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Arithmetic-Geometric Index in Various Signed Graphs

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Abstract: A topological index, also known as connectivity index, is a molecular structure descriptor calculated from a molecular graph of a chemical compound which characterizes its topology. Various topological indices are categorized based on their degree, distance, and spectrum. In this study, we calculated and analysed the degree-based topological indices such as positive arithmetic- geometric index ($(AG)^+$ index) and negative arithmetic- geometric index ($(AG)^-$ index). Further investigated the $(AG)^+$ index) and $(AG)^-$ index in regular graph, complete graph, complete bipartite graph, union of graphs and join of graphs are derived. Further explain the theorem by examples.
Keywords: Graphs, Topological index and arithmetic- geometric index .

Arithmetic-geometric index is defined as $AG(G) = \sum_{pq\in E(G)} \left(\frac{(d_p + d_q)}{2\sqrt{(d_p \cdot d_q)}} \right).$

INTRODUCTION

A signed graph is defined by an ordered pair $\Sigma = (G,\sigma)$ where $G = (V,E)$ is an underlying graph of Σ and $\sigma : E \to \{+,-\}$ is a function called a signature function.

The positive degree of the vertex u in the signed graph is defined by number of positive edges are incident in the vertex u and it is denoted by $d_+(u)$ .The negative degree of the vertex u in the signed graph is defined by number of negative edges are incident in the vertex u and it is denoted by $d_-(u)$.

The maximum positive degree of the signed graph Σ is maximum positive degree along the vertices in Σ it is denoted by $\Delta_+(G)$. The maximum negative degree of the signed graph Σ is maximum negative degree along the vertices in Σ it is denoted by $\Delta_-(G)$.

Note that the sum of positive degree and negative degree of a vertex in $u \in \Sigma$ is the degree of vertex in underlying graph $G = (V,E)$.

The positive degree of the edge uv in the signed graph is defined by number of positive edges are adjacent to the edge uv and it is denoted by $d_+(uv)$ .The negative degree of the edge uv in the signed graph is defined by number of negative edges are adjacent to the edge uv and it is denoted by $d_-(uv)$.

The minimum positive degree of the signed graph Σ is minimum positive degree along the edges in Σ it is denoted by $\delta_{E+}(G)$. The minimum negative degree of the signed graph Σ is minimum negative degree along the edges in Σ it is denoted by $\delta_{E-}(G)$.

One of the most investigated categories of topological indices used in mathematical chemistry is called degree-based topological indices, which are defined in terms of the degrees of the vertices of a graph. We can write the definition of such a topological index in the form given as

$$TI(G) = \sum_{pq\in G} F\big(d(p), d(q)\big)$$

Arithmetic-geometric index is defined as $AG(G) = \sum_{pq\in E(G)} \left(\frac{(d_p + d_q)}{2\sqrt{(d_p \cdot d_q)}} \right).$

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In this paper we define the Arithmetic-geometric index of signed graphs. Further we investigate some properties and bounds of the Arithmetic-geometric index of signed graphs.

1. ARITHMETIC-GEOMETRIC INDEX IN SIGNED GRAPHS

In this section we define the positive and negative AG index in signed graphs and investigate the properties and bounds of the AG index in signed graphs.

Definition 2.1: The positive Arithmetic-geometric (AG^+) index of signed graphs is defined as $AG^+(\Sigma) = \sum_{pq \in E(G)} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right).$

Definition 2.2: The negative Arithmetic-geometric (AG^-) index of signed graphs is defined as $AG^-(\Sigma) = \sum_{pq \in E(G)} \left(\frac{(d_P^- + d_q^-)}{2\sqrt{(d_P^- \cdot d_q^-)}} \right).$

Theorem 2.1: In a positive K regular signed graph with n vertices, then the (AG^+) index is $AG^+(\Sigma) \geq \left(\frac{nK}{2} \right)$

Proof: Let Σ be a positive K regular signed graph with n vertices. Therefore we get $d_-(v_i) = K$, *for every* $v_i \in \Sigma.$ The positive Arithmetic-geometric (AG^+) index of signed graphs is defined as $AG^+(\Sigma) = \sum_{pq \in E(G)} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right).$

$$AG^+(\Sigma) = \sum_{pq \in E(G)} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right).$$

$$= \sum_{pq \in E(G)} \left(\frac{(K + K)}{2\sqrt{(K \cdot K)}} \right).$$

$$= \sum_{pq \in E(G)} \left(\frac{(2K)}{2\sqrt{(K^2)}} \right).$$

$$AG^+(\Sigma) = \sum_{pq \in E(G)} (1).$$

In a negative K regular signed graph Σ with n vertices, there is minimum $\left(\frac{nK}{2} \right)$ edges in Σ.

