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A Third Order Runge-Kutta Method Based on Linear Combination Of Arithmetic Mean, Heronian Mean And Contra - Harmonic Mean For Hybrid Fuzzy Differential Equation

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ABSTRACT
In this paper, the third order Runge-Kutta method based on Arithmetic mean, Heronian mean and Contra Harmonic mean are implant into one method called the third order Runge-Kutta method based on linear combination of Arithmetic mean, Heronian mean and Contra Harmonic mean is proposed to find the approximate solution of hybrid fuzzy differential equation. Here the numerical algorithm for the proposed method is given to solve the hybrid fuzzy initial value problem. The validity of the proposed method is tested through illustration. Tables and figures show that the proposed method is applicable for solving any real life problems, which are modelled into Hybrid fuzzy differential equations.

Keywords: Hybrid fuzzy differential equations, Runge-Kutta method, Arithmetic mean, Heronian mean, Contra Harmonic mean, Triangular fuzzy number.

1. Introduction

Control systems that are capable of controlling complex systems which have discrete time dynamics and continuous time dynamics can be modeled by hybrid systems. The differential systems containing fuzzy valued functions and interaction with a discrete time controller are named as hybrid fuzzy differential systems.

Pederson and Sambandham used Euler and Runge-Kutta methods [6, 7] for solving hybrid fuzzy differential equations. Iterative procedure and Taylor series method were used by Narayana moorthy et al in [4, 5] to solve hybrid fuzzy differential equations. Fourth order Runge-Kutta method based on geometric mean for hybrid fuzzy initial value problem and centroidal mean for fuzzy and hybrid fuzzy initial value problem is solved by Gethsi Sharmila and Henry Amirtharaj in [1, 2]. Third order Runge-kutta method based on the linear combination of Arithmetic mean, Harmonic mean and Geometric mean is used to solve the Initial value problem by Gethsi Sharmila and Evangelin Diana Rajakumari in [3].

In this work, a third order Runge-Kutta Method based on linear combination of Arithmetic mean, Heronian mean and Contra - Harmonic mean is proposed to solve Hybrid fuzzy differential equations.

2. Preliminaries

Definition:2.1 (Fuzzy Set)

If X is a collection of objects denoted generally by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership of x in $\tilde{A}$.

Definition: 2.3 (Triangular fuzzy number)

It is a fuzzy number represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$, this representation is interpreted as membership functions and holds the following condition

(i) a_1 to a_2 is increasing function

(ii) a_2 to a_3 is decreasing function

(iii) $a_1 \le a_2 \le a_3$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & , a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & , a_2 \le x \le a_3 \\ 0 & , x > a_3 \end{cases}$$

Now, if you get crisp interval by α - cut operation, interval $\tilde{A}_\alpha$ shall be obtained as follows $\forall\ \alpha \in [0,1]$ from

$$\frac{a_1^{(\alpha)} - a_1}{a_2 - a_1} = \alpha \quad , \quad \frac{a_3 - a_3^{(\alpha)}}{a_3 - a_2} = \alpha$$

The Third order Runge-Kutta method based on a linear combination of Arithmetic mean (AM), Heronian mean (HeM) and Contra - Harmonic mean (CoM) for IVP was introduced by Khattri as follows

$$RM\,(k_1, k_2) = \frac{14\,AM\,(k_1, k_2) - HeM\,(k_1, k_2) + 32\,CoM\,(k_1, k_2)}{45}$$

