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Skolem Difference Odd Geometric Mean Labeling of Path and Cycle Related Graphs

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ABSTRACT
A function f is called a Skolem Difference Odd Geometric Mean labeling for the graph G(V,E) with p vertices and q edges, if it is possible to label the vertices $x \in V$ with different labeling f(x) from 1,3,5…….2q+1 such that the induced map $f^*: E(G) \to \{1,3,5 \ldots .2q-1\}$ defined by $f^*(e = uv) = \left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ or $\left\lceil \sqrt{f(u)f(v)} \right\rceil$ $\forall$ uv $\in E(G)$ is bijection. The graph which admits the Skolem Difference Odd Geometric Mean labeling is called Skolem Difference odd Geometric Mean graph. In this paper we investigate Skolem Difference odd Geometric mean labeling of some path and cycle related graphs.

Keywords: Skolem Difference odd Geometric mean labeling Cycle, Y-Tree, F-Tree, and comb graph.
AMS (2010): 05C78

1. Introduction:

Graph considered here are simple, finite and undirected graphs. For notations and terminology we follow [1]. In [2] Somasundaram and Ponraj introduced Mean labeling for some standard graphs in 2003.[3] The concept of Geometric Mean labeling has been introduced by S.Somasundaram, R.Ponraj and P.Vidhyarani in 2011. .[4] R.Vasuki, J.Venkateswari and G.Pooranam, introduced the Skolem Difference Odd Mean Labeling of Some Simple Graphs in 2015.[5] G.Muppidathi Sundari and K.Murugan introduced the concept of extra skolem difference mean labeling of some graphs.

Definition 1.1:

A graph with p vertices and q edges is said to be a Skolem Difference odd Geometric mean labeling if it is possible to label the vertices $x \in V$ with different labeling f(x) from 1,3,5…….2q+1 such that the induced map $f^*: E(G) \to \{1, 3, 5, \ldots .2q-1\}$defined as
$f^*(e=uv) = \left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ or $\left\lceil \sqrt{f(u)f(v)} \right\rceil$, is bijective. A graph that admits a skolem difference odd geometric mean labeling is called a skolem difference odd geometric mean graph.

Definition 1.2:

A cycle is a closed walk in which all vertices are distinct, except the last and the first.

Theorem 1.3:

The graph obtained by identifying a vertex of any two cycle C_m and C_n is a skolem difference odd geometric mean graph.

Proof:

Let $u_1\, u_2 \ldots .\, u_m$ and $v_1\, v_2 \ldots .\, v_n$ be a vertices of a cycle C_m and C_n respectively. Let G be a resultant graph obtained by identifying the vertex u_m of cycle C_m to the vertex v_n of cycle C_n

Define a function:

$f: V(G) \to \{1, 3, 5, \ldots .2(m+n)+1\}$ by

$f(u_i) = \{\, 2i-1 \qquad 1 \le i \le m-1$
$\qquad\quad 2i+1 \qquad i = m\}$

$f(v_i) = 2m+2i-1 \qquad 1 \le i \le n-1$

$f(v_n) = 2(m+n)+1$

Edge labels are

$f(u_1\, u_2) = 1$

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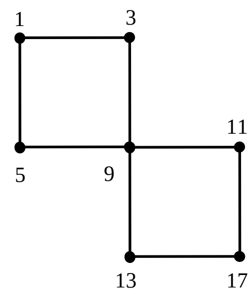
$f(u_i\ u_{i+2}) = 2i+1 \quad 1 \leq i \leq m-2$
$f(u_{m-1}\ u_m) = 2m-1$
$f(v_1 v_2) = f(u_m)$
$f(v_i v_{i+2}) = 2m+2i+1 \quad 1 \leq i \leq n-2$
$f(v_{n-1} v_n) = 2(m+n) - 1$

Edge labels are distinct

Example1.4:



Theorem 1.5:

$G_{m,n}$ is a connected g phs.

Proof:

In $G_{m,n}$, $u_1\ u_2\ ….\ u_n$ graph. $u_1\ u_2\ ….\ u_n$ be the vertices in the cycle. Second Cycle graph connected to the $_n$. 3rd Cycle graph connected to the 2nd Cycle graph at the vertex u_{2n}. In general, the kth Cycle graph connected to the (k-1)th graph at the vertex u_{nk}. Thus the graph has m copies of C_n graphs.

Define a function:

$f: V(G_{m,n}) \rightarrow \{1, 3, 5, ….2mn+1\}$ by

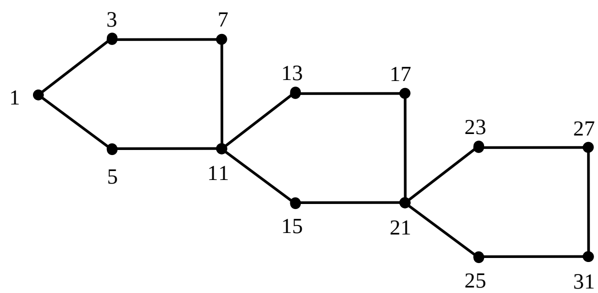
$f(u_i) = \{2i-1 \quad 1 \leq i \leq n-1$
$\quad 2i+1 \quad i = n, n+1, n+2….., 2n, 2n+1,….mn\ \}$

Edge labels are

$f(u_1\ u_2) = 1$
$f(u_i\ u_{i+2}) = 2i+1 \quad 1 \leq i \leq n-2$
$f(u_{n-1}\ u_n) = 2mn-1$

Edge labels are distinct

Example1.6:



Definition1.7:

A Y- tree is attained from a path P_n by attaching a pendant vertex to the (n-1)th vertex of P_n . Y tree on n+1 vertices is denoted by $Y_{n,.}$

Theorem 1.8:

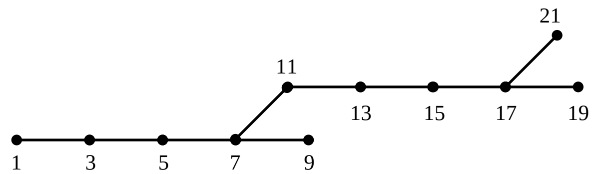
$G_{m,n}$ is a connected graph whose m blocks are Y_n graphs.

