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Application Of Sign Distance Ranking Technique In Hexagonal Intuitionistic Fuzzy Transporation Problem

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ABSTRACT: We propose the Sign Distance Ranking technique of Hexagonal Intuitionistic Fuzzy Number in this paper. This technique is used for solving the Intuitionistic Fuzzy Transportation Problem (IFTP) with the Hexagonal Intuitionistic Fuzzy Number (HIFN) as the membership and non-membership functions. Here, The HIFN are transformed to crisp data by a Sign Distance Ranking technique. And from this ranking crisp data, the initial and the optimal solution of the IFTP are obtained. The efficiency of this technique is illustrated with two numerical examples.

Keywords: Intuitionistic Fuzzy Number, Hexagonal Intuitionistic Fuzzy Number, Intuitionistic Fuzzy Transportation Problem, Sign Distance, Ranking Function

1. INTRODUCTION

The basic transportation model on crisp values with the transportation constraints was introduced in the year 1941. But nowadays, the transportation parameters like demand, supply and unit transportation cost are uncertain because of several uncontrolled reasons. Due to this, fuzzy and intuitionistic fuzzy transportation problem were formulated and solved by many researchers.O'heigeartaigh [6]was who proposed a method to solve the fuzzy transportation problem with fuzzy demand and supply. Lofti A. Zadeh [10] introduced the Fuzzy Set (FS) with the membership degree (μ) between 0 and 1. Later, Krasssimir T. Atanassov [2,3] proposed the Intuitionistic Fuzzy Set (IFS) with respect to both membership ($\mu \epsilon[0,1]$) and non-membership ($\theta \epsilon [0,1]$) such that $\mu + \theta <=1$.As the IFS represent the uncertainty more, it is suitable to handle problems with imprecision information nowadays.

As, in decision making problems, the ranking of intuitionistic fuzzy numbers plays an important role in comparison analysis referred by S. K. Bharati [5]. Intuitionistic Fuzzy Transportation Problem with the Hexagonal Intuitionistic Fuzzy Number was introduced by A.Thamariselvi& R.Santhi [7]. Also,G.Uthra [8] proposed the Generalized Intuitionistic Pentagonal, Hexagonal and Octagonal Fuzzy Numbers defined and a new Ranking formula by finding the area of the centroid from the origin method. Later,A. Anju [1] presented a solution for fractional transportation problem where the cost functions are hexagonal intuitionistic fuzzy numbers by the method of Ranking and Russell's. The distinctive representation, ranking and defuzzification technique of Hexagonal fuzzy number was illustrated by Avishek [4].

As the Sign Distance Ranking technique solves the Intuitionistic Fuzzy Transportation Problem (IFTP) in which the costs, supplies and demands are of the triangular intuitionistic fuzzy numbers was introduced earlier [9] andthis paper is an attempt to introduce the sign distance ranking technique to solve theIntuitionistic Fuzzy Transportation Problem (IFTP) with HexagonalIntuitionistic Fuzzy Number (HIFN).

The organization of this paper is as follows: In section 2, Fuzzy sets, Intuitionistic Fuzzy sets, Fuzzy Number, Intuitionistic Fuzzy Number, Hexagonal Intuitionistic Fuzzy Number, Ranking of Fuzzy and Intuitionistic Fuzzy Number are explained. The Intuitionistic Fuzzy Transportation problem and the procedure of the Sign Distance Ranking Technique in HIFN are proposed in the section 3. The section 4, analyses the numerical examples of IFTP underHIFN.

2. PRELIMINARIES

Definition : 2.1
Let X be a nonempty set. A fuzzy set $\bar{A}$ of X is defined as $\bar{A}= \{< x, \mu_{\bar{A}}(x) > / x \epsilon X\}$, where $\mu_{\bar{A}}(x)$ is called membership function and it maps each element of X to a value between 0 and 1.

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Definition : 2.2
A fuzzy number is a generalized of a regular real number. It does not refer to a single value but rather to a connected set of possible value, where each possible value has its weight between 0 and 1. The weight is called the membership function.

A fuzzy number $\bar{A}$ is a convex normalized fuzzy set on the real line R such that there exist at least one x ϵ R with $\mu_{\bar{A}}(x) = 1$ and $\mu_{\bar{A}}(x)$ is piece wise continuous.