3. The Hybrid Fuzzy Differential Equations

The hybrid fuzzy differential equation

$$\dot{y}(t) = f\big(t, y(t), \lambda_k(y_k)\big),\ t \in [\,t_k,\ t_{k+1}\,] \qquad k = 0, 1, 2, \ldots$$

$$y\,(t_0) = y_0 \tag{3.1}$$

where $t_k = 0$ is strictly increasing and unbounded, y_k denotes $y(t_k)$, $f : [t_0, \infty) \times R \times R \to R$ is continuous, and each $\lambda_k : R \to R$ is a continuous function. A solution to (3.1) will be a function $y : [t_0, \infty) \to R$ satisfying (3.1) . For k = 0, 1, 2, 3, … , let $f_k : [t_k, t_{k+1}) \times R \to R$ where, $f\,(t, y_k(t)) = f\,(t, y_k(t), \lambda_k(y_k))$.
The hybrid fuzzy differential equation in (3.1) can be written in expanded form as

$$\dot{y}(t) = \begin{cases} \dot{y}_0(t) = f\big(t, y_0(t), \lambda_0(y_0)\big) = f_0\big(t, y_0(t)\big), y_0(t_0) = y_0, t_0 \le t \le t_1 \\ \dot{y}_1(t) = f\big(t, y_1(t), \lambda_1(y_1)\big) = f_1\big(t, y_1(t)\big), y_1(t_1) = y_1, t_1 \le t \le t_2 \\ \vdots \\ \dot{y}_k(t) = f\big(t, y_k(t), \lambda_k(y_k)\big) = f_k\big(t, y_k(t)\big), y_k(t_k) = y_k, t_k \le t \le t_{k+1} \\ \vdots \end{cases} \tag{3.2}$$

and a solution of (3.1) can be expressed as

$$y\,(t) = \begin{cases} y_0(t) = y_0,\ t_0 \le t \le t_1 \\ y_1(t) = y_1,\ t_1 \le t \le t_2 \\ \vdots \\ y_k(t) = y_k,\ t_k \le t \le t_{k+1} \\ \vdots \end{cases} \tag{3.3}$$

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we note that the solution of (3.1) is continuous and piecewise differentiable over $[t_0, \infty)$ and differentiable on each interval (t_k, t_{k+1}) for any fixed $y_k \in R$ and k = 0, 1, 2, … .

4. A Third order Runge-Kutta method based on linear combination of Arithmetic mean, Heronian mean and Contra - Harmonic mean for hybrid fuzzy differential equation

Consider the IVP (3.1) with crisp initial condition $y(t_0) = y_0 \in R$ and $t \in [t_0, T]$. Let the exact solution $[Y(t)]_r = [\underline{Y}(t;r), \overline{Y}(t;r)]$ is approximated by some $[y(t)]_r = [\underline{y}(t;r), \overline{y}(t;r)]$ from the equation $y_{n+1} = y_n + \frac{h}{2}\left[\sum_{i=1}^{2} Means\right]$

where means includes Arithmetic mean(AM), Geometric mean (GM), Contra - Harmonic mean (CoM), Centroidal mean (CeM), Root mean Square (RM), Harmonic mean(HaM), and Heronian mean (HeM) which involves $k_{i,}\ 1 \le i \le 4$,
where,

$$k_1 = f(t_n, y_n)$$
$$k_2 = f(t_n + a_1 h, y_n + a_1 h k_1)$$
$$k_3 = f(t_n + (a_2 + a_3) h, y_n + a_2 h k_1 + a_3 h k_2)$$

where the parameters for the linear combination of Arithmetic mean, Heronian mean and Contra - Harmonic mean are

$$a_1 = \frac{2}{3},\ a_2 = \frac{-77}{810},\ a_3 = \frac{617}{810}$$

The third order formulae based on Runge- Kutta scheme using the Combination of Arithmetic mean, Heronian mean and Contra - Harmonic mean are

$$y_{n+1} = y_n + \frac{h}{90}\left\{7(k_1 + 2k_2 + k_3) - \frac{1}{3}\left(k_1 + 2k_2 + k_3 + \sqrt{k_1 k_2} + \sqrt{k_2 k_3}\right) + 32\left(\frac{k_1^2 + k_2^2}{k_1 + k_2} + \frac{k_2^2 + k_3^2}{k_2 + k_3}\right)\right\}$$

with the grid points $a = t_0 \le t_1 \le \ldots \le t_N = b$ and $h = \frac{(b-a)}{N} = t_{i+1} - t_i$