Proof :

In $G_{m,n}$, $u_1\ u_2\ ….\ u_n$ and v_1 be the vertices of first Y graph.$u_1\ u_2\ ….\ u_n$ be the vertices in the path and v_1 be the pendant vertices. Second Y graph connected to the first Y graph at the vertex v_1. 3rd Y graph connected to the 2nd Y graph at the vertex v_2.In general, the kth Y graph connected to the pendant vertex of (k-1)th graph. Thus the graph has m copies of Y_n graphs.

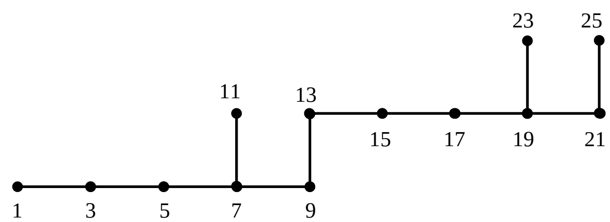
Define a function:

$f: V(G_{m,n}) \rightarrow \{1, 3, 5, \ldots .2mn+1\}$ by
$f(u_i) = 2i-1 \qquad 1\leq i \leq n$
$f(v_i) = 2ni + 1 \qquad 1\leq i \leq m$
$f(u_{i+1}) = f(u_i) + 2 \qquad i = n+1, n+2, \ldots\ldots. 2n-1, 2n, 2n+1, \ldots mn-1$
Edge labels are distinct.
Example1.9:



Definition 1.10:
A F- tree o y attaching exactly two pendant vertices of P_n, $(n-1)^{th}$

Theorem1.11:
$G_{m,n}$ is a connected graph whose m blocks are F_n graphs.
Proof:
In $G_{m,n}$, $u_1, u_2, \ldots. u_n$ & v_1, v_2 be the vertices of first F graph. $u_1, u_2 \ldots. u_n$ be the vertices in the path and v_1 & v_2 be the pendant vertices. Second F graph connected to the first F graph at the vertex v_2. In general, k^{th} F graph is connected to the 2^{nd} pendant vertex of $(k-1)^{th}$ graph. The graph has m copies of F_n graphs.
Define a Function:
$f : V(G_{m,n}) \rightarrow \{1, 3, 5, \ldots .2m(n+1)+1\}$ by
$f(u_i) = 2i-1 \qquad 1\leq i \leq n$
$f(v_1) = 2n+1$
$f(v_{2i+1}) = f(v_{2i-1}) + 2n+2, \qquad 1\leq i \leq m-1$
$f(u_{i+1}) = f(u_i) + 4, \qquad i=n, 2n, 3n \ldots.mn.$
$f(u_{i+1}) = f(u_i) + 2, \qquad i=n+1, n+2, n+3, \ldots.2n-1, 2n+1, 2n+2, \ldots..mn-1.$
The edge labels are distinct.
Example1.12:



Definition1.13:
The graph obtained by joining a single pendant edge to each vertex of a path is called a Comb.

Theorem1.14:
In $G_{m,n}$ is a connected graph. Whose m blocks are $P_n \Theta K_1$ graph.
Proof:
In $G_{m,n}$, $P_n \Theta K_1$ be a comb obtained from a path $P_n = u_1 u_2 \ldots.. u_n$ by joining a pendant vertex v_i $(1\leq i \leq n)$ to each vertex of P_n. $G_{m,n}$ is a connected graph whose m blocks are n copies of the comb graph $P_n \Theta K_1$. The graph $G_{m,n}$ has m(2n-1)+1 vertices and m(2n-1) edges. The 2^{nd} comb graph connected to the 1^{st} comb graph at the vertex v_n. The 3^{rd} comb graph connected to the 2^{nd} comb graph at the vertex v_{2n}. In general, the k^{th} comb graph is connected to the $(k-1)^{th}$ copy of comb graph at the vertex $v_{(k-1)n}$.
Define a function:
$f : V(G_{m,n}) \rightarrow \{1, 3, 5, \ldots .2m(2n-1)+1\}$ by
$f(u_i) = 4i - 3 \qquad 1\leq i \leq n$
$f(v_i) = 4i-1 \qquad 1\leq i \leq n$
$f(u_{i+1})=\{ f(u_i) + 2 \qquad i= 2n,3n,4n\ldots.mn$

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$f(u_i)+4$ $\quad$ $i= n+1,n+2,\ldots 2n-1,2n+1,2n+2....mn-1\}$

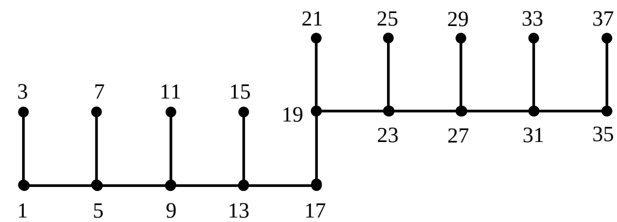
$f(v_{n+1})= 4n+1$

$f(v_{i+1})=\{ f(v_i)+2$ $\quad$ $i= 2n,3n,4n\ldots.mn$

$f(v_i)+4$ $\quad$ $i= n+1,n+2,\ldots 2n-1,2n+1,2n+2\ldots..mn-1\}$

Then the edge labels are distinct.

Example1.15:



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