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INTUITIONISTIC FUZZY IN NANO TOPOLOGICAL SPACES

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Abstract: *The scope of this paper is to give introduction and basic on intuitionistic fuzzy nano topology, intuitionistic fuzzy nano topological spaces, Basic properties of intuitionistic fuzzy nano open sets and intuitionistic fuzzy nano closed sets are studied.*

Keywords and Phrases: *Lower approximation, Upper approximation, Boundary region, Equivalence relation, Intuitionistic fuzzy set, Intuitionistic fuzzy topology and Intuitionistic fuzzy Nano-open sets, Intuitionistic fuzzy Nano-closed sets, Intuitionistic fuzzy Nano-closure and Intuitionistic fuzzy Nano interior in topological spaces.*

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1.Introduction

The concept of nano topology was first introduced by Lellis Thivagar[5] in which the terms of approximations and boundary region and equivalence relation of a subset are discussed and defined the nano closed sets, nano interior and nano-closure in topological space. Fuzzy set was proposed by Zadeh[10] in 1965, it shows a degree of membership for each member of the set to a subset of it. The

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degree of non-membership is added to fuzzy set, Atanassov [1] proposed intuitionistic fuzzy set in 1986 which deals with accuracy to uncertainty, related on the existing proofs and results. In this paper, we use the concept of intuitionistic nano fuzzy set and space. Engineering education exemplifies that mathematics plays a pivotal role in the development of the configure domain of an individual

2.Preliminaries

Definition 2.1.[5] The set valued information is defined in quadruple $S = (\bigcup, A, V, f)$ where $\bigcup$ is a non empty finite set of objects, A is a non empty finite set of attributes, $V = \cup V_a$ where V_a is a domain of the attribute 'a', $f : \bigcup \times A \to P(V)$ is a function such that for every $x \in \bigcup$ and $a \in A, f(x,a) \subseteq V_a$. Also we assume that $f(x,a) \geq 1$. The attribute set A is classified into two types of sets a set C is called condition attributes and d is called decision attribute, where $C \cap \{d\} = \phi$.

Definition 2.2.[5] Given a set-valued ordered information system $S = (\bigcup, A, V, f)$ and a subset X of $\bigcup$, the upper approximation of a set X is defined as $\left\{x \in \bigcup : [x]_A^{\geqq} \cap X \neq \phi\right\}$ and is defined by $U_A^{\geqq}(X)$ and the lower approximation is denoted as $\left\{x \in U : [x]_A^{\geqq} \subseteq X\right\}$ and is given by $L_A^{\geqq}(X)$. The boundary region of the set X, defined by $B_A^{\geqq}(X) = U_A^{\geqq}(X) - L_A^{\geqq}(X)$.

Definition 2.3.[5] A set-valued ordered information system S, a subset B of A is said to be a criterion reduction of S if $R_A^{\geqq} = R_B^{\geqq}$ and $R_M^{\geqq} \neq R_A^{\geqq}$ for any $M \subset B$. That is, a criterion reduction is a minimal attribute set B such that $R_A^{\geqq} = R_B^{\geqq}$.

Definition 2.4.[5] CORE(A) is given by $\left\{a \in A / R_A^{\geqq} \neq R_{A-\{a\}}^{\geqq}\right\}$.

Definition 2.5.[4] Let U be an universe of a non-empty finite set of objects and R be an equivalence relation defined on U called discernibility relation. Then U as disjoint equivalence classes. Elements involving to the same equivalence class are named to be indiscernible with each other. The order pair(U, R) is called as the approximation space.

Let X $\subseteq$ U. Then,

(1) The lower approximation of X with R of all objects for further classified as

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X and is denoted by L_R(X). $L_R(X) = \cup\{R(X) : R(X) \subseteq X, x \in U\}$ where R(x) defines the equivalence class denoted by $x \in$ U.