We define

$$\underline{y}_{k,n+1}(r) - \underline{y}_{k,n}(r) = \sum_{i=1}^{3} w_i \underline{k}_i(t_{k,n}; y_{k,n}(r))$$
$$\overline{y}_{k,n+1}(r) - \overline{y}_{k,n}(r) = \sum_{i=1}^{3} w_i \overline{k}_i(t_{k,n}; y_{k,n}(r))$$

where w_1, w_2, w_3 are constants,

$$\underline{k}_1(t_{k,n}; y_{k,n}(r)) = \min\left\{f(t_{k,n}, u, \lambda_k(u_k)) \backslash u \in [\underline{y}_{k,n}(r), \overline{y}_{k,n}(r)], u_k \in [\underline{y}_{k,0}(r), \overline{y}_{k,0}(r)]\right\}$$
$$\overline{k}_1(t_{k,n}; y_{k,n}(r)) = \max\left\{f(t_{k,n}, u, \lambda_k(u_k)) \backslash u \in [\underline{y}_{k,n}(r), \overline{y}_{k,n}(r)], u_k \in [\underline{y}_{k,0}(r), \overline{y}_{k,0}(r)]\right\}$$

$$\underline{k}_2(\mathrm{t_{k,n}}; y_{k,n}(r)) = \min\left\{\begin{array}{l} f(\mathrm{t_{k,n}} + \frac{2}{3}h_k, u, \lambda_k(u_k)) \backslash u \in [\underline{z}_{k_1}(\mathrm{t_{k,n}}; y_{k,n}(r)), \overline{z}_{k_1}(\mathrm{t_{k,n}}; y_{k,n}(r))], \\ u_k \in [\underline{y}_{k,0}(r), \overline{y}_{k,0}(r)] \end{array}\right\}$$

JOURNAL OF ALGEBRAIC STATISTICS
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$$\overline{k}_2(\mathrm{t_{k,n}}\,; y_{k,n}(r)) = \max\left\{\begin{array}{l} f(\mathrm{t_{k,n}} + \frac{2}{3}h_k, u, \lambda_k(u_k)) \setminus u \in [\underline{z}_{k_1}(\mathrm{t_{k,n}}\,; y_{k,n}(r)), \overline{z}_{k_1}(\mathrm{t_{k,n}}\,; y_{k,n}(r))], \\ u_k \in [\underline{y}_{k,0}(r), \overline{y}_{k,0}(r)] \end{array}\right\}$$

$$\underline{k}_3(\mathrm{t_{k,n}}\,; y_{k,n}(r)) = \min\left\{\begin{array}{l} f(\mathrm{t_{k,n}} + \frac{2}{3}h_k, u, \lambda_k(u_k)) \setminus u \in [\underline{z}_{k_2}(\mathrm{t_{k,n}}\,; y_{k,n}(r)), \overline{z}_{k_2}(\mathrm{t_{k,n}}\,; y_{k,n}(r))], \\ u_k \in [\underline{y}_{k,0}(r), \overline{y}_{k,0}(r)] \end{array}\right\}$$

$$\overline{k}_3(\mathrm{t_{k,n}}\,; y_{k,n}(r)) = \min\left\{\begin{array}{l} f(\mathrm{t_{k,n}} + \frac{2}{3}h_k, u, \lambda_k(u_k)) \setminus u \in [\underline{z}_{k_2}(\mathrm{t_{k,n}}\,; y_{k,n}(r)), \overline{z}_{k_2}(\mathrm{t_{k,n}}\,; y_{k,n}(r))], \\ u_k \in [\underline{y}_{k,0}(r), \overline{y}_{k,0}(r)] \end{array}\right\}$$

