \sigma)$ is $\Delta_{E-}(\Sigma) = 4$.
- The positive edge dominating set of the signed graph $\Sigma = (G, \sigma)$ is $E_- = \{ab, de\}$.
- The positive dominating number of the signed graph $\Sigma = (G, \sigma)$ is $\gamma_{E-}(\Sigma) = 2$.
- Note that $\gamma_{E-}(\Sigma) \leq S(G) - \Delta_{E-}(\Sigma)$.

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3. Conclusion

In this paper we define the edge domination parameters like positive edge domination number and negative edge domination number in signed graph. Further investigate the properties and bound of these domination parameters. In addition explain the theorem with examples. In future try to define various domination parameters in signed graphs.

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