$$AG^+(\Sigma) \geq 1 + 1 + 1 + \ldots \left(\frac{nK}{2} \right) Times$$

$$AG^+(\Sigma) \geq \left(\frac{nK}{2} \right).$$

Hence $AG^+(\Sigma) \geq \left(\frac{nK}{2} \right)$

Illustration 2.1: Positive 2-regular signed graph $\Sigma(G, \sigma)$.

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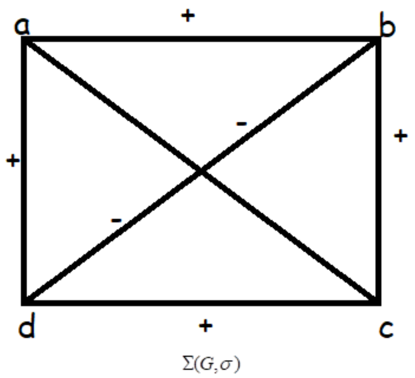

Figure 2.1: Positive 2-regular signed graph $\Sigma(G,\sigma)$.

In a positive 2-regular signed graph $\Sigma(G,\sigma)$ the positive degree of the every vertices in $\Sigma(G,\sigma)$ is 2. . The order $\Sigma(G,\sigma)$ is $O(\Sigma) = n = 4, S(G) = 6$. The (AG^{+}) index) $AG^{+}(\Sigma) = 4 \geq \left(\frac{nK}{2}\right)$.

$$AG^{+}(\Sigma) = \sum_{pq \in E(G)} \left(\frac{(d_P^{+} + d_q^{+})}{2\sqrt{(d_P^{+} \cdot d_q^{+})}} \right).$$

$$AG^{+}(\Sigma) \geq \left(\frac{(d_a^{+} + d_b^{+})}{2\sqrt{(d_a^{+} \cdot d_b^{+})}} \right) + \left(\frac{(d_b^{+} + d_c^{+})}{2\sqrt{(d_c^{+} \cdot d_c^{+})}} \right) + \left(\frac{(d_c^{+} + d_d^{+})}{2\sqrt{(d_c^{+} \cdot d_d^{+})}} \right) + \left(\frac{(d_a^{+} + d_d^{+})}{2\sqrt{(d_a^{+} \cdot d_d^{+})}} \right)$$

$$AG^{+}(\Sigma) \geq \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) + \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) + \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) + \left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right)$$

$$= 4\left(\frac{(2+2)}{2\sqrt{(2 \cdot 2)}} \right) = 4\left(\frac{4}{2\sqrt{4}} \right)$$

$$AG^{+}(\Sigma) \geq 4$$

Theorem 2.2: In a negative K regular signed graph with n vertices, then the (AG^{-}) index is $AG^{-}(\Sigma) \geq \left(\frac{nK}{2}\right)$

Proof: Let Σ be a negative K regular signed graph with n vertices. Therefore we get $d_{-}(v_i) = K,$ *for every* $v_i \in \Sigma.$ The negative Arithmetic-geometric (AG^{-}) index of signed graphs is defined as

$$AG^{-}(\Sigma) = \sum_{pq \in E(G)} \left(\frac{(d_P^{-} + d_q^{-})}{2\sqrt{(d_P^{-} \cdot d_q^{-})}} \right).$$

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$$AG^-(\Sigma) = \sum_{pq\in E(G)} \left(\frac{(d_P^- + d_q^-)}{2\sqrt{(d_P^- \cdot d_q^-)}} \right).$$

$$= \sum_{pq\in E(G)} \left(\frac{(K+K)}{2\sqrt{(K \cdot K)}} \right).$$

$$= \sum_{pq\in E(G)} \left(\frac{(2K)}{2\sqrt{(K^2)}} \right).$$

$$AG^-(\Sigma) = \sum_{pq\in E(G)} (1).$$

In a negative K regular signed graph Σ with n vertices, there is minimum $\left(\frac{nK}{2}\right)$ edges in Σ.

$$AG^-(\Sigma) \geq 1+1+1+\ldots\left(\frac{nK}{2}\right) Times$$

$$AG^-(\Sigma) \geq \left(\frac{nK}{2}\right).$$

Hence $AG^-(\Sigma) \geq \left(\frac{nK}{2}\right)$

Illustration 2.2: Negative 2-regular signed graph $\Sigma(G,\sigma)$.