Definition : 2.3
A hexagonal fuzzy number $\bar{A}_H$ is specified by 6 tuples$\bar{A}_{H}=(a_1, a_2, a_3, a_4, a_5, a_6)$
where a_1, a_2, a_3, a_4, a_5 and a_6 are real number and $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6$ with membership function are given below

$$\mu_{\bar{A}^I}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & \text{for } a_2 \le x \le a_3 \\ 1 & \text{for } a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & \text{for } a_5 \le x \le a_6 \\ 0 & \text{for otherwise} \end{cases}$$

Definition : 2.4
Let X is non empty. An intuitionistic fuzzy set $\bar{A}^I$ of X is defined as,
$\bar{A}^I = \{ < x, \mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x) > / x \in X \}$. where $\mu_{\bar{A}^I}(x)$ and $\vartheta_{\bar{A}^I}(x)$ are the membership and the non-membership function such that $\mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x) : X \to [0,1]$ and
$0 \le \mu_{\bar{A}^I}(x) + \vartheta_{\bar{A}^I}(x) \le 1$ for all x ∈ X.

Definition : 2.5
An intuitionistic fuzzy subset $\bar{A}^I = \{ < x, \mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x) > / x \in X \}$ of the real line R is called an intuitionistic fuzzy number, if the following condition hold,

(i) There exist m in R such that $\mu_{\bar{A}^I}(m) = 1$ and $\vartheta_{\bar{A}^I}(m) = 0$
(ii) $\mu_{\bar{A}^I}$ is a continuous function from R→ [0,1] such that $0 \le \mu_{\bar{A}^I}(x) + \vartheta_{\bar{A}^I}(x) \le 1$ for all $x \in X$.

Definition : 2.6
A hexagonal intuitionistic fuzzy number is specified by
$\bar{A}_H^I = (a_1, a_2, a_3, a_4, a_5, a_6)(a_1', a_2', a_3, a_4, a_5', a_6')$ where
$a_1, a_2, a_3, a_4, a_5, a_6, a_1', a_2', a_3, a_4, a_5', a_6'$ are real number $a_1' \le a_1 \le a_2' \le a_2 \le a_3 \le a_4 \le a_5 \le a_5' \le a_6 \le a_6'$ and its membership and non membership function are given below

$$\mu_{\bar{A}^I}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & \text{for } a_2 \le x \le a_3 \\ 1 & \text{for } a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & \text{for } a_5 \le x \le a_6 \\ 0 & \text{for otherwise} \end{cases}$$

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$$\vartheta_{\bar{A}^{I}}(x)=\begin{cases}1-\frac{1}{2}\left(\frac{x-a_1'}{a_2'-a_1'}\right) & \text{for } a_1'\le x\le a_2'\\ \frac{1}{2}\left(\frac{a_3-x}{a_3-a_2'}\right) & \text{for } a_2'\le x\le a_3\\ 0 & \text{for } a_3\le x\le a_4\\ \frac{1}{2}\left(\frac{x-a_4}{a_5'-a_4}\right) & \text{for } a_4\le x\le a_5'\\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_5'}{a_6'-a_5'}\right) & \text{for } a_5'\le x\le a_6'\\ 1 & \text{for otherwise}\end{cases}$$

Definition : 2.7

Let a,b € ℝ be two real numbers, then sign distance between a and b, is defined as D(a, b) = a-b.
As, D(a,0) = a and D(b,0) = b, we have D(a,b) =D(a,b) + D(b,0)

(i) If a> 0, D(a,0) = a <=> a is the right – hand side of 0 with sign distance a.
(ii) If a< 0,D(a,0) = a <=> a is the left-hand side of 0 with sign distance -a.

Definition : 2.8

Let a, b € ℝ, then ranking function for real numbers a, b is defined as

i. D (a, b) > 0 ↔ D (a,0) > D(b,0) ↔ a > b
ii. D (a, b) < 0 ↔ D (a,0) < D(b,0) ↔ b<a
iii. D (a, b) = 0 ↔ D(a,0) = D(b,0) ↔ b=a

Definition : 2.9

Let $\bar{A}^I=\{(x,\mu_{\bar{A}}(x),\ \vartheta_{\bar{A}}(x)): x\epsilon\mathbb{R}\}$and $\bar{A}^{\,I}=\{(x,\mu_B(x),\vartheta_B(x)): x\epsilon\mathbb{R}\}$ be two intuitionistic fuzzy number, the distance between two intuitionistic fuzzy is given by $D^S(\bar{A}^I,B^I)$ =

$$\frac{1}{4}\left[\int_0^1\left(\bar{A}^{I}{}_L^{+}(\alpha)-B_L^{+}(\alpha)\right)d\alpha+\int_0^1\left(\bar{A}^{I}{}_U^{+}(\alpha)-B_U^{+}(\alpha)\right)d\alpha+\int_0^1\left(\bar{A}^{I}{}_L^{-}(\alpha)-B_L^{-}(\alpha)\right)d\alpha+\int_0^1\left(\bar{A}^{I}{}_U^{-}(\alpha)-B_U^{-}(\alpha)\right)d\alpha\right]$$

