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Cartesian Product of Path Semigraphs with 2 mid vertices and its a-Domination

D. Narmatha,
Department of Mathematics, Sri Ramakrishna Engineering College, Coimbatore.
K. P. Thilagavathy, Department of Mathematics, Kumaraguru College of Technology, Coimbatore.
N. Nithya Devi,
Department of Science and Humanities, Faculty of Engineering, Karpagam Academy of Higher Education, Coimbatore.
D. Indhumathy,
Department of Mathematics, Sri Ramakrishna Engineering College, Coimbatore.

ABSTRACT

The formation of more complex structures from the well-known simplest structures is a general way of thought in all endeavours, and the extension of the live properties of easiest structures to the toughest structures is an usual attempt. In this paper, a - domination number of the Cartesian product of elementary semigraphs with several edges and two middle vertices are discussed.

Keywords: Semigraph, Path Semigraph, Cartesian Product, Dominating set, Domination number.

1. Introduction

A $a-$ dominating set is that, subset C of A in which if for every $b \in A-C$ there exists $a \in C$ such that a and b are adjacent. The minimum cardinality of such a set C is called $a-$domination number of the semigraph P. It is denoted as $\gamma_a(P)$.

In 1990, S. T. Hedetniemi et.al [3] discussed some basic definitions of domination parameters. In 2003, E. S. S. Kamath and R. S. Bhat [2] studied domination in semigraphs. In [4, 5, 6] N. Murugesan and D. Narmatha studied domination number of Cartesian product of path semigraphs.

2. Definition

Consider two path semigraphs P_1 and P_2 with vertex set A_1 and A_2 and edge set B_1 and B_2 respectively. The Cartesian product of P_1 and P_2 ie., $P_1 \square P_2$ is defined as

$$P_1 \square P_2 = \left(A_1 \times A_2, B_1 \times B_2\right) \text{ such that } A_1 \times A_2 = \left\{(a_i, a_j) / a_i \in A_1, a_j \in A_2\right\} \text{ and}$$

i. Any vertex $a \in A_1$ and any edge $B = (b_1, b_2, ..., b_t)$ in B_2, $((a, b_1), (a, b_2), ..., (a, b_t))$ is an element of $B_1 \times B_2$ and also
ii. Any edge $B = (a_1, a_2, ..., a_r)$ in B_1 and for any vertex $b \in A_2$, $((a_1, b), (a_2, b), ..., (a_r, b))$ is an element of $B_1 \times B_2$.

Dominations in semigraphs was discussed in [1].

2.1 Theorem

$$\gamma_a\left[P_{s(1m(1))} \square P_{s(nm(1))}\right] = 3$$

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Proof:

Let $P_{s(1m(1))}$ be a path semigraph with single edge having only one middle vertex. When $n = 1$, $P_{s(nm(1))}$ becomes $P_{s(1m(1))}$.

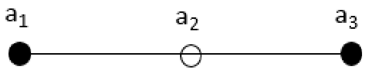

Fig. 2.1 $P_{s(1m(1))}$

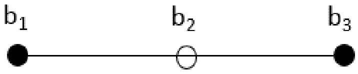

Fig. 2.2 $P_{s(1m(1))}$

The following figure represents $P_{s(1m(1))} \square P_{s(1m(1))}$

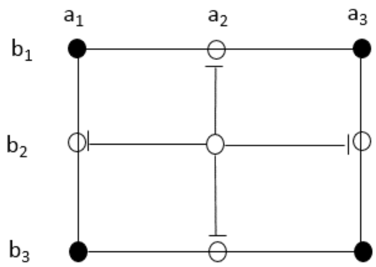

Fig. 2.3 $P_{s(1m(1))} \square P_{s(1m(1))}$

In the above figure, if we select any three vertices from each row otherwise in each column forms a minimal a-dominating set. i.e., from the above fig., the semigraph which contains minimum number of vertices that vertices are enough to dominate all the other vertices in that graph. Hence $\gamma_a [P_{s(1m(1))} \square P_{s(1m(1))}] = 3$.

Next put $n = 2$, $P_{s(1m(1))} \square P_{s(nm(1))}$ becomes $P_{s(1m(1))} \square P_{s(2m(1))}$.