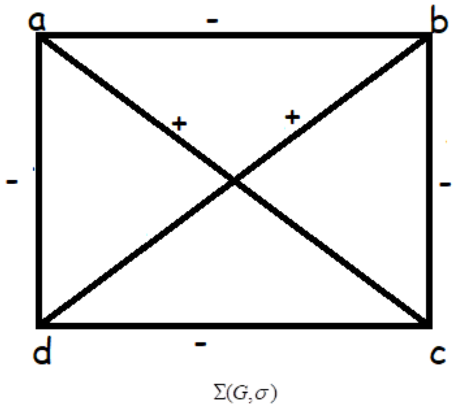

Figure 2.2: Negative 2-regular signed graph $\Sigma(G,\sigma)$.

In a negative 2-regular signed graph $\Sigma(G,\sigma)$ the negative degree of the every vertices in $\Sigma(G,\sigma)$ is 2. . The order $\Sigma(G,\sigma)$ is $O(\Sigma) = n = 4, S(G) = 6$. The (AG^-) index) $AG^-(\Sigma) = 4 \geq \left(\frac{nK}{2}\right)$.

$$AG^-(\Sigma) = \sum_{pq\in E(G)} \left(\frac{(d_P^- + d_q^-)}{2\sqrt{(d_P^- \cdot d_q^-)}} \right).$$

$$AG^+(\Sigma) \geq \left(\frac{(d_P^- + d_q^-)}{2\sqrt{(d_P^- \cdot d_q^-)}} \right) + \left(\frac{(d_P^- + d_q^-)}{2\sqrt{(d_P^- \cdot d_q^-)}} \right) + \left(\frac{(d_P^- + d_q^-)}{2\sqrt{(d_P^- \cdot d_q^-)}} \right) + \left(\frac{(d_P^- + d_q^-)}{2\sqrt{(d_P^- \cdot d_q^-)}} \right)$$

$$AG^{-}(\Sigma) \geq \left(\frac{(2+2)}{2\sqrt{(2\cdot 2)}}\right)+\left(\frac{(2+2)}{2\sqrt{(2\cdot 2)}}\right)+\left(\frac{(2+2)}{2\sqrt{(2\cdot 2)}}\right)+\left(\frac{(2+2)}{2\sqrt{(2\cdot 2)}}\right)$$

$$=4\left(\frac{(2+2)}{2\sqrt{(2\cdot 2)}}\right)=4\left(\frac{4}{2\sqrt{4}}\right)$$

$$AG^{-}(\Sigma) \geq 4$$

Theorem 2.3: For a positive complete signed graph of n vertices, then the (AG) index $AG^{+}(\Sigma) \geq \frac{n(n-1)}{2}$.

Proof: Let $\Sigma(G,\sigma)$ be a positive complete signed graph this implies the induced sub graph $< V^{+} >$ is a complete graph of order $O(G)=n$. This implies the positive degree of every vertices in G is (n-1) and n number of vertices in the induced sub graph $< V^{+} >$. In a (n-1) regular graph there is $\frac{n(n-1)}{2}$ edges in a complete graph of n vertices. The positive Arithmetic-geometric (AG^{+}) index of signed graphs is defined as

$$AG^{+}(\Sigma) = \sum_{pq\in E(G)} \left(\frac{(d_P^{+}+d_q^{+})}{2\sqrt{(d_P^{+}\cdot d_q^{+})}}\right)$$

$$AG^{+}(\Sigma) = \sum_{\substack{pq\in E^{+}\\ p,q\in V^{+}}} \left(\frac{(d_P^{+}+d_q^{+})}{2\sqrt{(d_P^{+}\cdot d_q^{+})}}\right) + \sum_{\substack{pq\notin E^{+}\\ p,q\notin V^{+}}} \left(\frac{(d_P^{+}+d_q^{+})}{2\sqrt{(d_P^{+}\cdot d_q^{+})}}\right)$$