3. SIGN DISTANCE RANKING OF HEXAGONAL INTUITIONISTIC FUZZY NUMBER

Let $\bar{A}_{HEXA}=((a_1,a_2,a_3,a_4,a_5,a_6),(b_1,b_2,a_3,a_4,b_5,b_6))$ be hexagonal intuitionistic fuzzy number. Then sign distance of A can be calculated as

$$D^S(\bar{A}_{HEXA})=\frac{1}{4}\left[\int_0^1\left\{\overline{A}^{I^+}{}_L(\alpha)\right\}d\alpha+\int_0^1\left\{\overline{A}^{I^+}{}_U(\alpha)\right\}d\alpha+\int_0^1\left\{\overline{A}^{I^-}{}_L(\alpha)\right\}d\alpha+\int_0^1\left\{\overline{A}^{I^-}{}_U(\alpha)\right\}d\alpha\right]$$

$$D^S(\bar{A}_{HEXA})=\left(\frac{a_1+a_2+4a_3+4a_4+a_5+a_6+b_1+b_2+b_5+b_6}{16}\right)$$

Let $((a_1,a_2,a_3,a_4,a_5,a_6),(b_1,b_2,a_3,a_4,b_5,b_6))$ be the hexagonal intuitionistic fuzzy number.FromS. K. Bharati [3], The Sign Distance Ranking of Hexagonal Intuitionistic Fuzzy Number is defined as

$$R\,(\bar{A}_{\text{HEXA}})=\left(\frac{a_6-a_1}{b_6-b_1}\right)D^S(\bar{A}_{\text{HEXA}})$$

where, $D^S(\bar{A}_{HEXA})=\left(\frac{a_1+a_2+4a_3+4a_4+a_5+a_6+b_1+b_2+b_5+b_6}{16}\right)$

4. HEXAGONAL INTUITIONISTIC FUZZY TRANSPORTATION PROBLEM

As the Transportation problem is associated with day-to-day activities in our real life.Transportation

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Problem helps insolving problem on distribution and transportation of resources from one place toanother and
themainobjectiveistominimizethetransportationcostpossiblebysatisfyingthedemandatdestinationfromsupply constraint. But in present situation, due to severaluncontrolled reasons, the transportation parameters like demand, supply and unittransportationcostareuncertain.Hence,thefuzzyandintuitionisticfuzzytransportation problem were formulated and solved. Here, we consider the IntuitionisticFuzzyTransportationProblem (IFTP) of Hexagonal Intuitionistic Fuzzy Number (HIFN) and intend to describe a methodology and solve.

The Intuitionistic FuzzyTransportation Problem with m origins (rows) and n destinations (columns) is considered with C_{ij}, the cost of transporting one unit of the product from i^{th} origin to j^{th}destination.

$\overline{a}^{HEXA}{}_{i} = \left(a_i^1, a_i^2, a_i^3, a_i^4, a_i^5, a_i^6\right)\left(a_i^{1'}, a_i^{2'}, a_i^3, a_i^4, a_i^{5'} a_i^{6'}\right)$ be the quantity of commodity available at origin i. of HIFN
$\overline{b}^{HEXA}{}_{j} = \left(b_j^1, b_j^2, b_j^3, b_j^4, b_j^5, b_j^6\right)\left(b_j^{1'}, b_j^{2'}, b_j^3, b_j^4, b_j^{5'}, b_j^{6'}\right)$the quantity of commodity needed at intuitionistic fuzzy destination j of HIFN
$\overline{x}^{HEXA}{}_{ij} = \left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6\right)\left(x_{ij}^{1'}, x_{ij}^{2'}, x_{ij}^3, x_{ij}^4, x_{ij}^{5'}, x_{ij}^{6'}\right)$is the quantity transported from i^{th} origin to j^{th}destination, and using these the minimization of theHIFN transportation cost is formulated as follows,

$$Minimum\ \overline{Z}^{HEXA} = \sum_{i=1}^{m}\sum_{j=1}^{n} \overline{C}^{I}{}_{ij} \otimes \overline{x}^{HEXA}{}_{ij}$$

Subject to, $$\sum_{j=1}^{n} \overline{x}^{HEXA}{}_{ij} = \overline{a}^{HEXA}{}_{i} \quad for\ i = 1,2,\dots,m$$

$$\sum_{j=1}^{n} \overline{x}^{HEXA}{}_{ij} = \overline{b}^{HEXA}{}_{j}\ for\ j = 1,2,\dots,n$$

$$\left(x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4, x_{ij}^5, x_{ij}^6\right)\left(x_{ij}^{1'}, x_{ij}^{2'}, x_{ij}^3, x_{ij}^4, x_{ij}^{5'}, x_{ij}^{6'}\right) \geq \overline{0}^{I}$$

$for\ i = 1,2,\dots,m$ and $for\ j = 1,2,\dots,n$ where m is the number of supplies and n is the number of demands.