where

$$\underline{z}_{k_1}(\mathrm{t_{k,n}}\,; y_{k,n}(r)) = \underline{y}_{k,n}(r) + \frac{2}{3}h_k\, \underline{k}_1(\mathrm{t_{k,n}}\,; y_{k,n}(r))$$

$$\overline{z}_{k_1}(\mathrm{t_{k,n}}\,; y_{k,n}(r)) = \overline{y}_{k,n}(r) + \frac{2}{3}h_k\, \overline{k}_1(\mathrm{t_{k,n}}\,; y_{k,n}(r))$$

$$\underline{z}_{k_2}(\mathrm{t_{k,n}}\,; y_{k,n}(r)) = \underline{y}_{k,n}(r) - \frac{77}{810}h_k\, \underline{k}_1(\mathrm{t_{k,n}}\,; y_{k,n}(r)) + \frac{617}{810}h_k\, \underline{k}_2(\mathrm{t_{k,n}}\,; y_{k,n}(r))$$

$$\overline{z}_{k_2}(\mathrm{t_{k,n}}\,; y_{k,n}(r)) = \overline{y}_{k,n}(r) - \frac{77}{810}h_k\, \overline{k}_1(\mathrm{t_{k,n}}\,; y_{k,n}(r)) + \frac{617}{810}h_k\, \overline{k}_2(\mathrm{t_{k,n}}\,; y_{k,n}(r))$$

define

$$S_k\left[t_{k,n}\,; \underline{y}_{k,n}(r), \overline{y}_{k,n}(r)\right] = \left\{\begin{array}{l} 7\left[\underline{k}_1\left(t_{k,n}, y_{k,n}(r)\right) + 2\underline{k}_2\left(t_{k,n}, y_{k,n}(r)\right) + \underline{k}_3\left(t_{k,n}, y_{k,n}(r)\right)\right] \\ -\frac{1}{3}\left[\begin{array}{l}\underline{k}_1\left(t_{k,n}, y_{k,n}(r)\right) + 2\underline{k}_2\left(t_{k,n}, y_{k,n}(r)\right) + \underline{k}_3\left(t_{k,n}, y_{k,n}(r)\right) \\ + \sqrt{\underline{k}_1\left(t_{k,n}, y_{k,n}(r)\right)\underline{k}_2\left(t_{k,n}, y_{k,n}(r)\right)} + \sqrt{\underline{k}_2\left(t_{k,n}, y_{k,n}(r)\right)\underline{k}_3\left(t_{k,n}, y_{k,n}(r)\right)}\end{array}\right] \\ +32\left[\frac{\underline{k}_1^2\left(t_{k,n}, y_{k,n}(r)\right) + \underline{k}_2^2\left(t_{k,n}, y_{k,n}(r)\right)}{\underline{k}_1\left(t_{k,n}, y_{k,n}(r)\right) + \underline{k}_2\left(t_{k,n}, y_{k,n}(r)\right)} + \frac{\underline{k}_2^2\left(t_{k,n}, y_{k,n}(r)\right) + \underline{k}_3^2\left(t_{k,n}, y_{k,n}(r)\right)}{\underline{k}_2\left(t_{k,n}, y_{k,n}(r)\right) + \underline{k}_3\left(t_{k,n}, y_{k,n}(r)\right)}\right] \end{array}\right\}$$