$$AG^{+}(\Sigma) \geq \sum_{\substack{pq\in E^{+}\\ p,q\in V^{+}}} \left(\frac{(d_P^{+}+d_q^{+})}{2\sqrt{(d_P^{+}\cdot d_q^{+})}}\right)$$

$$AG^{+}(\Sigma) \geq \sum_{\substack{pq\in E^{+}\\ p,q\in V^{+}}} \left(\frac{((n-1)+(n-1))}{2\sqrt{((n-1)^2)}}\right)$$

$$AG^{+}(\Sigma) \geq \sum_{\substack{pq\in E^{+}\\ p,q\in V^{+}}} \left(\frac{(2(n-1))}{2\sqrt{((n-1)^2)}}\right) = \sum_{\substack{pq\in E^{+}\\ p,q\in V^{+}}} \left(\frac{(2(n-1))}{2(n-1)}\right)$$

$$AG^{+}(\Sigma) \geq \sum_{\substack{pq\in E^{+}\\ p,q\in V^{+}}} (1) = 1+1+1+\ldots\frac{n(n-1)}{2}\,times$$

$$AG^{+}(\Sigma) \geq \frac{n(n-1)}{2}$$

Hence $AG^{+}(\Sigma) \geq \frac{n(n-1)}{2}$.

Illustration 2.3: Positive complete signed graph $\Sigma(G,\sigma)$.

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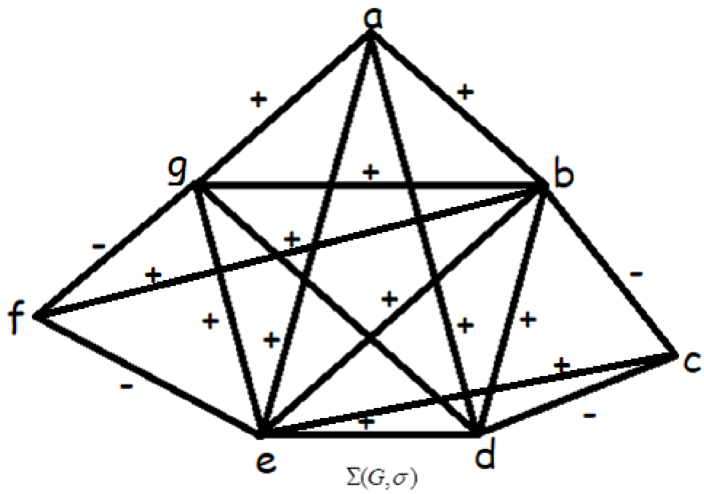

Figure 2.3: Positive complete signed graph $\Sigma(G,\sigma)$.

In a positive complete signed graph $\Sigma(G,\sigma)$ the positive degree of the every vertices in $<V^+>$ is 4. . The order $\Sigma(G,\sigma)$ is $O(\Sigma)=n=7, S(\Sigma)=14$. The (AG^+) index) $AG^+(\Sigma)=15\geq\left(\frac{n(n-1)}{2}\right)$.

$$AG^+(\Sigma)=\sum_{pq\in E(G)}\left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+\cdot d_q^+)}}\right)$$

$$AG^+(\Sigma)=\sum_{\substack{pq\in E^+(G)\\ p,q\in V^+(G)}}\left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+\cdot d_q^+)}}\right)+\sum_{\substack{pq\notin E^+(G)\\ p,q\notin V^+(G)}}\left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+\cdot d_q^+)}}\right)$$

$$AG^+(\Sigma)=\sum_{\substack{pq\in E^+(G)\\ p,q\in V^+(G)}}\left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+\cdot d_q^+)}}\right)+$$

$$\left(\frac{(d_b^+ + d_c^+)}{2\sqrt{(d_b^+\cdot d_c^+)}}\right)+\left(\frac{(d_c^+ + d_d^+)}{2\sqrt{(d_c^+\cdot d_d^+)}}\right)+\left(\frac{(d_e^+ + d_f^+)}{2\sqrt{(d_e^+\cdot d_f^+)}}\right)+\left(\frac{(d_f^+ + d_g^+)}{2\sqrt{(d_f^+\cdot d_g^+)}}\right)$$