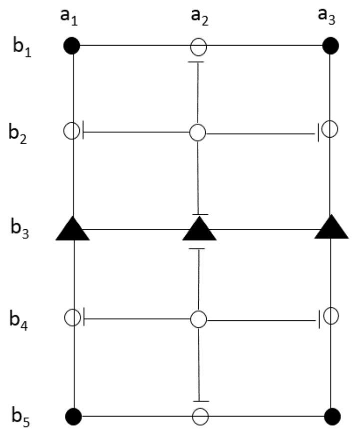

Fig.2.4 $P_{s(1m(1))} \square P_{s(2m(1))}$

From the above figure (triangles) it can be easily observed that $\gamma_a [P_{s(1m(1))} \square P_{s(2m(1))}] = 3$. Similarly $\gamma_a [P_{s(1m(1))} \square P_{s(nm(1))}] = 3$.

2.2 Note

i. $\gamma_a [P_{s(nm(1))} \square P_{s(1m(1))}] = 3$.

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ii. $\gamma_a\,[\,P_{s(nm(1))} \square P_{s(rm(1))}\,] = \gamma_a\,[\,P_{s(rm(1))} \square P_{s(nm(1))}\,] = r$, if $r < n$.

2.3 Lemma

i. $\gamma_a\,[\,P_{s(2m(1))} \square P_{s(2m(1))}\,] = 4$

ii. $\gamma_a\,[\,P_{s(2m(2))} \square P_{s(2m(1))}\,] = 4$

iii. $$\gamma_a\,[\,P_{s(2m(n))} \square P_{s(2m(1))}\,] = \begin{cases} \dfrac{6n}{3}+1 & if\;\; n=3p \\ \dfrac{6(n-1)}{3}+2 & if\;\; n=3p+1 \\ \dfrac{6(n-2)}{3}+4 & if\;\; n=3p+2 \end{cases}$$

where $p = 1,2,3.....$

Proof:

Consider a path semigraph with single edge having exactly two middle vertices, it is denoted as $P_{s(2m(1))}$. For calculating the minimal a-domination number for the Cartesian product graph $P_{s(2m(1))}$ and $P_{s(2m(1))}$, consider the above mentioned two graphs with labeling $a_i\,, i = 1,2,3,4$ and $b_j\,, j = 1,2,3,4$ as shown below.

$P_{s(2m(1))}$ $P_{s(2m(1))}$

Fig. 2.5 Single edge path semigraph with 2 middle vertices

$P_{s(2m(1))} \square P_{s(2m(1))}$ represents the Cartesian product of the above two graphs. It is also a graph containing the vertex set

$$V = \begin{Bmatrix} (a_1,b_1),(a_2,b_1),(a_3,b_1),(a_4,b_1),(a_1,b_2),(a_2,b_2),(a_3,b_2),(a_4,b_2),(a_1,b_3),(a_2,b_3), \\ (a_3,b_3),(a_4,b_3),(a_1,b_4),(a_2,b_4),(a_3,b_4),(a_4,b_4) \end{Bmatrix}$$

and edge set

$$E = \begin{Bmatrix} [(a_1,b_1),(a_2,b_1),(a_3,b_1),(a_4,b_1)],[(a_1,b_2),(a_2,b_2),(a_3,b_2),(a_4,b_2)], \\ [(a_1,b_3),(a_2,b_3),(a_3,b_3),(a_4,b_3)],[(a_1,b_4),(a_2,b_4),(a_3,b_4),(a_4,b_4)], \\ [(a_1,b_1),(a_1,b_2),(a_1,b_3),(a_1,b_4)],[(a_2,b_1),(a_2,b_2),(a_2,b_3),(a_2,b_4)], \\ [(a_3,b_1),(a_3,b_2),(a_3,b_3),(a_3,b_4)],[(a_4,b_1),(a_4,b_2),(a_4,b_3),(a_4,b_4)] \end{Bmatrix}$$

The following figure represents the Cartesian product graphs of the above figure.

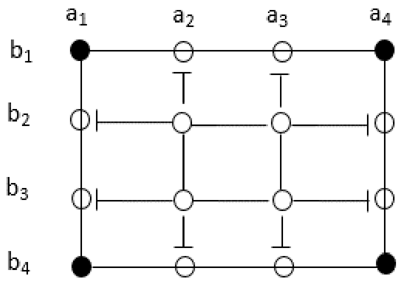

Fig. 2.6 $P_{s(2m(1))} \square P_{s(2m(1))}$ **Semigraph**

In the above semigraph $(a_1,b_1),(a_4,b_1),(a_1,b_4),(a_4,b_4)$ are end vertices, $(a_2,b_1),(a_3,b_1),(a_1,b_2),(a_4,b_2),(a_1,b_3),(a_4,b_3),(a_2,b_4),(a_3,b_4)$ are middle-end vertices and $(a_2,b_2),(a_3,b_2),(a_2,b_3),(a_3,b_3)$ are middle vertices.