(2) The upper approximation of X with R of all objects are classified as X is denoted by $U_R(X) = \cup\{R(X) : R(X) \cap X \neq \phi, x \in U\}$.

(3) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not - X with respect to R and is denoted by B_R(X).

B_R(X) = U_R(X) - L_R(X)

Definition 2.6.[4] Let U be the universe of a set, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where X$\subseteq$ U. Then $\tau_R(X)$ satisfies the following :

(1) U and $\phi \in \tau_R(X)$.

(2) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on universe called the nano topology to X. (U, τ_R(X)) is called the nano topological space. Elements of the nano topology are known as nano open sets in U.

Definition 2.7. [4]In terms of basis of a nano topological space, a criterion reduction of a set valued ordered information system is a minimal attribute subsets B of A such that $\beta_B \neq \beta_A$ and CORE(A) = $\left\{a \in A : \beta_A \neq \beta_{A-\{a\}}\right\}$ (or) CORE(A) = $\cap$ RED(A) where RED(A) denotes a criterion reduction.

3. INTUITIONISTIC FUZZY NANO TOPOLOGICAL SPACES

Definition 3.1 An intuitionistic fuzzy nano topology (IFNT in short) on the set X is a family $\tau_R(X)$ of IFNSs in X satisfying the following

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(i) $0_\sim \in \tau_R(X)$, $1_\sim \in \tau_R(X)$

(ii) $G_1 \cap G_2 \in \tau_R(X)$ for any $G_1, G_2 \in \tau_R(X)$

(iii) $\cup G_i \in \tau_R(X)$ for any family $\{G_i/i \in J\} \subseteq \tau_R(X)$

The pair $(X, \tau_R(X))$ is called intuitionistic fuzzy nano topological space (IFNTS for short) and IFNTS in $\tau_R(X)$ is said to be an intuitionistic fuzzy nano open set (IFNOS in short) in X. The complement of A^c of an IFNOS A in IFNTS $(X, \tau_R(X))$ is called an intuitionistic fuzzy nano closed set (IFNCS in short) in X.

Definition 3.2 Let $(X, \tau_R(X))$ be an IFNTS and $A = \langle x, \mu_A(x), \nu_A(x)\rangle$ be an IFNS in X. Then the intuitionistic fuzzy nano interior and intuitionistic fuzzy nano closure are denoted by

$int(A) = \cup\{G/G$ is an IFNOS in X and G $\subseteq A\}$

$cl(A)= \cap\{G/G$ is an IFNCS in X and A $\subseteq$K $\}$

Data are presented as a table, columns of which are labeled by attributes, rows by objects of interest and entries of the table are attribute values. For example, in a table containing information about subjects from a engineering are students, attributes can be Mathematics, Physics and Chemistry. Let us take that the students score in examinations over 100 marks total on the above mentioned subjects to determine their future career. We relate intuitionistic fuzzy as source since it compare the membership degree(i.e. the marks of the question answered by the student), the non-membership degree (i.e. the marks of the student failed). Columns of the table below are labeled by attributes(subjects) and rows by objects(students), entries of the table are attribute values. Thus each row of the table represented as information about specific student. The example for information table is shown below.

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Students	Mathematics (a_1)	Physics (a_2)	Chemistry (a_3)	Engineering(d)
S_1	(0.9, 0.0)	(0.8,0.1)	(0.7,0.2)	Yes
S_2	(0.8, 0.1)	(0.6, 0.3)	(0.9, 0.0)	No
S_3	(0.9, 0.1)	(0.7, 0.1)	(0.8, 0.1)	Yes
S_4	(0.8, 0.1)	(0.5,0.4)	(0.7,0.1)	No
S_5	(0.9, 0.0)	(0.6,0.3)	(0.8,0.0)	Yes