$$AG^+(\Sigma)=\sum_{pq\in E^+(G)}\left(\frac{(4+4)}{2\sqrt{(4\cdot 4)}}\right)+\left(\frac{(4+1)}{2\sqrt{(4)}}\right)+\left(\frac{(4+1)}{2\sqrt{(4)}}\right)+\left(\frac{(4+1)}{2\sqrt{(4)}}\right)+\left(\frac{(4+1)}{2\sqrt{(4)}}\right)$$

$$AG^+(\Sigma)=\sum_{pq\in E^+(G)}(1)+\left(\frac{5}{4}\right)+\left(\frac{5}{4}\right)+\left(\frac{5}{4}\right)+\left(\frac{5}{4}\right)$$

$$AG^+(\Sigma)=1+1+1+..\left(\frac{5(5-1)}{2}\right)times+4\left(\frac{5}{4}\right)$$

$$AG^+(\Sigma)=10+5=15$$

Theorem 2.3: For a negative complete signed graph of n vertices, then the (AG^-) index $AG^-(\Sigma)\geq\frac{n(n-1)}{2}$.

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Proof: Let $\Sigma(G,\sigma)$ be a negative complete signed graph this implies the induced sub graph $< V^{-} >$ is a complete graph of order $O(G)=n$. This implies the positive degree of every vertices in G is (n-1) and n number of vertices in the induced sub graph $< V^{-} >$. In a (n-1) regular graph there is $\frac{n(n-1)}{2}$ edges in a complete graph of n vertices. The positive arithmetic-geometric (AG^{-}) index of signed graphs is defined as

$$AG^{-}(\Sigma)=\sum_{pq\in E(G)}\left(\frac{\left(d_P^{-}+d_q^{-}\right)}{2\sqrt{\left(d_P^{-}\cdot d_q^{-}\right)}}\right)$$

$$AG^{-}(\Sigma)=\sum_{\substack{pq\in E^{-}\\ p,q\in V^{-}}}\left(\frac{\left(d_P^{-}+d_q^{-}\right)}{2\sqrt{\left(d_P^{-}\cdot d_q^{-}\right)}}\right)+\sum_{\substack{pq\notin E^{-}\\ p,q\notin V^{-}}}\left(\frac{\left(d_P^{-}+d_q^{-}\right)}{2\sqrt{\left(d_P^{-}\cdot d_q^{-}\right)}}\right)$$

$$AG^{-}(\Sigma)\geq\sum_{\substack{pq\in E^{-}\\ p,q\in V^{-}}}\left(\frac{\left(d_P^{-}+d_q^{-}\right)}{2\sqrt{\left(d_P^{-}\cdot d_q^{-}\right)}}\right)$$

$$AG^{-}(\Sigma)\geq\sum_{\substack{pq\in E^{-}\\ p,q\in V^{-}}}\left(\frac{((n-1)+(n-1))}{2\sqrt{\left((n-1)^2\right)}}\right)$$

$$AG^{-}(\Sigma)\geq\sum_{\substack{pq\in E^{-}\\ p,q\in V^{-}}}\left(\frac{(2(n-1))}{2\sqrt{\left((n-1)^2\right)}}\right)=\sum_{\substack{pq\in E^{-}\\ p,q\in V^{-}}}\left(\frac{(2(n-1))}{2(n-1)}\right)$$

$$AG^{-}(\Sigma)\geq\sum_{\substack{pq\in E^{-}\\ p,q\in V^{-}}}(1)=1+1+1+\ldots\frac{n(n-1)}{2}\,times$$

$$AG^{-}(\Sigma)\geq\frac{n(n-1)}{2}$$

Hence $AG^{-}(\Sigma)\geq\frac{n(n-1)}{2}$.

Illustration 2.4: Negative complete signed graph $\Sigma(G,\sigma)$.

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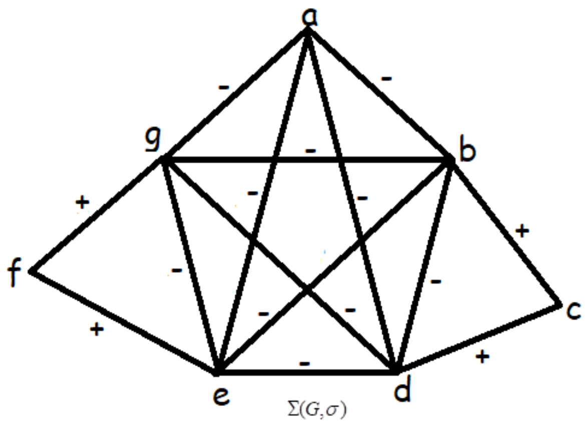

Figure 2.4: Negative complete signed graph $\Sigma(G,\sigma)$.