$$T_k\left[t_{k,n}\,; \underline{y}_{k,n}(r), \overline{y}_{k,n}(r)\right] = \left\{\begin{array}{l} 7\left[\overline{k}_1\left(t_{k,n}, y_{k,n}(r)\right) + 2\overline{k}_2\left(t_{k,n}, y_{k,n}(r)\right) + \overline{k}_3\left(t_{k,n}, y_{k,n}(r)\right)\right] \\ -\frac{1}{3}\left[\begin{array}{l}\overline{k}_1\left(t_{k,n}, y_{k,n}(r)\right) + 2\overline{k}_2\left(t_{k,n}, y_{k,n}(r)\right) + \overline{k}_3\left(t_{k,n}, y_{k,n}(r)\right) \\ + \sqrt{\overline{k}_1\left(t_{k,n}, y_{k,n}(r)\right)\overline{k}_2\left(t_{k,n}, y_{k,n}(r)\right)} + \sqrt{\overline{k}_2\left(t_{k,n}, y_{k,n}(r)\right)\overline{k}_3\left(t_{k,n}, y_{k,n}(r)\right)}\end{array}\right] \\ +32\left[\frac{\overline{k}_1^2\left(t_{k,n}, y_{k,n}(r)\right) + \overline{k}_2^2\left(t_{k,n}, y_{k,n}(r)\right)}{\overline{k}_1\left(t_{k,n}, y_{k,n}(r)\right) + \overline{k}_2\left(t_{k,n}, y_{k,n}(r)\right)} + \frac{\overline{k}_2^2\left(t_{k,n}, y_{k,n}(r)\right) + \overline{k}_3^2\left(t_{k,n}, y_{k,n}(r)\right)}{\overline{k}_2\left(t_{k,n}, y_{k,n}(r)\right) + \overline{k}_3\left(t_{k,n}, y_{k,n}(r)\right)}\right] \end{array}\right\}$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1454-1461
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The exact solutions at $t_{k,n+1}$ is given by

$$\underline{Y}_{k,n+1}(r) \approx \underline{Y}_{k,n}(r) + \frac{h_k}{90} S_k \left[t_{k,n}; \underline{Y}_{k,n}(r), \overline{Y}_{k,n}(r)\right]$$
$$\overline{Y}_{k,n+1}(r) \approx \overline{Y}_{k,n}(r) + \frac{h_k}{90} T_k \left[t_{k,n}; \underline{Y}_{k,n}(r), \overline{Y}_{k,n}(r)\right]$$

The approximate solutions at $t_{k,n+1}$ is given by

$$\underline{y}_{k,n+1}(r) = \underline{y}_{k,n}(r) + \frac{h_k}{90} S_k \left[t_{k,n}; \underline{y}_{k,n}(r), \overline{y}_{k,n}(r)\right]$$
$$\overline{y}_{k,n+1}(r) = \overline{y}_{k,n}(r) + \frac{h_k}{90} T_k \left[t_{k,n}; \underline{y}_{k,n}(r), \overline{y}_{k,n}(r)\right]$$

5. Numerical Examples
Example : 5.1

consider the fuzzy Initial Value Problem

$$\begin{cases} y'(t) = y(t), t \in [0,1] \\ y(0;r) = [0.75 + 0.25r, 1.125 - 0.125r], 0 \le r \le 1 \end{cases}$$

The exact solution is given by

$$Y(t;r) = [(0.75 + 0.25r)e^t, (1.125 - 0.125r)e^t], 0 \le r \le 1$$

at t = 1 we get

$$Y(1;r) = [(0.75 + 0.25r)e^1, (1.125 - 0.125r)e^1], 0 \le r \le 1$$

By the third order Runge – Kutta method based on the combination of Arithmetic mean, Heronian mean and Contra - Harmonic mean with N = 2 , gives

$$y(1.0;r) = [(0.75 + 0.25r)(C_{0,1})^2, (1.125 - 0.125r)(C_{0,1})^2], 0 \le r \le 1$$

Where

$$C_{0,1} = 1 + \frac{h}{90}\left\{\begin{array}{l} 7\left[4 + 2h + \frac{617}{1215}h^2\right] - \frac{1}{3}\left[4 + 2h + \frac{617}{1215}h^2 + \sqrt{\left(1 + \frac{2}{3}h\right)} + \sqrt{\left(1 + \frac{2}{3}h\right)\left(1 + \frac{2}{3}h + \frac{617}{1215}h^2\right)}\right] \\ +32\left[\frac{1 + \left(1 + \frac{2}{3}h\right)^2}{\left(2 + \frac{2}{3}h\right)} + \frac{\left(1 + \frac{2}{3}h\right)^2 + \left(1 + \frac{2}{3}h + \frac{617}{1215}h^2\right)^2}{\left(2 + \frac{4}{3}h + \frac{617}{1215}h^2\right)}\right] \end{array}\right\}$$