A set valued ordered information system is shown in the above table, where $U = \{S_1, S_2, S_3, S_4, S_5\}$ and $A = \{a_1, a_2, a_3, d\}$, a_1= Mathematics, a_2 = Physics, a_3 = Chemistry of the subject groups and d is the decision as to whether a student got engineering seat or not. The attribute set A is divided into two classes-class C, of condition attributes, namely, a_1,a_2,a_3 and class D of decision attribute d. The set of attribute values is given by $V = \{Maths, Physics, Chemistry\}$ and M, P and C respectively stand for Mathematics, Physics and Chemistry. From the table,

$f(S_2, a_1)$ = (0.8, 0.1) and $f(S_3, a_1)$ = (0.9, 0.1)and hence $f(S_4, a_1)$ = (0.8, 0.1), $f(S_5, a_1)$ = (0.9, 0.0), $f(S_2, a_1) \subseteq f(S_3, a_1)$ and hence $f(S_2, a_1) \subseteq f(S_5, a_1)$. The family of dominance class is given by $U_C^{\geq} = \{\{S_2, S_3\}\{S_2, S_5\}, \{S_4, S_5\}, \{S_1\}, \{S_3\}\}$ R be the equivalence relation on U and $\tau_R(X) = \{0_\sim, 1_\sim, L_R(X), U_R(X), B_R(X)\}$.

Case 1. Let $X = \{S_1, S_2, S_5\}$, the set of students got engineering. Then the lower and upper approximations of X are given by $L_C^{\geq}(X) = \{S_1\}$ and $U_C^{\geq}(X) = \{S_1, S_2, S_3, S_4, S_5\}$ and hence the boundary region of X is $B_C^{\geq}(X) = \{S_1, S_2, S_3, S_4, S_5\}$. Then the corresponding intuitionistic fuzzy nano topology with respect to X is given by $\tau_C^{\geq}(X) = \{0_\sim, 1_\sim, (0.9, 0.0), (0.0, 0.9)\}$. The basis of $\tau_C^{\geq}(X)$ is given by $\beta_C^{\geq}(X) = \{1_\sim, (0.9, 0.0), (0.0, 0.9)\}$.

Step 1. Let $B_1 = C - \{maths\}$.

Then $\bigcup_C^{\geq}(B_1) = \{\{S_1, S_5\}, \{S_4, S_5\}, \{S_1\}, \{S_3\}\}$ and the corresponding intuitionistic fuzzy nano topology is given by $\tau_C^{\geq}(X) = \{0_\sim, 1_\sim, (0.8, 0.1), (0.8, 0.0), (0.0, 0.8)\}$. Therefore, $\beta_{B_1}^{\geq}(X) \neq \beta_C^{\geq}(X)$.

Step 2. If $B_2 = C - \{physics\}$.

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hen $\bigcup_C^{\geq}(B_2) = \{\{S_2, S_3, S_5\}, \{S_4, S_5\}, \{S_1\}\}$ and the corresponding intuitionistic fuzzy nano topology is given by $\tau_C^{\geq}(X) = \{0_\sim, 1_\sim, (0.9, 0.0), (0.0, 0.9)\}$. The basis of $\tau_C^{\geq}(X)$ is given by $\beta_C^{\geq}(X) = \{1_\sim, (0.9, 0.0), (0.0, 0.9)\}$. Therefore, $\beta_{B_3}^{\geq}(X) = \beta_C^{\geq}(X)$.

Step 3. If $B_3 = C - \{chemistry\}$.

hen $\bigcup_C^{\geq}(B_3) = \{\{S_2, S_3, S_5\}, \{S_2\}, \{S_1\}, \{S_4\}\}$ and the corresponding intuitionistic fuzzy nano topology is given by $\tau_C^{\geq}(X) = \{0_\sim, 1_\sim, (0.9, 0.0), (0.0, 0.9)\}$. The basis of $\tau_C^{\geq}(X)$ is given by $\beta_C^{\geq}(X) = \{1_\sim, (0.9, 0.0), (0.0, 0.9)\}$. Therefore, $\beta_{B_3}^{\geq}(X) = \beta_C^{\geq}(X)$.