In a negative complete signed graph $\Sigma(G,\sigma)$ the negative degree of the every vertices in $< V^- >$ is 4. . The order $\Sigma(G,\sigma)$ is $O(\Sigma) = n = 7, S(\Sigma) = 14$. The (AG^-) index) $AG^-(\Sigma) = 15 \geq \left(\frac{n(n-1)}{2}\right)$.

$$AG^-(\Sigma) = \sum_{pq\in E(G)} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right)$$

$$AG^-(\Sigma) = \sum_{pq\in E^+(G)} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right) + \sum_{pq\in E^-(G)} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right)$$

$$AG^-(\Sigma) = \left(\frac{(d_b^+ + d_c^+)}{2\sqrt{(d_b^+ \cdot d_c^+)}} \right) + \left(\frac{(d_c^+ + d_d^+)}{2\sqrt{(d_c^+ \cdot d_d^+)}} \right) + \left(\frac{(d_e^+ + d_f^+)}{2\sqrt{(d_e^+ \cdot d_f^+)}} \right) + \left(\frac{(d_f^+ + d_g^+)}{2\sqrt{(d_f^+ \cdot d_g^+)}} \right) +$$

$$\sum_{pq\in E^-(G)} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right)$$

$$AG^-(\Sigma) = \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \left(\frac{(4+1)}{2\sqrt{(4)}} \right) + \sum_{pq\in E^-(G)} \left(\frac{(4+4)}{2\sqrt{(4\cdot 4)}} \right) +$$

$$AG^-(\Sigma) = \left(\frac{5}{4}\right) + \left(\frac{5}{4}\right) + \left(\frac{5}{4}\right) + \left(\frac{5}{4}\right) + \sum_{pq\in E^-(G)} (1)$$

$$AG^-(\Sigma) = 4\left(\frac{5}{4}\right) + \left(1+1+1+..\left(\frac{5(5-1)}{2}\right) times \right)$$

$$AG^-(\Sigma) = 5 + 10 = 15$$

Theorem 2.5: For a positive complete bipartite signed graph $K_{m,n}$, then the (AG^+) index $AG^+(\Sigma) \geq \frac{1}{2}(n+m)\sqrt{mn}$.

Proof: Let $\Sigma(G,\sigma)$ be a positive complete bipartite signed graph of vertex sets $V_m{}^+ \& V_n{}^+$ respectively .This implies the induced sub graph $< V_{m,n}^+ >$ is a positive complete bipartite graph of vertex sets $V_m \& V_n$. This implies the

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positive degree of every vertices in V_m and V_n are $n \& m$ respectively such that $d^+(v_i) = n, \forall v_i \in V_m$ and $d^+(v_j) = m, \forall v_j \in V_n$, there is mn edges in a positive complete bipartite signed graph $K_{m,n}$ of (m,n) vertices. Therefore (AG^+) index

$$AG^+(\Sigma) = \sum_{pq \in E(G)} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right)$$

$$AG^+(\Sigma) = \sum_{\substack{pq \in E^+(G) \\ p,q \in V^+(G)}} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right) + \sum_{\substack{pq \notin E^+(G) \\ p,q \notin V^+(G)}} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right)$$

$$AG^+(\Sigma) = \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \& q \in V_n}} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right) + \sum_{pq \notin K_{m,n}} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right)$$

$$AG^+(\Sigma) \geq \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \& q \in V_n}} \left(\frac{(d_P^+ + d_q^+)}{2\sqrt{(d_P^+ \cdot d_q^+)}} \right)$$

$$= \sum_{\substack{pq \in K_{m,n} \\ p \in V_m \& q \in V_n}} \left(\frac{(n+m)}{2\sqrt{(n \cdot m)}} \right)$$

$$= \frac{(n+m)}{2\sqrt{(n \cdot m)}} + \frac{(n+m)}{2\sqrt{(n \cdot m)}} + \frac{(n+m)}{2\sqrt{(n \cdot m)}} + \ldots mn\,times$$

$$= \frac{(n+m)}{2\sqrt{(n \cdot m)}} (mn)$$

$$AG^+(\Sigma) \geq \frac{1}{2}(n+m)\sqrt{mn}$$

Hence $AG^+(\Sigma) \geq \frac{1}{2}(n+m)\sqrt{mn}$.