Now consider the Hybrid fuzzy initial value problem

$$\begin{cases} y'(t) = y(t) + m(t)\lambda_k(y(t_k)),\ t \in [t_k, t_{k+1}],\ t_k = k, k = 0,1,2,\ldots \\ y(t;r) = [(0.75 + 0.25r)e^t,\ (1.125 - 0.125r)e^t],\ 0 \le r \le 1 \end{cases} \tag{5.1}$$

where

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1454-1461
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$$m(t)=\begin{cases} 2\,(t\ (\bmod\ 1)) & , if\ t\ (\bmod\ 1)\le 0.5,\\ 2\,(1-t\ (\bmod\ 1))\ , & if\ t\ (\bmod\ 1)\ge 0.5\end{cases}$$

$$\lambda_k\,(y(t_k))=\begin{cases}0 & , if\ k=0,\\ \mu & , if\ k\in 1,2,\ldots\end{cases}$$

The hybrid fuzzy Initial value problems is equivalent to the following system of fuzzy Initial value problems.
The exact solution for $t\in[0,2]$ is given by

$$\begin{cases} y_0'(t)=y_0(t),\ t\in[0,1]\\ y_0(0;\ r)=[(0.75+0.25r)e,(1.125-0.125r)e],\ 0\le r\le 1\\ y_i'(t)=y_i(t)+m(t)\,y_{i-1}(t), t\in[t_i,t_{i+1}], y_i(t)=y_{i-1}(t_i),\ i=1,2,\ldots\end{cases}$$

$y(t)+m(t)\lambda_k(y(t_k))$ is continuous function of t, x and $\lambda_k(y(t_k))$, for each k = 0, 1, 2, …, the fuzzy Initial value problem

$$\begin{cases} y'(t)=y(t)+m(t)\,\lambda_k(y(t_k)), t\in[t_k,t_{k+1}], t_k=k\\ y(t_k)=y_{t_k}\end{cases}$$

has a unique solution on $[t_k, t_{k+1}]$. To numerically solve the hybrid fuzzy IVP the modified third order Runge – Kutta method based on Arithmetic mean, Heronian mean and Contra - Harmonic mean is applied for solving hybrid fuzzy differential equation with N = 2 to obtain, $y_{1,2}(r)$ approximating x (2.0; r) .

Let $f:[0,\infty)\times R\times R\to R$ be given by

$$f(t,y,\lambda_k(y(t_k)))=y(t)+m(t)\lambda_k(y(t_k)), t_k=k, k=0,1,2,\ldots,$$

Where $\lambda_k:R\to R$ is given by

$$\lambda_k(y)=\begin{cases}0,\ if\ k=0\\ y,\ if\ k\in\{1,2,\ldots,\}\end{cases}$$

Since the exact solution for t ϵ [1, 1.5] is

$$Y(t;\ r)=Y(1;\ r)(3e^{t-1}-2t), 0\le r\le 1,\ Y(1.5;\ r)=Y(1;\ r)(3\sqrt{e}-3), 0\le r\le 1$$