From fig. 2 any one vertex taken in each row or any one vertex taken in each column ie., 4 vertices form a minimal a-dominating set.
$\therefore\ \gamma_a$ [$P_{s(2m(1))} \square P_{s(2m(1))}$] = 4.

Consider the path semigraphs

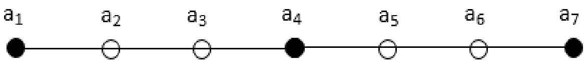

Fig. 2.7 $P_{s(2m(1))}$

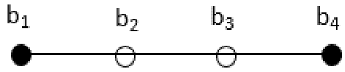

Fig. 2.8 $P_{s(2m(1))}$

The Cartesian product of the above two graphs is given below.

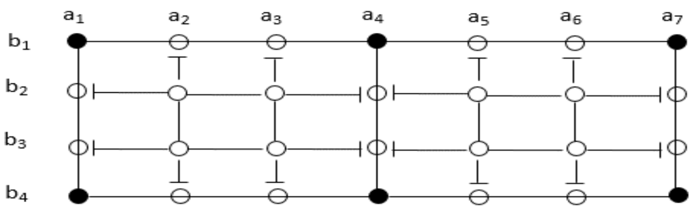

Fig. 2.9 $P_{s(2m(2))} \square P_{s(2m(1))}$

In the above graph the vertex set $\{(a_4,b_1),(a_4,b_2),(a_4,b_3),(a_4,b_4)\}$ form a minimal a- dominating set.

$\therefore\ \gamma_a$ [$P_{s(2m(2))} \square P_{s(2m(1))}$] = 4.

To prove (iii), let us assume $n = 3p,\ p = 1,2,3,4,....$ can be noted that the semigraph $P_{s(2m(2))} \square P_{s(2m(1))}$ is of order $36p+4$ and of size $21p+1$.

Consider the path semigraph with 2 edges and 4 middle vertices.

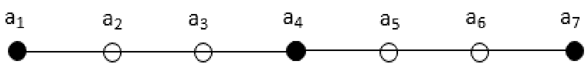

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Fig. 2.10 $P_{s(2m(2))}$ Semigraph

In fig.2.10 the vertex a_4 dominates the adjacent vertices a_1, a_2, a_3, a_5, a_6, a_7. It is noted that the grid $P_{s(2m(2))} \square P_{s(2m(1))}$ containing 40 vertices with exactly 4 copies of $P_{s(2m(2))}$. Therefore $(a_4,b_1),(a_4,b_2),(a_4,b_3),(a_4,b_4)$ are the exactly 4 vertices dominating the other adjacent vertices in that edge. Also the semigraph $P_{s(2m(n))} \square P_{s(2m(1))}$, $n = 3p, p = 1,2,3,4,....$ containing p copies of $P_{s(2m(2))} \square P_{s(2m(1))}$. Hence the set $U = \{(a_i, b_j)/ i = 4,13,22,...(9p-5), p = 1,2,....j = 1,2,3,4\}$

with 4k (may be end or middle-end) vertices construct a minimal a-dominating set which dominates all the other vertices in $P_{s(2m(n))} \square P_{s(2m(1))}$ apart from $(a_{9t-1},b_1),(a_{9t-1},b_2),(a_{9t-1},b_3),(a_{9t-1},b_4)$ and $(a_{9t},b_1),(a_{9t},b_2),(a_{9t},b_3),(a_{9t},b_4)$, $t = 1,2,3,4,......,p$ vertices and the vertices $(a_{9r+1},b_1),(a_{9r+1},b_2),(a_{9r+1},b_3),(a_{9r+1},b_4)$. Note that for all $t = 1,2,3,4,......,p$ the vertices $(a_{9t-1},b_1),(a_{9t-1},b_2),(a_{9t-1},b_3),(a_{9t-1},b_4)$ form an edge E_{9t-1} (say) and $(a_{9t},b_1),(a_{9t},b_2),(a_{9t},b_3),(a_{9t},b_4)$ form an edge E_{9t} (say) with $(a_{9t},b_1),(a_{9t},b_4),(a_{9t-1},b_1),(a_{9t-1},b_4)$ middle-end vertices and $(a_{9t},b_2),(a_{9t},b_3),(a_{9t-1},b_2),(a_{9t-1},b_3)$ middle vertices in which any one vertex from the edge E_{9t-1} and one vertex from the edge E_{9t} dominates all the other vertices in that edge. Hence p vertices must be taken i.e., any one vertex from each edge to dominates the vertices $(a_{9t-1},b_1),(a_{9t-1},b_2),(a_{9t-1},b_3),(a_{9t-1},b_4)$ and $(a_{9t},b_1),(a_{9t},b_2),(a_{9t},b_3),(a_{9t},b_4)$, $t = 1,2,3,4,......,p$. At the end if we select only one vertex from $E_{9r+1} = ((a_{9r+1},b_1),(a_{9r+1},b_2),(a_{9r+1},b_3),(a_{9r+1},b_4))$ the corresponding set containing 6p+1 vertices, where $n = 3p$ which is a minimal a-dominating set in $P_{s(2m(n))} \square P_{s(2m(1))}$. Hence $\gamma_a [P_{s(2m(n))} \square P_{s(2m(1))}] = 6\left(\frac{n}{3}\right) + 1$ if $n = 3k$.