Methodology

Step1: In Intuitionistic Fuzzy Transportation Problem (IFTP) with Hexagonal Intuitionistic Fuzzy Number (HIFN), the HIFN quantities are reduced into an integer using the sign distance ranking number.
Step 2 :VAM method is used to find the initial basic feasible solution
Step 3 : MODI method is applied to find the optimal solution.

NUMERICAL EXAMPLES

Example 3.1:
We consider 3X4 Intuitionistic Fuzzy Transportation Problem (IFTP) with supplies and demands as the Hexagonal Intuitionistic Fuzzy Number (HIFN).

	D1	D2	D3	D4	Supply
S1	5	6	12	9	(7,9,11,13,16,20) (5,7,11,13,19,23)
S2	3	2	8	4	(6,8,11,14,19,25) (4,7,11,14,21,27)
S3	7	11	20	9	(9,11,13,15,18,20) (8,10,13,15,19,22)
Demand	(3,4,5,6,8,10) (2,4,5,6,10,12)	(3,5,7,9,11.15) (2,4,7,9,13,17)	(6,7,9,11,13,16) (5,6,9,11,16,18)	(10,12,14,16,20,24) (8,10,14,16,20,25)	

SOLUTION:

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Here, we reduce the Hexagonal Intuitionistic Fuzzy Number (HIFN) to a crisp data by the sign distance ranking measure, found to be an Unbalanced Transportation Problem

	D1	D2	D3	D4	SUPPLY
S1	5	6	12	9	9.1
S2	3	2	8	4	11.2
S3	7	11	20	9	11.3
DEMAND	4.2	6.9	8	13.3	

STARTING TABLE : This is obtained by introducing the S4 Origin for balancing the Intuitionistic Fuzzy Transportation Problem (IFTP)

	D1	D2	D3	D4	SUPPLY
S1	5	6	12	9	9.1
S2	3	2	8	4	11.2
S3	7	11	20	9	11.3
S4	0	0	0	0	0.8
DEMAND	4.2	6.9	8	13.3	

INITIAL SOLUTION:Using the VAM method, the initial solution obtained is represented in the following table

	D1		D2		D3		D4		SUPPLY
S1	5		6	1.9	12	7.2	9		9.1
S2	3		2		8		4	11.2	11.2
S3	7	4.2	11	5	20		9	2.1	11.3
S4	0		0		0	0.8	0		0.8
DEMAND	4.2		6.9		8		13.3		

Initial solution cost is 245.9. By MODI Method the OPTIMAL SOLUTION of the IFTP is given below and by checking the optimality for all dij> 0, the solution is found to be optimum and unique as 194.7 Rs

	D1		D2		D3		D4		SUPPLY
S1	5	4.2	6		12	4.9	9		9.1
S2	3		2	6.9	8	2.3	4	2	11.2
S3	7		11		20		9	8.3	11.3
S4	0		0		0	0.8	0		0.8
DEMAND	4.2		6.9		8		13.3		

Example 3.2:
In this example, We consider a 3X3 IFTP withcost values as the Hexagonal Intuitionistic Fuzzy Number.

	D1	D2	D3	Supply
S1	(2,4,5,7,10,11) (1,3,5,7,12,13)	(3,4,5,6,8,10) (2,4,5,6,10,13)	(7,9,11,13,16,20) (5,7,11,13,19,24)	15
S2	(6,8,11,14,19,25) (4,7,11,14,21,26)	(9,11,13,15,18,20) (8,10,13,15,19,22)	(10,12,14,16,20,24) (8,10,14,16,20,25)	20
S3	(3,5,6,8,9,10) (2,3,6,8,11,13)	(4,7,9,12,15,17) (3,6,9,12,16,18)	(10,12,14,16,20,24) (8,10,14,16,20,25)	35
Demand	25	35	10	

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SOLUTION:
Using the sign distance ranking measure, the hexagonal intuitionistic fuzzy number (HIFN) is reduced to a crisp data.

	D1	D2	D3	Supply
S1	4.9	4.97	8.7	15
S2	11.7	11.3	12.8	20
S3	4.5	9.2	12.8	35
Demand	25	35	10	

SOLUTION TABLE:
As the IFTP is a balanced one, we use VAM for the initial feasible basic solution determination and found the initial cost as 520. Here,the optimal solution also remains the same from the MODI method

	D1		D2		D3		Supply
S1	4.9		4.97	15	8.7		15
S2	11.7		11.3	10	12.8	10	20
S3	4.5	25	9.2	10	12.8		35
Demand	25		35		10		

5. CONCULSION

As far in this paper, we have discussed the initial and the optimal solution for Intuitionistic Fuzzy Transportation Problem (IFTP)of theHexagonal Intuitionistic Fuzzy Number (HIFN) with the Sign Distance Ranking technique. We have solved the few problems on IFTP with HIFP and heading to discuss more about it future.

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