Then Y(1.5 ; r) is approximately 5.29 and the exact solution for t ϵ [1.5, 2] is

$$Y(t;\ r)=Y(1;\ r)(2t-2+e^{t-1.5}(3\sqrt{e}-4)), 0\le r\le 1$$

Therefore

$$Y(2.0;\ r)=Y(1;\ r)(2+3e-4\sqrt{e})$$

Then Y(2.0 ; r) is approximately 9.6749
The approximate solution, exact solution and absolute error using the third order Runge-Kutta method based on a linear combination of Arithmetic mean (RK3AM), Heronian mean (RK3HM) and Contra - Harmonic mean (RK3CoM) for the r-level set with h = 0.1 and t = 2 of the example 5.1 is given below.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1454-1461
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Table 5.1 : Solution of third order Runge-Kutta method based on a linear combination of RK3AM, RK3HM, RK3CoM for the r-level set with h = 0.1 and t = 2

r	t	approximate solution		exact solution		absolute error	
0	2	7.256207	10.88431	7.257732	10.886598	1.53E-03	2.29E-03
0.1	2	7.49808	10.763373	7.499656	10.765635	1.58E-03	2.26E-03
0.2	2	7.739954	10.642437	7.741581	10.644673	1.63E-03	2.24E-03
0.3	2	7.981827	10.5215	7.983505	10.523711	1.68E-03	2.21E-03
0.4	2	8.223701	10.400563	8.225429	10.402749	1.73E-03	2.19E-03
0.5	2	8.465575	10.279626	8.467354	10.281787	1.78E-03	2.16E-03
0.6	2	8.707448	10.158689	8.709278	10.160824	1.83E-03	2.14E-03
0.7	2	8.949322	10.037753	8.951202	10.039862	1.88E-03	2.11E-03
0.8	2	9.191195	9.916816	9.193127	9.9189	1.93E-03	2.08E-03
0.9	2	9.433069	9.795879	9.435051	9.797938	1.98E-03	2.06E-03
1	2	9.674942	9.674942	9.676976	9.676976	2.03E-03	2.03E-03

The following is the graphical representation for the absolute error of the third order Runge-Kutta method based on the RK3AM, RK3HM, RK3CoM and the linear combination of RK3AM, RK3HM, RK3CoM for the above example is given below, where t = 2, h = 0.1

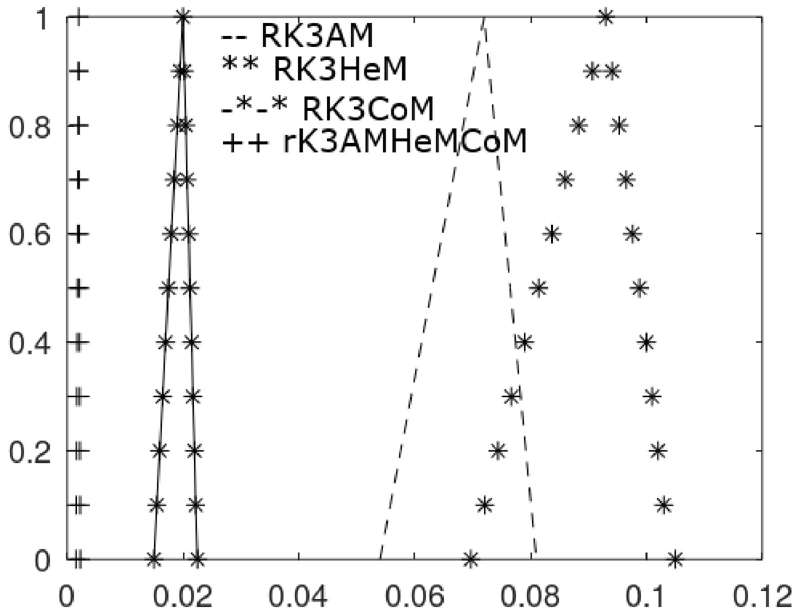

Fig: 5.1 (h = 0.1, t = 2)

6. Conclusion

In this paper, the first order hybrid fuzzy differential equation is solved using the third order Runge-Kutta method based on the linear combination of Arithmetic Mean, Heronian Mean and Contra Harmonic Mean. The solution of the proposed method is compared with the Arithmetic , Heronian and Contra Harmonic Mean methods. Hence the proposed method is suitable for solving hybrid fuzzy initial value problem.

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