Illustration 2.5: Positive complete bipartite signed graph $\Sigma(G,\sigma)$.

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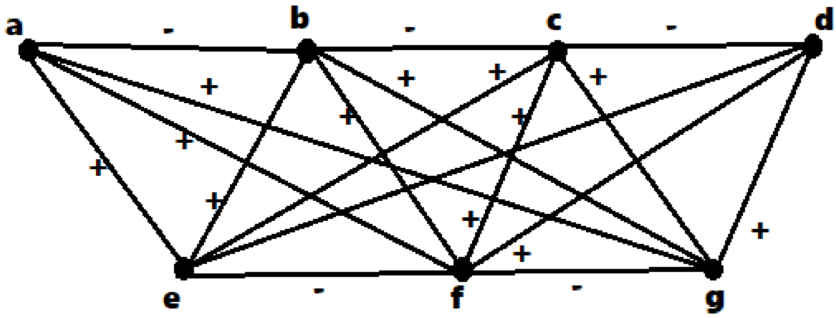

Figure 2.5: Positive complete bipartite signed graph $\Sigma(G,\sigma)$.

In a positive complete bipartite signed graph $\Sigma(G,\sigma)$ the positive degree of the every vertices in V_1^{+} and V_2^{+} are m=4 and n=3 respectively. The order $\Sigma(G,\sigma)$ is $O(\Sigma)=7$. There is 12 edges in positive complete bipartite signed graph $\Sigma(G,\sigma)$. The (AG^{+}) index) $AG^{+}(\Sigma)=7\sqrt{3}\geq\frac{1}{2}(n+m)\sqrt{mn}$.

$$AG^{+}(\Sigma)\geq\sum_{\substack{pq\in K_{m,n}\\ p\in V_m \& q\in V_n}}\left(\frac{(d_P^{+}+d_q^{+})}{2\sqrt{(d_P^{+}\cdot d_q^{+})}}\right)$$

$$=\sum_{\substack{pq\in K_{m,n}\\ p\in V_m \& q\in V_n}}\left(\frac{(4+3)}{2\sqrt{(4\cdot 3)}}\right)$$

$$=\frac{(7)}{2\sqrt{(12)}}+\frac{(7)}{2\sqrt{(12)}}+\frac{(7)}{2\sqrt{(12)}}+\ldots mn\,times$$

$$=\frac{(7)}{4\sqrt{3}}(12)$$

$$AG^{+}(\Sigma)\geq 7\sqrt{3}$$

Theorem 2.6: For a negative complete bipartite signed graph $K_{m,n}$, then the (AG^{-}) index $AG^{-}(\Sigma)\geq\frac{1}{2}(n+m)\sqrt{mn}$.

Proof: Let $\Sigma(G,\sigma)$ be a negative complete bipartite signed graph of vertex sets $V_m^{-}\,\&\,V_n^{-}$ respectively .This implies the induced sub graph $<V_{m,n}^{=}>$ is a negative complete bipartite graph of vertex sets $V_m \& V_n$. This implies the negative degree of every vertices in V_m and V_n are $n\,\&\,m$ respectively such that $d^{-}(v_i)=n,\ \forall v_i\in V_m$ and $d^{-}(v_j)=m,\ \forall v_j\in V_n$. clearly there is mn edges in a negative complete bipartite signed graph $K_{m,n}$ of (m,n) vertices. Therefore (AG^{-}) index

$$AG^{-}(\Sigma)=\sum_{pq\in E(G)}\left(\frac{\left(d_P^{-}+d_q^{-}\right)}{2\sqrt{\left(d_P^{-}\cdot d_q^{-}\right)}}\right)$$

$$AG^{-}(\Sigma)=\sum_{\substack{pq\in E^{+}(G)\\ p,q\in V^{+}(G)}}\left(\frac{\left(d_P^{-}+d_q^{-}\right)}{2\sqrt{\left(d_P^{-}\cdot d_q^{-}\right)}}\right)+\sum_{\substack{pq\notin E^{+}(G)\\ p,q\notin V^{+}(G)}}\left(\frac{\left(d_P^{-}+d_q^{-}\right)}{2\sqrt{\left(d_P^{-}\cdot d_q^{-}\right)}}\right)$$