Case 2. Let $X = \{S_2, S_4\}$, the set of students ts not getting engineering. Then the lower and upper approximations of X are given by $L_C^{\geq}(X) = \{0_\sim\}$ and $U_C^{\geq}(X) = \{0_\sim, 1_\sim, (0.9, 0.0), (0.0, 0.9)\}$. Then intuitionistic fuzzy nano topology with respect to X is defined by $\tau_C^{\geq}(X) = \{0_\sim, 1_\sim, (0.9, 0.0), \}$. Therefore, $\beta_C^{\geq}(X) = \{1_\sim, (0.9, 0.0)\}$.

Step 1. If $B_1 = C - \{maths\}$.Then the nano topology is defined by $\tau_{B_1}^{\geq}(X) = \{0_\sim, 1_\sim, (0.9, 0.0), (0.0, 0.9)\}$. The basis of $\tau_C^{\geq}(X)$ is defined by $\beta_C^{\geq}(X) = \{1_\sim, (0.8, 0.0), (0.0, 0.8)\}$. Hence, $\beta_{B_1}^{\geq}(X) \neq \beta_C^{\geq}(X)$.

Step 2. If $B_2 = C - \{physics\}$.

Then the corresponding nano topology is given by $\tau_{B_2}^{\geq}(X) = \{0_\sim, 1_\sim, (0.9, 0.0), (0.0, 0.9)\}$. Therefore, $\beta_{B_2}^{\geq}(X) = \beta_C^{\geq}(X)$.

Step 3. If $B_3 = C - \{physics\}$.

Then nano topology is defined by $\tau_{B_2}^{\geq}(X) = \{0_\sim, 1_\sim, (0.9, 0.0), (0.0, 0.9)\}$. Therefore, $\beta_{B_3}^{\geq}(X) = \beta_C^{\geq}(X)$.

Thus B_2 and B_3 are the reducts. As in case 1, we show that these two reductions cannot further be reduced and hence they are the criterion reductions. Thus CORE (A)= $B_2 \cap B_3 = \{maths\}$ Thus $B_1 = \{a_1, a_3\}$ is the criterion reduction of

Observation: It is concluded as per the analysis based on intuitionistic fuzzy nano topology that mathematics stands as a unique subject in comparision with other subjects such as physics and chemistry in engineering education.

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4. Applications of Intuitionistic fuzzy sets

Parkhedkar and Dubey (2013) proposed segmentation and detection of tumor of MRI brain images using a method provided by Attanassov IFS theory. Segmentation is important in detecting different type of tumor, stroke, paralysis etc which are developed inside brain. Segmentation becomes very difficult in medical images which are not properly illuminated. An image segmentation approach IFS theory and a membership function called restricted equivalence function from automorphisms, for finding the membership values of the pixels of the image is proposed. An intuitionistic fuzzy image is constructed using Sugeno type intuitionistic fuzzy generator. A distance measure Intuitionistic Fuzzy Divergence is used. From this Intuitionistic Fuzzy Divergence edge detection is carried out. The results showed a much better performance on poor illuminated medical images, where the brain tumor is detected properly.

Chaira (2010) proposed IFS for extracting region of images. Experiment results were compared with fuzzy sets and found better to the results using fuzzy sets. This was due to consideration of uncertainity while defining membership function in IFS.

Chaira(2009) addressed an issue of intuitionistic fuzzy c means color clustering using intuitionistic fuzzy set theory. The non membership degree helps to converge the cluster center to a desirable location than the cluster centers obtained by fuzzy C means algorithm. Experiments are performed on 40 images and five sets on blood cell images. The results showed the clustered images are almost clearly and properly clustered than conventional FCM and also the number of iterations required are much less than that of the conventional FCM. The reason for obtaining a better result is that the membership function is always not accurately defined due to the personal error and so IFS set gives better result where

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uncertainty in the form of hesitation degree is used.

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