Next, $n = 3p+1,\ p = 1,2,3,4,....$ The Cartesian product graph $P_{s(2m(n))} \square P_{s(2m(1))}$ when $n = 3p+1$ contains all the vertices of $P_{s(2m(n))} \square P_{s(2m(1))}$ when $n = 3p$ and also the vertices $(a_{9r+2},b_1),(a_{9r+2},b_2),(a_{9r+2},b_3),(a_{9r+2},b_4),(a_{9r+3},b_1),(a_{9r+3},b_2),(a_{9r+3},b_3),(a_{9r+3},b_4),$

$(a_{9r+4},b_1),(a_{9r+4},b_2),(a_{9r+4},b_3),(a_{9r+4},b_4)$. Hence for selecting vertices from the edges $E_{9r+1} = ((a_{9r+1},b_1),(a_{9r+1},b_2),(a_{9r+1},b_3),(a_{9r+1},b_4))$, the corresponding set form a minimal a-dominating set. Hence $\gamma_a [P_{s(2m(n))} \square P_{s(2m(1))}]$ when $n = 3p+1$ is $\gamma_a [P_{s(2m(n))} \square P_{s(2m(1))} + 1]$.

Therefore $\gamma_a [P_{s(2m(n))} \square P_{s(2m(1))}] = 6p + 1 + 1 = 6p + 2 = 6\left(\frac{n-1}{3}\right) + 2$.

At the end, let $n = 3p+2$. It can be observed that $\gamma_a [P_{s(2m(n))} \square P_{s(2m(1))}]$ is same for $n = 3p, 3p+1, 3p+2$ from th edge $E_1 = ((a_1,b_1),(a_1,b_2),(a_1,b_3),(a_1,b_4))$ to $E_{9p-2} = ((a_{9p-3},b_1),(a_{9p-3},b_2),(a_{9p-3},b_3),(a_{9p-3},b_4))$. For various values of n, γ_a changes based on the remaining edges. The list of remaining edges is given below.

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S.No.	n	Edges	Minimal a- dominating vertices
1	$3p$	$E_{9p-4}, E_{9p-3}, E_{9p-2}, E_{9p-1}, E_{9p}, E_{9p+1}$	$(a_{9p-1}, b_4), (a_{9p}, b_4), (a_{9p+1}, b_4)$
2	$3p+1$	$E_{9p-4}, E_{9p-3}, E_{9p-2}, E_{9p-1}, E_{9p}, E_{9p+1}, E_{9p+2}, E_{9p+3}, E_{9p+4}$	$(a_{9p+1}, b_1), (a_{9p+1}, b_2), (a_{9p+1}, b_3), (a_{9p+1}, b_4)$
3	$3p+2$	$E_{9p-4}, E_{9p-3}, E_{9p-2}, E_{9p-1}, E_{9p}, E_{9p+1}, E_{9p+2}, E_{9p+3}, E_{9p+4}, E_{9p+5}, E_{9p+6}, E_{9p+7}$	$(a_{9p-1}, b_4), (a_{9p}, b_4), (a_{9p+4}, b_1), (a_{9p+4}, b_2), (a_{9p+4}, b_3), (a_{9p+4}, b_4)$

Table: Minimal a-dominating vertices

Therefore from the second and third row of the above table, it can be easily understood that, when n increases by one, γ_a increases by two.

Therefore γ_a [$P_{s(2m(n))} \square P_{s(2m(1))}$] ,when $n = 3p + 2$ is

γ_a [$P_{s(2m(n))} \square P_{s(2m(1))}$ +2]= $6p + 2 + 2 = 6p + 4 = 6\left(\frac{n-2}{3}\right) + 4$. Hence the lemma.

Conclusion

In this research work, γ_a [$P_{s(2m(n))} \square P_{s(2m(1))}$] was discussed briefly.

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