$$AG^{+}(\Sigma)=\sum_{\substack{pq\in K_{m,n}\\ p\in V_m\,\&\,q\in V_n}}\left(\frac{\left(d_P^{-}+d_q^{-}\right)}{2\sqrt{\left(d_P^{-}\cdot d_q^{-}\right)}}\right)+\sum_{pq\notin K_{m,n}}\left(\frac{\left(d_P^{-}+d_q^{-}\right)}{2\sqrt{\left(d_P^{-}\cdot d_q^{-}\right)}}\right)$$

$$AG^{+}(\Sigma)\geq\sum_{\substack{pq\in K_{m,n}\\ p\in V_m\,\&\,q\in V_n}}\left(\frac{\left(d_P^{-}+d_q^{-}\right)}{2\sqrt{\left(d_P^{-}\cdot d_q^{-}\right)}}\right)$$

$$=\sum_{\substack{pq\in K_{m,n}\\ p\in V_m\,\&\,q\in V_n}}\left(\frac{(n+m)}{2\sqrt{(n\cdot m)}}\right)$$

$$=\frac{(n+m)}{2\sqrt{(n\cdot m)}}+\frac{(n+m)}{2\sqrt{(n\cdot m)}}+\frac{(n+m)}{2\sqrt{(n\cdot m)}}+\ldots mn\,times$$

$$=\frac{(n+m)}{2\sqrt{(n\cdot m)}}(mn)$$

$$AG^{-}(\Sigma)\geq\frac{1}{2}(n+m)\sqrt{mn}$$

Hence $AG^{-}(\Sigma)\geq\frac{1}{2}(n+m)\sqrt{mn}$.

Illustration 2.6: Negative complete bipartite signed graph $\Sigma(G,\sigma)$.

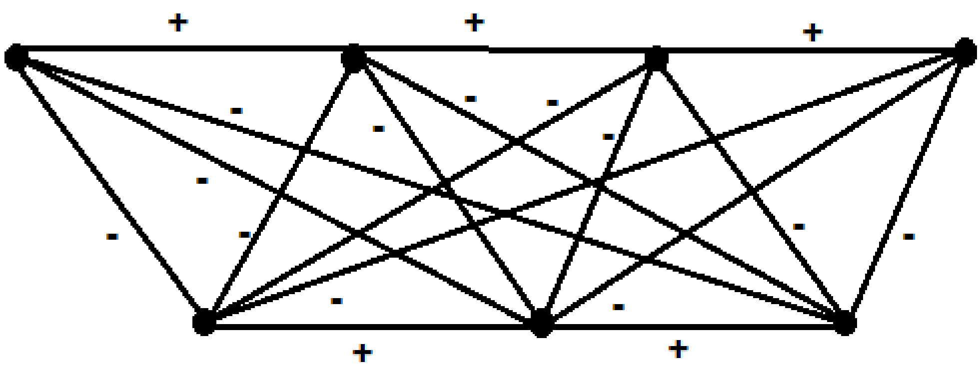

Figure 2.6: Negative complete bipartite signed graph $\Sigma(G,\sigma)$.

In a Negative complete bipartite signed graph $\Sigma(G,\sigma)$ the Negative degree of the every vertices in V_1^{+} and V_2^{+} are m=4 and n=3 respectively. The order $\Sigma(G,\sigma)$ is $O(\Sigma)=7$. There is 12 edges in Negative complete bipartite signed graph $\Sigma(G,\sigma)$. The (AG^{+}) index) $AG^{-}(\Sigma)=7\sqrt{3}\geq\frac{1}{2}(n+m)\sqrt{mn}$.

2. Conclusion

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In this study, we calculated and analysed the degree-based topological indices such as positive arithmetic-geometric index ($(AG)^{+}$ index) and negative arithmetic- geometric index ($(AG)^{-}$ index). Further investigated the $(AG)^{+}$ index) and $(AG)^{-}$ index in regular graph, complete graph, complete bipartite graph, union of graphs and join of graphs are derived. Further explain the theorem by examples. In future we will analysed the different degree-based topological indices

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